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Sheet 00: Differentiation and Integration

Solution Example Problem 1: Differentiation of polynomials [1]

$$(a) f'(x) = 9x^2 + 2, \quad f''(x) = 18x.$$

$$(b) f'(x) = 4x^3 - 4x, \quad f''(x) = 12x^2 - 4.$$

Solution Example Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [1]

Using $\sin' x = \cos x$, $\cos' x = -\sin x$, and the product and chain rules, we obtain:

$$(a) f'(x) = \sin x + x \cos x \quad (b) f'(x) = -\sin[\pi(x^2 + x)]\pi(2x + 1)$$

$$(c) f'(x) = \frac{2x}{(7 - x^2)^2} \quad (d) f'(x) = \frac{1}{x+1} - \frac{x-1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Solution Example Problem 3: Differentiation of powers, exponentials, logarithms [2]

$$(a) f'(x) = \frac{1}{2\sqrt{2x^3}} \quad (b) f'(x) = \frac{1}{2} \frac{1}{x^{1/2}(x+1)^{1/2}} - \frac{1}{2} \frac{x^{1/2}}{(x+1)^{3/2}} = \frac{1}{2} \frac{1}{x^{1/2}(x+1)^{3/2}}$$

$$(c) f'(x) = e^x(2x - 1) \quad (d) f'(x) = \frac{d}{dx} e^{\ln 3^x} = \frac{d}{dx} e^{x \ln 3} = e^{x \ln 3} \ln 3 = 3^x \ln 3$$

$$(e) f'(x) = \ln x + \frac{x}{x} = \ln x + 1 \quad (f) f'(x) = \ln(9x^2) + x \frac{1}{9x^2} 18x = \ln(9x^2) + 2$$

Solution Example Problem 4: Elementary integrals [1]

$$(a) I(x) = \int_1^x dy(2y^3 - 2y + 3) = \left[\frac{1}{2}y^4 - y^2 + 3y \right]_1^x = \boxed{\frac{1}{2}x^4 - x^2 + 3x - \frac{5}{2}}.$$

$$(b) I(x) = \int_0^x dy e^{3y} = \left[\frac{1}{3}e^{3y} \right]_0^x = \boxed{\frac{1}{3}(e^{3x} - 1)}.$$

[Total Points for Example Problems: 5]
