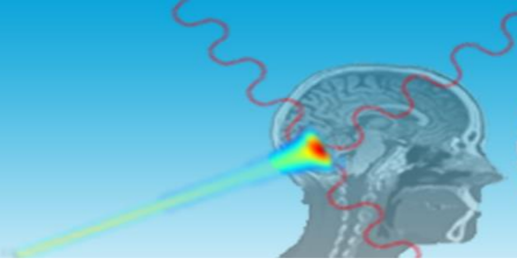

Inverse problems and machine learning in medical physics

Tomographic image reconstruction for ion imaging

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19/11/2024
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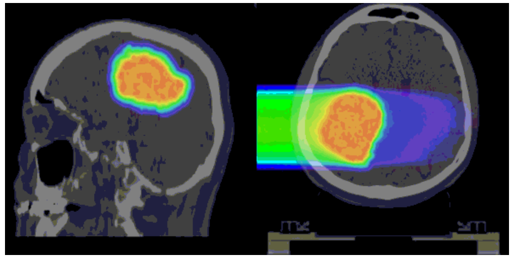
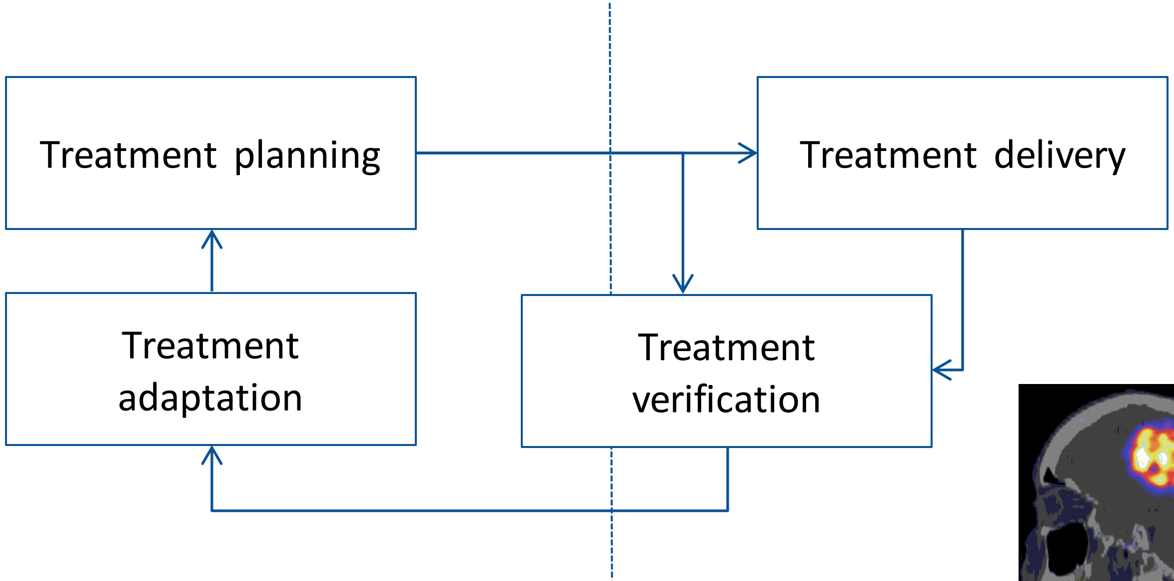
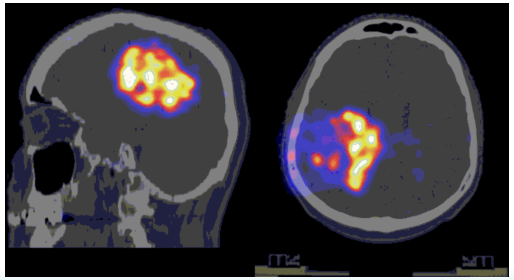
Ion imaging in ion beam therapy

- Ion imaging can in principle enable direct assessment of the tissue stopping power and its variations due to inter-fractional anatomical changes

- Ion imaging can thus play a role not only as imaging technique for treatment planning but also for treatment verification and adaptation in ion beam therapy



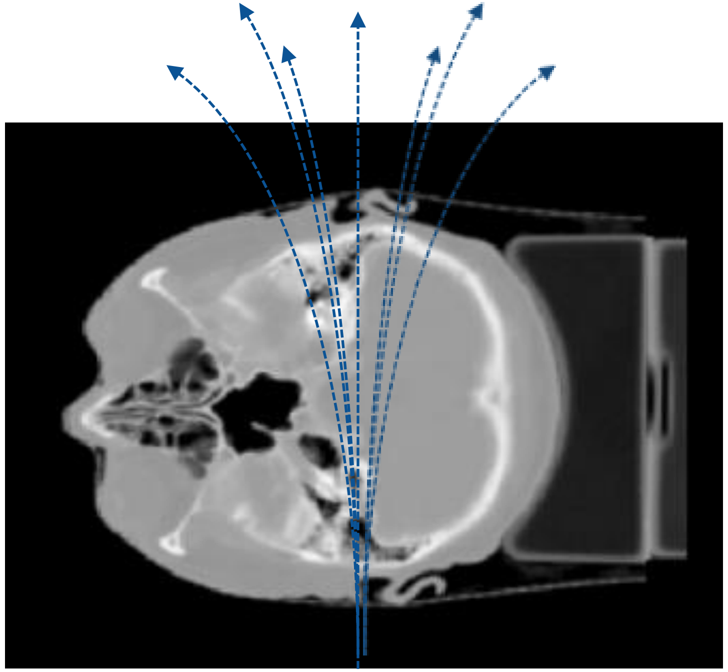




Ion imaging model

- Ion imaging offers the promise of eliminating these inaccuracies by measuring the water equivalent thickness (WET) and reconstructing the relative stopping power (RSP) of the object of interest

- In ion imaging, the forward-projection model describes the measured WET of the traversed object of interest as an integral of the RSP along a certain concept of ion trajectory that depends on the detector configuration



single ion or pencil beam

$$WET_i = \sum_j a_{ij} RSP_j$$

forward-projection model

the ion radiography

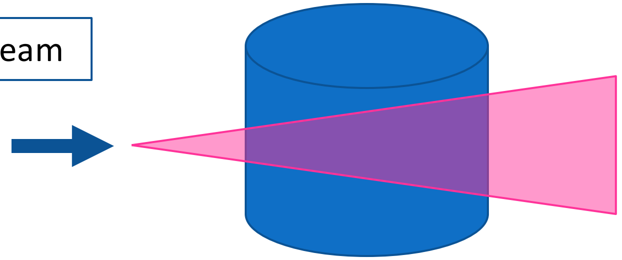
the ion image

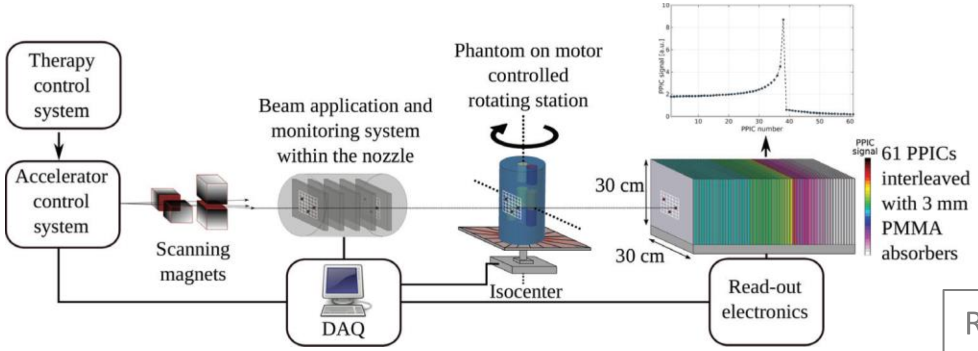
- The a_{ij} is the coefficient of the system matrix that describes the intersection length/area/volume of the trajectory i (single ion or pencil beam) with the voxel j

Ion imaging detectors

- Detectors are mainly distinguished in list-mode and integration-mode configurations
  - For integration-mode detectors, the concept of ion trajectory can be statistically described by the multiple Coulomb scattering model

pencil beam

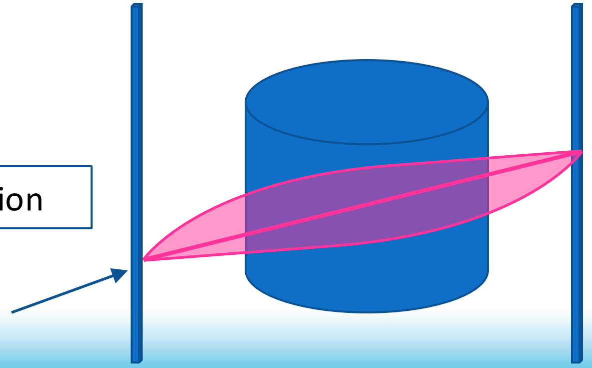


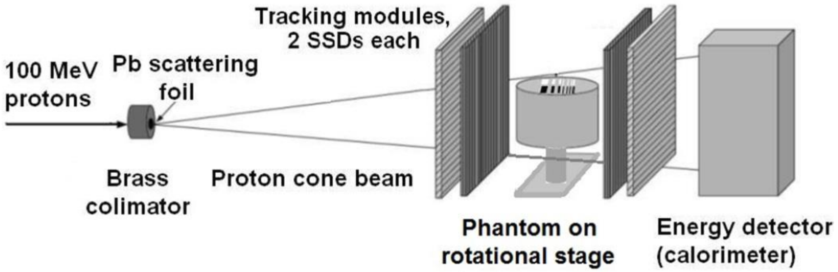


Rinaldi et al. 2013 *Phys. Med. Biol.*
Meyer, Gianoli, … et al. 2017 *Phys. Med. Biol.*

  - For list-mode detectors, the concept of ion trajectory can be statistically described by combining the multiple Coulomb scattering model and the Bayesian inference, as the most likely path algorithm

single ion





Bashkirov et al. 2016 *Nucl. Instrum. Methods Phys. Res. A*

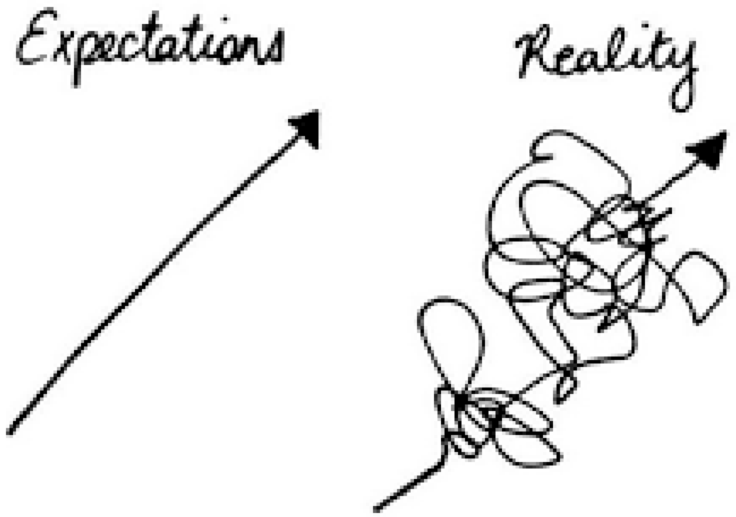
Detector configuration and ion trajectories

- The concept of ion trajectory for different detector configuration plays a crucial role in the forward-projection model, which is a foundation in ion imaging

$$\overrightarrow{WET} \neq A * \overrightarrow{RSP_t} \quad (t \text{ for "ground truth"})$$

- However, in clinical scenarios the intrinsic inconsistencies of the forward-projection model are in the same order of magnitude of the inaccuracies of the semi-empirical calibration of the X-ray CT

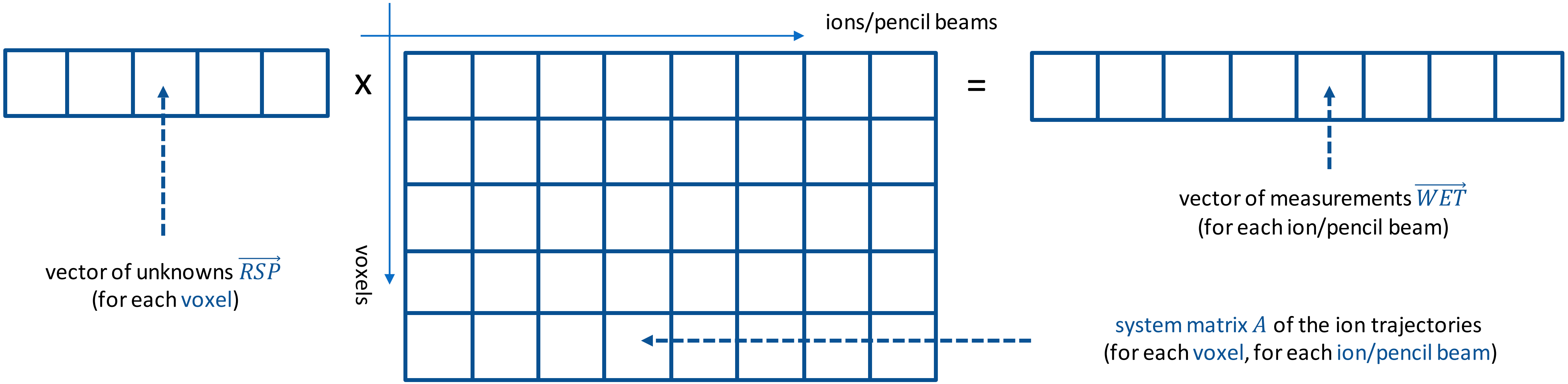
  - Relying on Monte Carlo simulations, the normalized root mean square error between the ion radiography and the forward-projection of the ground truth ion CT image is 1-2.5% for list-mode detector configuration and up to 2.5-5% for integration-mode detector configuration



Gianoli et al. *2020 Phys. Med. Biol.*

Tomographic image reconstruction

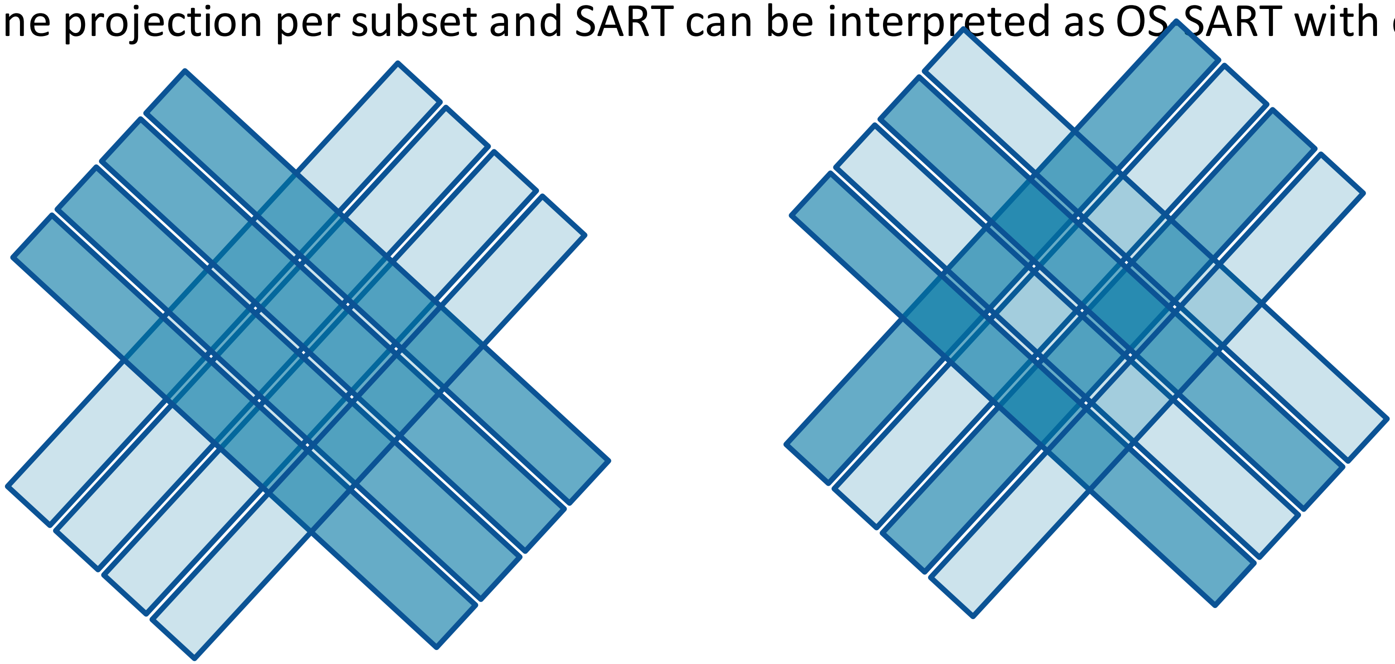
- Tomographic image reconstruction is applied to several ion radiographies, with projection angles covering 180°
  - The ordered subsets simultaneous algebraic reconstruction technique (OS-SART) coupled with total variation superiorization currently represents the state-of-the-art in ion imaging[1,2,3]
  - Information redundancy mitigates the intrinsic inaccuracies of the forward-projection model[4]



[1]Penfold et al. 2010 Med. Phys. [2]Meyer et al. 2019 *Phys. Med. Biol.* [3]Meyer et al. 2021 *Phys. Med. Biol.*
[4]Gianoli et al. 2019 *Phys. Med.*

Ordered Subsets

- The ordered subsets (OS) approach is introduced to accelerate numerical image reconstruction and reduce the memory requirement for reconstruction
- In OS approach, instead of accessing all projections simultaneously for updating the image, the image is updated relaying on a subset of projections
- ART can be interpreted as OS-SART with only one projection per subset and SART can be interpreted as OS-SART with only one subset
  - An update performed using a single subset is called a sub-iteration
  - An iteration is completed when all subsets have been processed once
  - The convergence acceleration is expressed in terms of number of iterations (not sub-iterations)



Ordered Subsets

- The idea of OS was originally proposed for emission tomography and then transferred to transmission tomography (the SART and the ML-EM produce the maximum likelihood estimate in the Gaussian and Poisson data, respectively)

- The subset of projections $S(s)$ is employed for updating of the image, and this update, together with a different subset of projections $S(s+1)$, is then used for calculating the next update

$$\overrightarrow{f^{(s+1)}} = \overrightarrow{f^{(s)}} - \frac{\sum_{i \in S(s)} \left(a_{ij} \cdot \left(\frac{\overrightarrow{f^{(s)}} \cdot \overrightarrow{a_{ij}}^T - \vec{g}}{\sum_j a_{ij}} \right) \right)}{\sum_i a_{ij}}$$

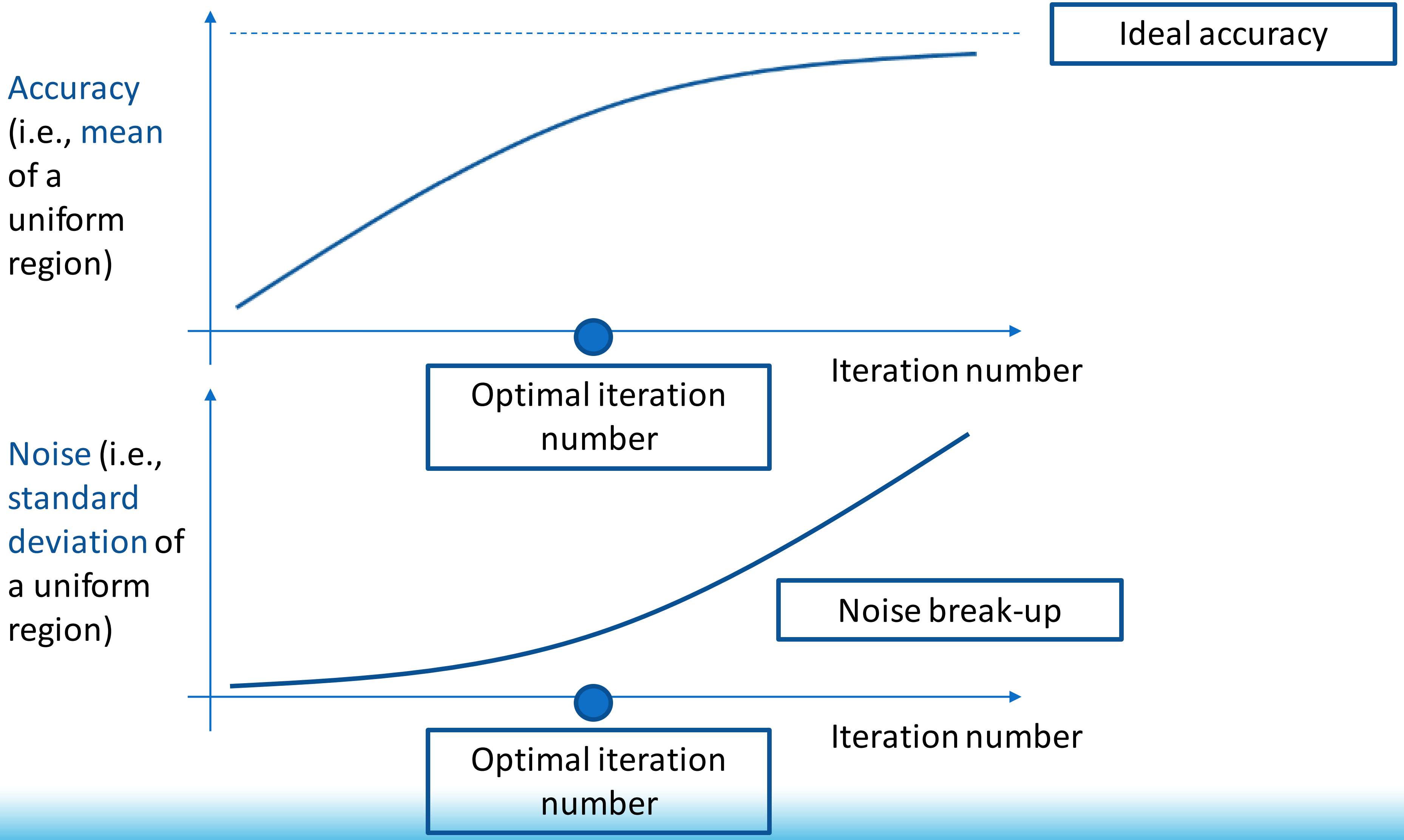
- The best ordering of the subsets is defined according to the maximum angular distance ("as orthogonal as possible") from the previously used projections

    - This ordering further accelerates convergence as compared to sequential or random orderings

    - The increased convergence speed (in function of the number of iterations) and the reduced memory requirement (due to a reduced dimension of the system matrix) comes at the cost of an increased noise of the reconstructed image

Hudson, H. M., & Larkin, R. S. (1994). Accelerated image reconstruction using ordered subsets of projection data. IEEE transactions on medical imaging, 13(4), 601-609.

Outlook


Ideal accuracy
Accuracy (i.e., mean of a uniform region)
Optimal iteration number
Iteration number
Noise (i.e., standard deviation of a uniform region)
Noise break-up
Optimal iteration number
Iteration number

Tomographic image reconstruction

- Because of the intrinsic inconsistencies of the forward-projection model, the optimal solution minimizing the objective function of the tomographic image reconstruction algorithm is not necessarily the solution that best reconstructs the ground truth image

- The fundamental approach for the mitigation of the intrinsic inconsistencies of the forward-projection model is to stop the tomographic image reconstruction algorithm before the solution diverges (albeit with a lower objective function), thus being referred to as semi-convergence

- Another approach is to use superiorization techniques to shift the solution at each iteration to one that is superior to the current solution

    - A superior solution is defined in terms of a certain merit function φ

algorithm $$x^{k+1} = f(x^k)$$

superiorized algorithm $$x^{k+1} = f\left(x^k + \beta_k v^k\right) \quad \text{so that} \quad \varphi(x^k + \beta_k v^k) \leq \varphi(x^k)$$

Tomographic image reconstruction

- In transmission imaging (i.e., ion imaging), the merit function φ is typically the total variation

- For a two-dimensional (2D) image representation in i and j of the image vector x^k is defined as:

$x_{i-1,j-1}$	$x_{i,j-1}$	$x_{i+1,j-1}$
$x_{i-1,j}$	$x_{i,j}$	$x_{i+1,j}$
$x_{i-1,j+1}$	$x_{i,j+1}$	$x_{i+1,j+1}$

$$\varphi(x^k) = \sum_i \sum_j \sqrt{(x^k_{i+1,j} - x^k_{i,j})^2 + (x^k_{i,j+1} - x^k_{i,j})^2}$$

- $\beta_k v^k$ is the perturbation term, with β_k real non-negative numbers (typically $0 < \beta_k < 1$) and v^k the perturbation vector, typically defined as non-ascending direction of the merit function φ at x^k

$$v^k = -\frac{\nabla\varphi(x^k)}{\|\nabla\varphi(x^k)\|_2} = -\frac{s^k}{\|s^k\|}$$

  - The perturbation vector is calculated as the negative of the normalized sub-gradient s^k (generalized concept of derivative for convex functions which are not necessarily differentiable) of the total variation $\varphi(x^k)$ for the image vector x^k

Penfold, S. N., Schulte, R. W., Censor, Y., & Rosenfeld, A. B. (2010). Total variation superiorization schemes in proton computed tomography image reconstruction. Medical physics, 37(11), 5887-5895.

Tomographic image reconstruction

- Another approach is to use regularization technique to add a penalty function to the objective function
  - The penalty function enforces a desired feature on the reconstructed image

$$x_{\min} = \operatorname{argmin}_x F(x) + \lambda\phi(x)$$

  - $\phi(x)$ is the penalty function (i.e., total variation or smoothness-related functions) and λ is a parameter controlling its weighting
  - The regularization changes the problem!

Defrise, M., Vanhove, C., & Liu, X. (2011). An algorithm for total variation regularization in high-dimensional linear problems. Inverse Problems, 27(6), 065002.

  - The optimal value of λ depends on the level of noise in the data
    - λ too small under-regularizes (i.e., not substantially improve image quality)
    - λ too large over-regularizes, resulting typically in an oversmoothed image

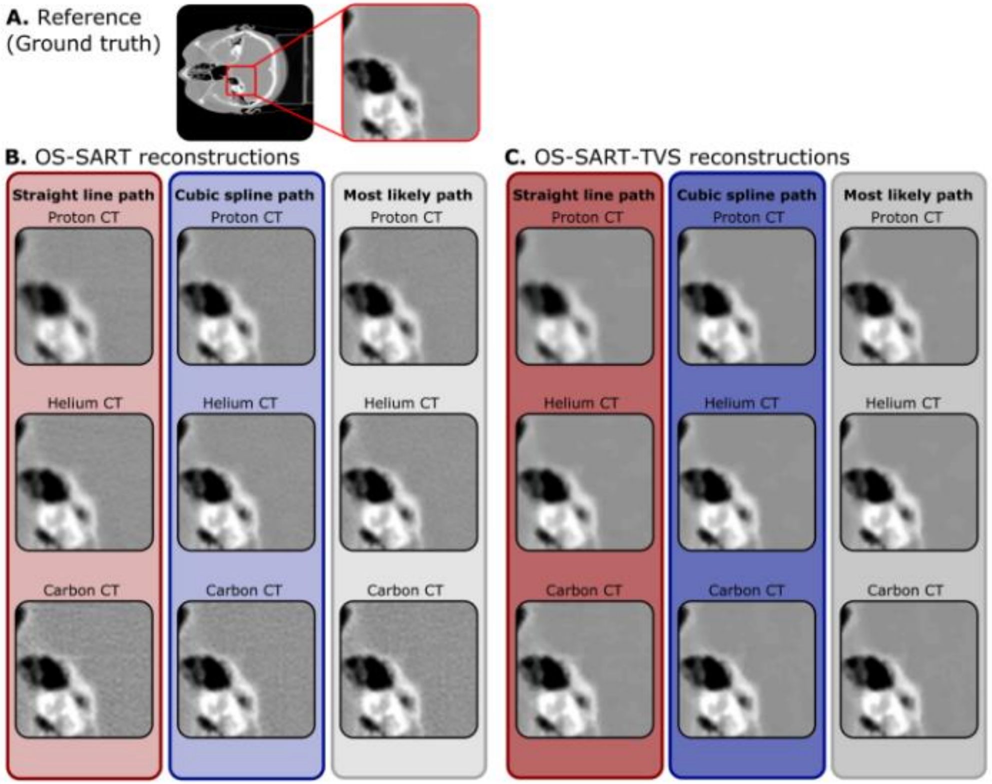
Tomographic image reconstruction for list-mode detector configuration

- The superiorization of the algorithm for tomographic image reconstruction introduces a “perturbation” of the solution in tomographic domain in order to reduce, and not necessarily minimize, a merit function φ (i.e., the total variation)

- The superiorization changes the algorithm by adding a shifting step but it does not change the problem!

set $k = 0$, $l = 0$, $\beta_k = 1$;
while *repeat for each subset over the requested number of cycles* **do**
  if $\|s^k\| \geq 0$ **then**
    $v^k = -\frac{s^k}{\|s^k\|}$;
  else
    $v^k = s^k$;
  end
  while *loop* **do**
    set $y^k = x^k + \beta_k v^k$;
    if $\Phi(y^k) \leq \Phi(x^k)$ **then**
      $x^k = y^k$;
      set $loop = false$;
    end
    $\beta_k = \beta_k / 2$;
  end
  $x^{k+1} = \mathbf{P}_Q x^k$;
  $k = k + 1$;
end

P_Q is the operator that updates the OS-SART algorithm

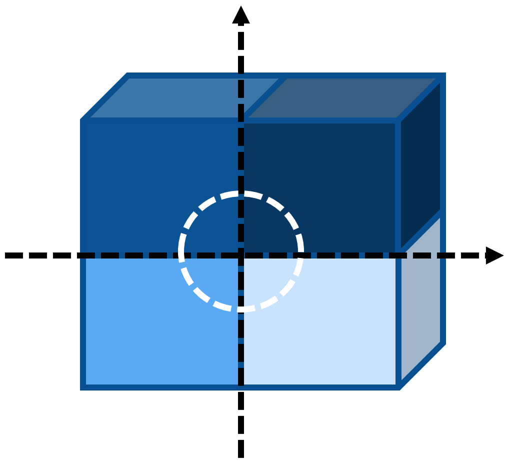


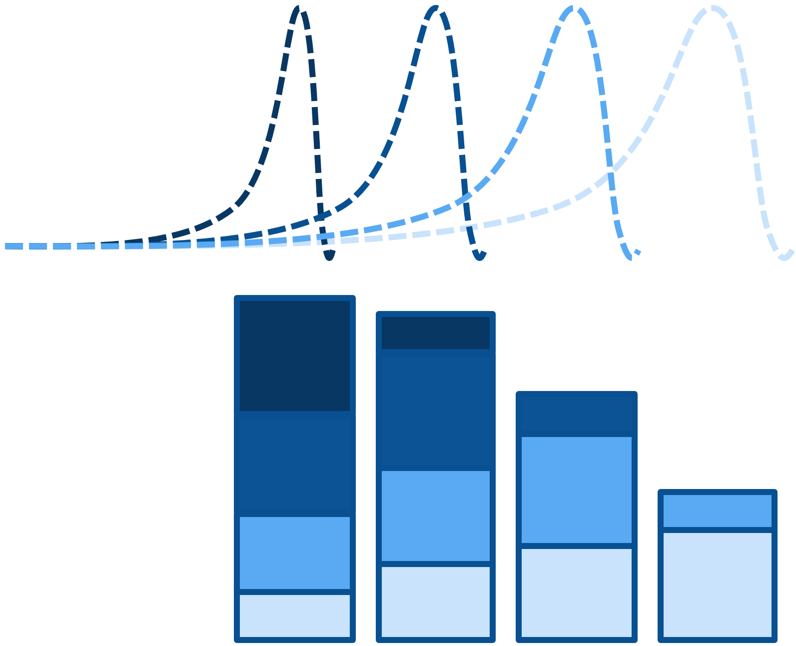
Meyer, S., Pinto, M., Parodi, K., & Gianoli, C. (2021). The impact of path estimates in iterative ion CT reconstructions for clinical-like cases. Physics in Medicine & Biology, 66(9), 095007.

Integration-mode detector configuration for pencil beams

- In integration-mode detector configuration, the Bragg peak signal for each pencil is discretized according to the multiple layers (i.e., channels) or according to the multiple initial energies in a single layer

- Due to lateral inhomogeneity traversed by the pencil beam, the Bragg peak signal results in a linear combination of elementary Bragg peak signals





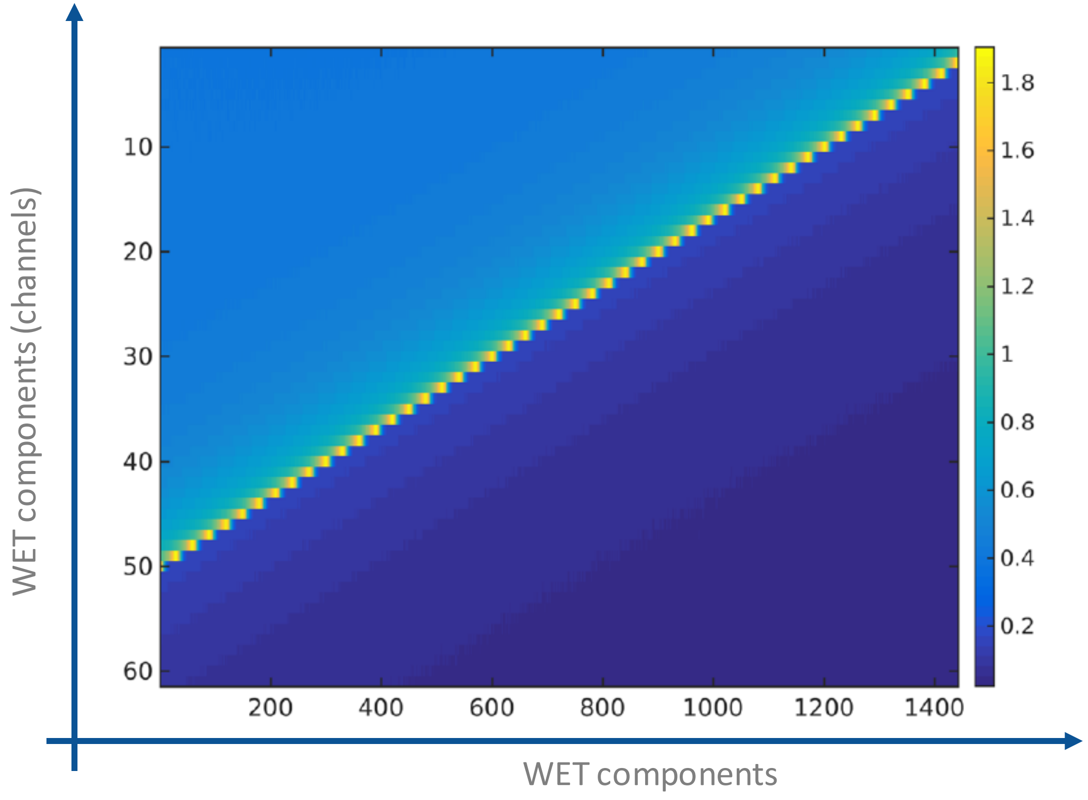
- The Bragg peak of the component with the larger WET (i.e., the shorter range) takes advantages from the Bragg peaks of the components with smaller WET

Integration-mode detector configuration for pencil beams

- Linear decomposition[1,2] (inverse problem) is applied to retrieve the WET histogram as WET occurrence for each WET component by solving the system of linear equations $\overrightarrow{BP} = LUT * \overrightarrow{WET}$
  - $\overrightarrow{BP}$ is the discretized Bragg peak signal
    - $\overrightarrow{WET}$ is the unknown vector of WET occurrences
    - LUT is the look-up-table of individual Bragg peak signals for each WET component
  - The least square optimization is based on Euclidean distance minimization

$$argmin_{\overrightarrow{WET}} \frac{1}{2} \left\| LUT * \overrightarrow{WET} - \overrightarrow{BP} \right\|_2^2$$

  - An histogram of WET occurrences for each WET component is obtained



[1]Krah et al. 2015 *Phys. Med. Biol.*
[2]Meyer et al. 2017 *Phys. Med. Biol.*

Tomographic image reconstruction for integration-mode detector configuration

- Defining p_{ik} the WET components and w_{ik} the WET occurrences, the tomographic image reconstruction for integration-mode detector configuration deals with the additional channel dimension of the WET histogram (WET_{hist})

- The typical approach, the WET histogram is reduced to a single WET component

  - $WET_i = \max\{p_{ik}\}$ the WET component with maximum WET occurrence (WET_{mode} or WET_{max})

  - $WET_i = mean(p_{ik} w_{ik})$ the weighted averaged WET components (WET_{mean})

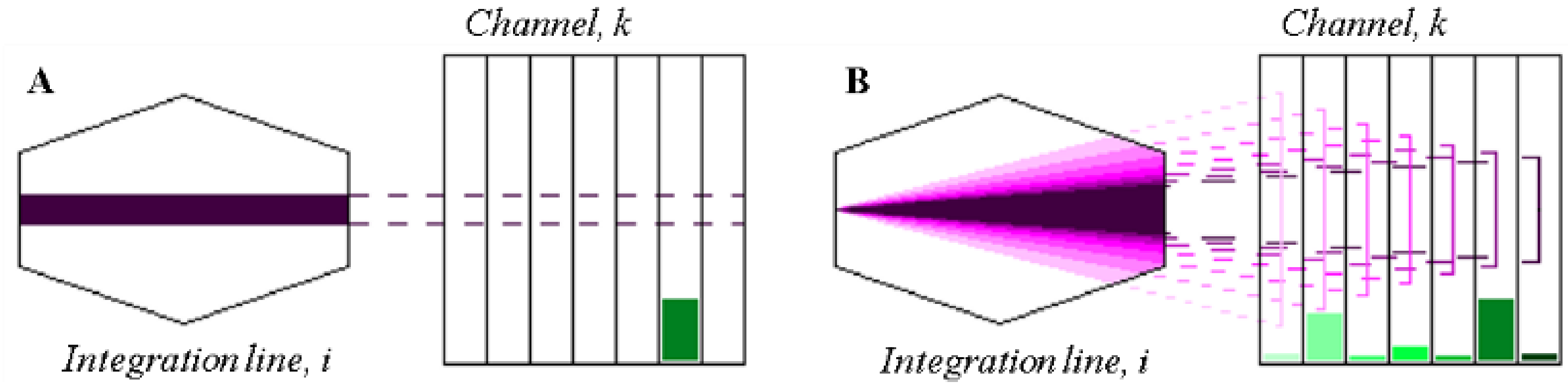
$$WET_i = \sum_j a_{ij} RSP_j$$

- Therefore, the integration line is assumed as straight or coinciding to the mean ion trajectory of the pencil beam

  - Analytical or numerical algorithms for tomographic image reconstruction are applied

Tomographic image reconstruction for integration-mode detector configuration

- Alternatively, WET_{hist} is handled within numerical tomographic image reconstruction (ART&SART)
  - WET components and WET occurrents are entirely exploited
  - The integration line is defined according to the scattering model (conical Gaussian) for each WET component
    - The WET components are spatially assigned (the *mean ion trajectory* is valid only for WET_{max} and WET_{mean})



Tomographic image reconstruction for integration-mode detector configuration

- The SART algorithm is considered for numerical tomographic image reconstruction

$$f_j^{n+1} = f_j^n + \frac{\sum_i a_{ij} \cdot \frac{g_i - \sum_j a_{ij} \cdot f_j^n}{\sum_j a_{ij}}}{\sum_i a_{ij}}$$

- The SART algorithm is modified to handle the additional channel dimension

$$m_{ik} = \sum_k w_{ik} a_{(ik)j} \qquad g_i = \sum_k w_{ik} g_{ik}$$

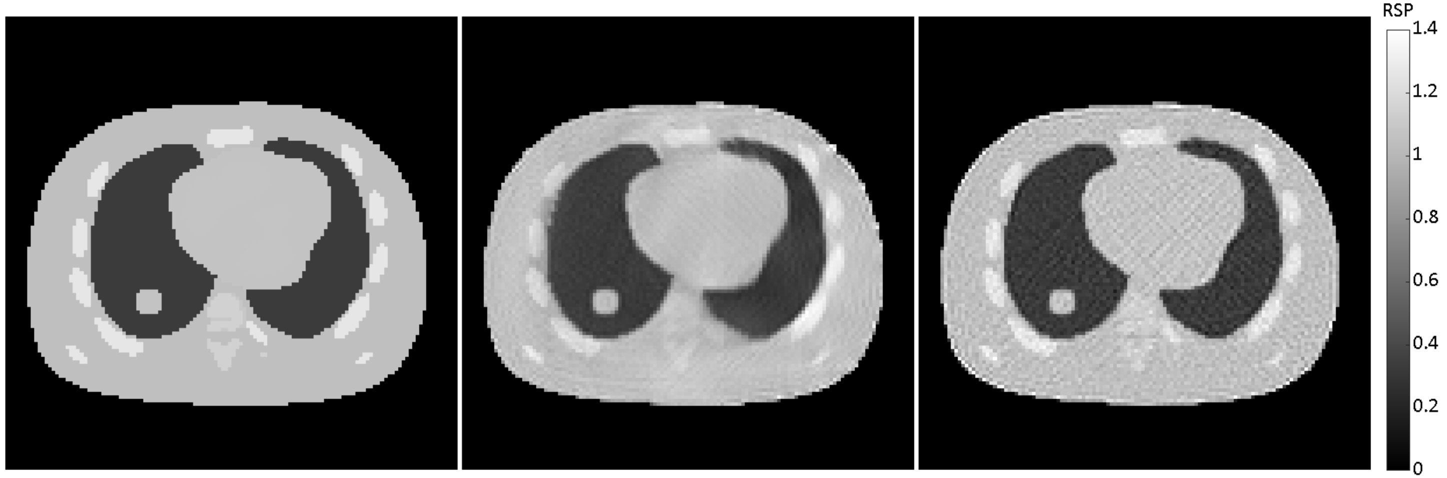
where $a_{(ik)j}$ describe the conical Gaussian for each channel
(indexing j the pixel/voxel, k the channel and i the measurement)

$$f_j^{n+1} = f_j^n + \frac{\sum_i m_{ij} \cdot \frac{g_i - \sum_j m_{ij} \cdot f_j^n}{\sum_j m_{ij}}}{\sum_i m_{ij}}$$

Seller Oria et al. 2018 *IEEE NSS-MIC*

Tomographic image reconstruction for integration-mode detector configuration

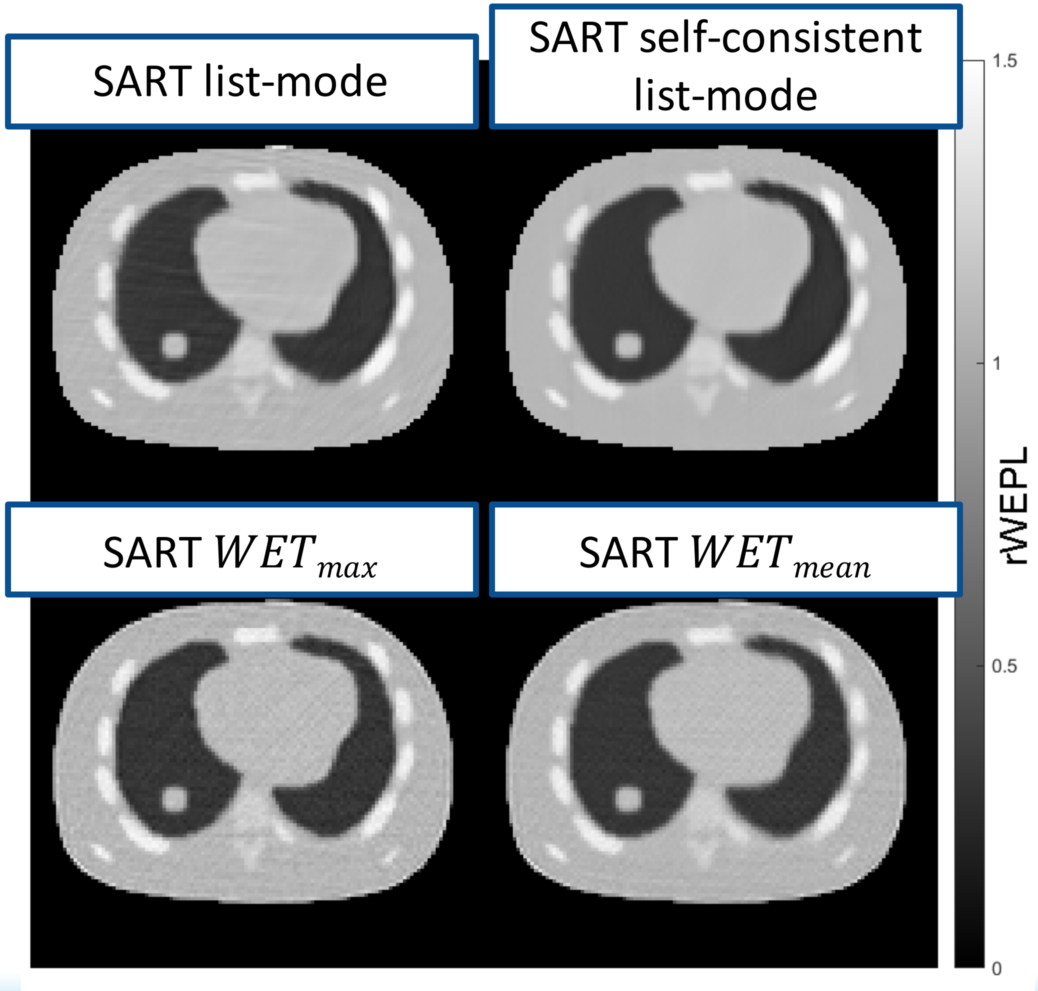
Ground truth	SART WET_{hist}	SART WET_{max}



Tomographic image reconstruction for integration-mode detector configuration

- Comparison between list-mode and integration-mode detector configurations

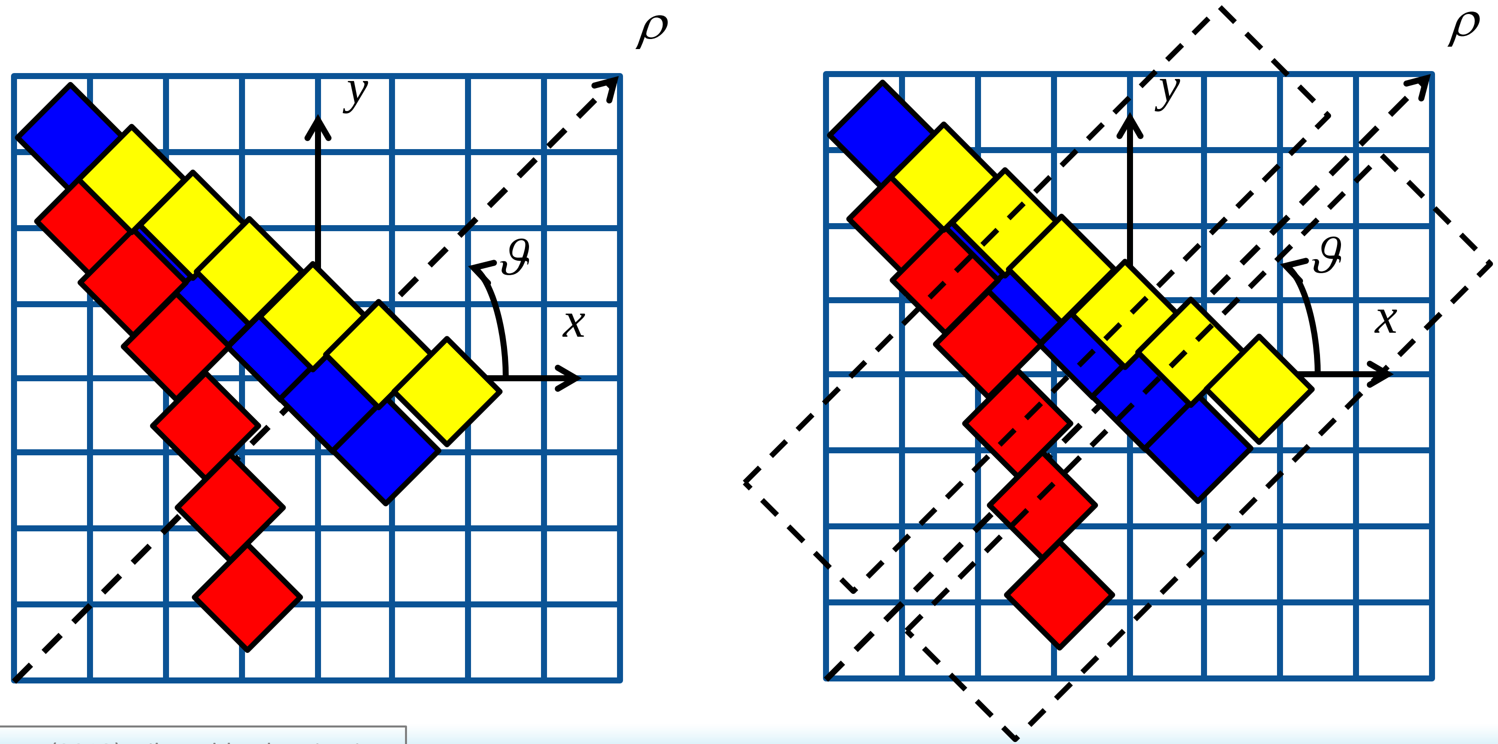
- The self-consistent tomographic image reconstruction pretends to know the exact trajectories of the protons (to overcome the ill-posed nature of the inverse problem in ion imaging)



Gianoli et al. 2019 *Phys. Med.*

Tomographic image reconstruction (list-mode detector configuration)

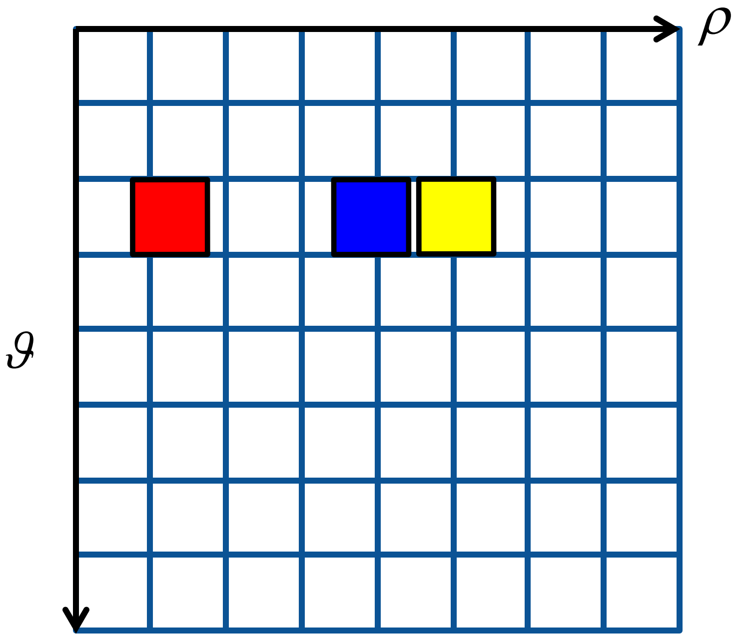
- The MLP is the estimation of the proton trajectory that can be easily adopted in the system matrix of numerical reconstruction
- Alternatively, the estimation of the proton trajectory can be used to implement a “modified” FBP, based on a distance-driven binning of projections for individual source positions

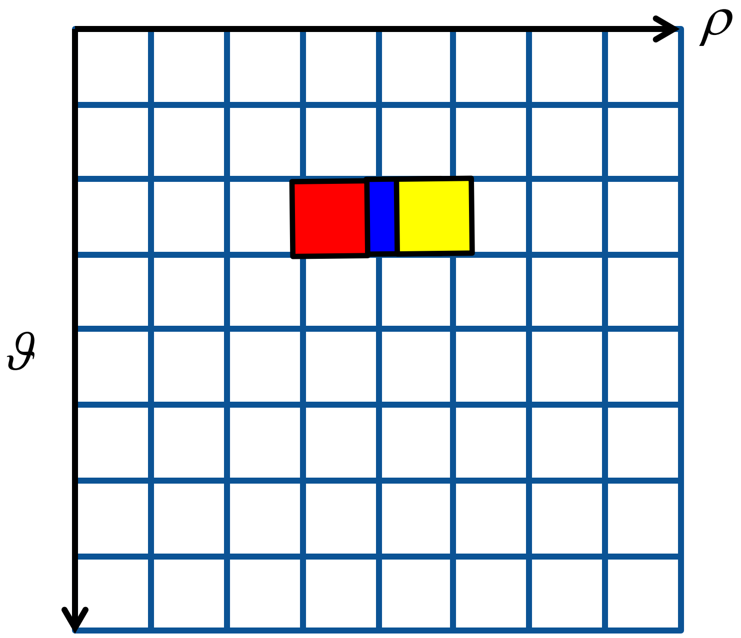


Rit, S., Dedes, G., Freud, N., Sarrut, D., & Létang, J. M. (2013). Filtered backprojection proton CT reconstruction along most likely paths. Medical physics, 40(3), 031103.

Tomographic image reconstruction (list-mode detector configuration)

- The projection is binned (subdivided) according to the source to detector distance
- The binning provides the sinogram an additional dimension, whose size is defined by the number of bins

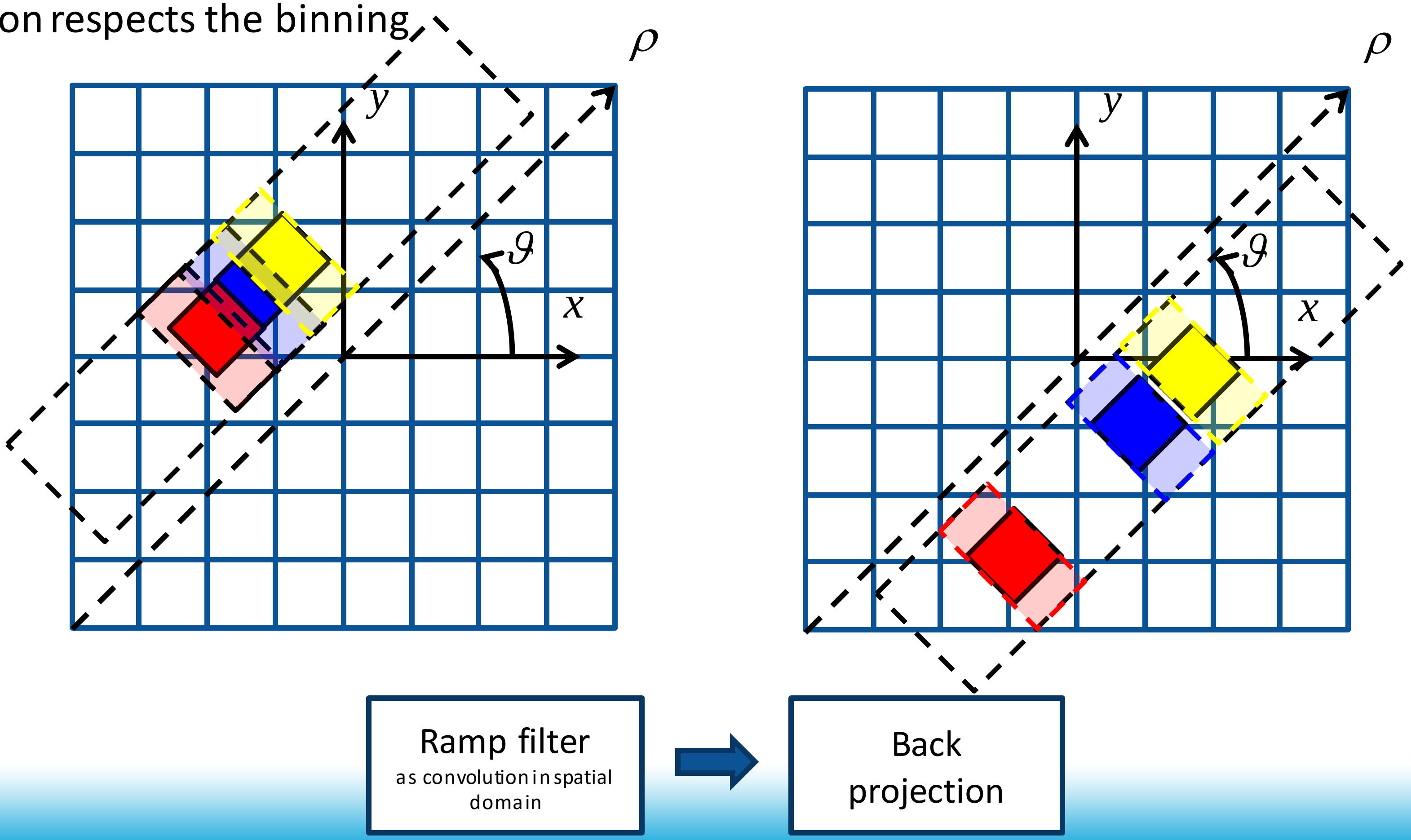




Tomographic image reconstruction (list-mode detector configuration)

- The filtering, enabled by the distance-driven binning, is applied to each binned projection of the sinogram (without filtering the method is simply a back-projection along the MLP)

- The back-projection respects the binning



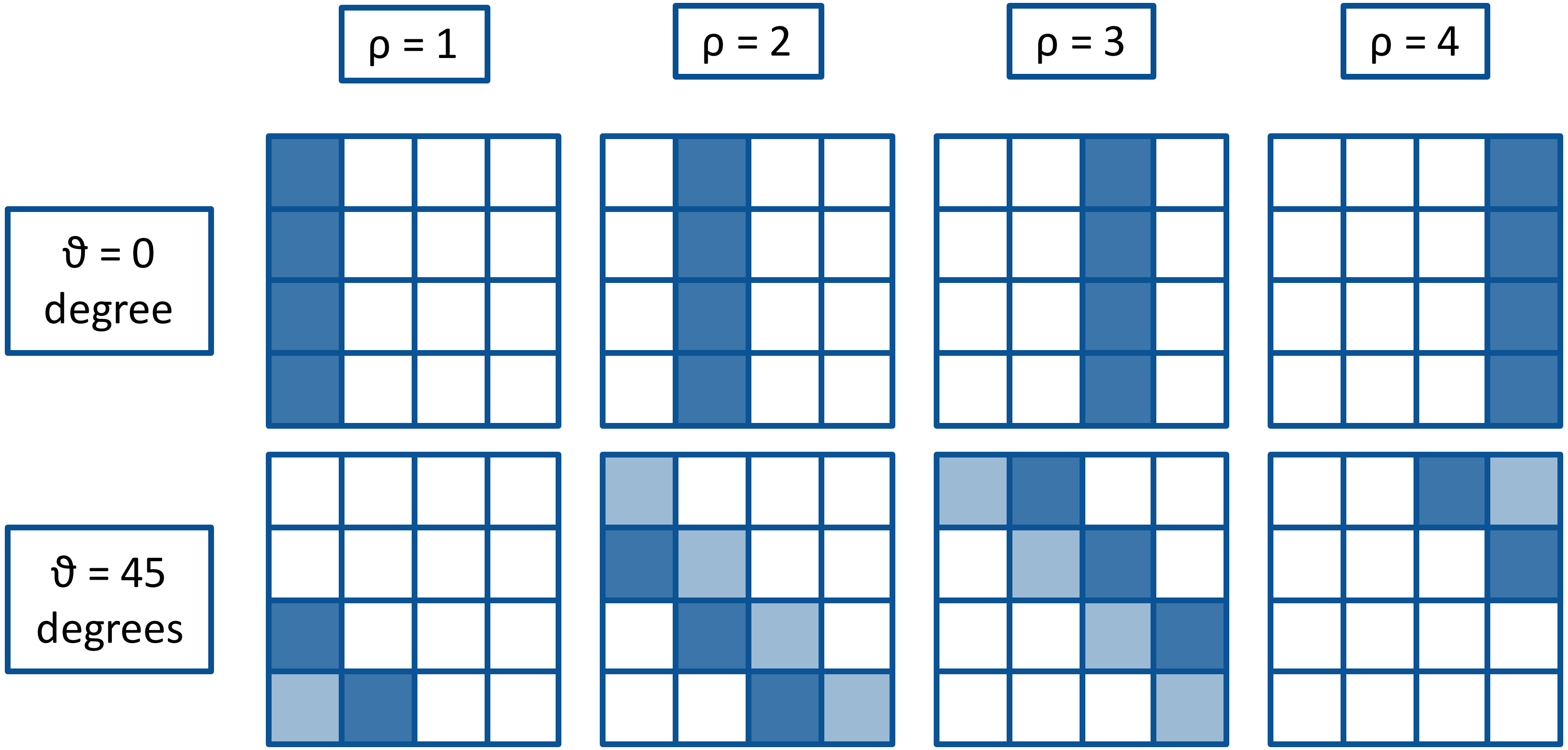
Exercise #1

- Analytical calculation of the sinogram or Radon Transform where each projection is calculated as line integrals of the image intensity
  - Choose the image (phantom.png)
  - Choose the number of projection lines ($n\rho = 128$) and the number of projection angles ($n\vartheta = 180$ with spacing $\Delta\vartheta = 1$ degree)
  - For loop over projection angles
    - Rotate the image matrix according to the projection angle and integrate the image matrix along the straight integration lines (instead of rotating the integration lines)
    - Store the resulting vector as a column in the sinogram matrix

Exercise #1

- Analytical calculation of the system matrix
  - Choose the number of projection lines ($n\rho = 128$) and the number of projection angles ($n\vartheta = 180$ with spacing $\Delta\vartheta = 1$ degree)
  - For loop over projection angles
    - For loop over projections
      - Create image matrix made of a column in correspondence of the projection
      - Rotate the image matrix according to the projection angle
      - Store the resulting vector as a column in the system matrix
- Analytical calculation of the sinogram or Radon Transform as forward-projection of the image to be compared to the previous sinogram
  - Are they different? Why?

Exercise #1



Exercise #1

- Implementation of the numerical tomographic image reconstruction algorithm SART
  - Take a sinogram and the system matrix
    - Initialize the vectorized image as vector of zero
    - For loop over iterations
    - Updating formula of the SART

$$\overrightarrow{f^{(n+1)}} = \overrightarrow{f^{(n)}} - \frac{\sum_i \left(a_{ij} \cdot \left(\frac{\overrightarrow{f^{(n)}} \cdot \overrightarrow{a_{ij}}^T - \vec{g}}{\sum_j a_{ij}} \right) \right)}{\sum_i a_{ij}}$$

Forward projection (vector of length I)

Back projection (vector of length J)