LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WiSe 2023/24
TA1: Condensed matter physics
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_24_25/TA1_theoretical_condensed_matter/index.html

Problem set 5

Problem 1 The d-dimensional Debye model

In the lecture we discussed the Debye model for the 3-dimensional case making use of the density of states (as a function of the momentum $|\vec{q}|$). Show that in the general, d-dimensional case, the density of states, now as a function of the normal mode energies ε, is of the form

$$f(\varepsilon) \sim \varepsilon^{d-1}\theta(k_B\Omega_D - \varepsilon)\ , \tag{1}$$

where $\theta(x)$ denotes the heaviside step function. Determine the corresponding heat capacity at low temperatures $T \ll \Theta_D$.

Problem 2 Non-linear phonons

Normally the Debye–Waller factor vanishes in 1D and 2D at finite temperature, and in 1D at zero temperature as well. The situation can be different for cases with long-range coupling between atoms, which can result in non-linear phonon dispersion of the form $\Omega(q) \sim q^\alpha$ $(\alpha < 1)$ for small q.

(2.a) Show that the Debye–Waller factor does not vanish for a 2D lattice at finite temperature, as long as $0 < \alpha < 1$. Show that the same is true for a 1D lattice at zero temperature.

(2.b) Find the range of α within which the Debye-Waller factor does not vanish for a 1D lattice at finite temperature.