LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WiSe 2023/24
TA1: Condensed matter physics
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_24_25/TA1_theoretical_condensed_matter/index.html

Problem set 1

Problem 1 Radiation of an oscillating electric dipole

In the lecture we used the electric field emitted by an oscillating dipole at position $\vec{R}$

$$\vec{E}_{\text{em}}(\vec{r}, t) = \frac{e^2}{m_e c^2} \left[\vec{n} \times \left(\vec{n} \times \vec{E}_{\text{in}}\right)\right] \mathrm{e}^{\mathrm{i}(\vec{k}\vec{r} - \omega t)} \frac{\mathrm{e}^{\mathrm{i}k|\vec{R}-\vec{r}|}}{|\vec{R} - \vec{r}|}. \tag{1}$$

We assumed that it is generated by an incoming electric field $\vec{E}_{\text{in}}\mathrm{e}^{\mathrm{i}(\vec{k}\vec{r}-\omega t)}$ interacting with an electron at $\vec{R}$. Derive this equation and compute the power emitted by the oscillating electron.

Problem 2 Distribution functions

Consider the gaussian probability distribution function $p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$ for some $\sigma > 0$ and $x \in \mathbb{R}$.

(2.a) Show that $p(x)$ is normalized, i.e., it is indeed a probablity distribution.

(2.b) Calculate the expectation values $\langle x^{2n} \rangle$ for $n \in \mathbb{N}$.

(2.c) For $\alpha \in \mathbb{R}$, using the Taylor expansion of $\mathrm{e}^{\mathrm{i}\alpha x}$ and computing the expectation values $\langle \frac{(\mathrm{i}\alpha x)^k}{k!} \rangle$ for every order k, show the relation $\langle \mathrm{e}^{\mathrm{i}\alpha x} \rangle = \mathrm{e}^{-\frac{1}{2}\alpha^2 \langle x^2 \rangle}$.

Problem 3 Cumulant expansion

Let $p(x_1, x_2, \ldots)$ be a probability distribution function for random variables $x_1, x_2, \ldots$ and $A \equiv A(x_1, x_2, \ldots$ an observable depending on the random variables. The expectation value of the exponential of A can always be written in the form $\langle \mathrm{e}^A \rangle = \mathrm{e}^C$. Assuming $\langle A \rangle = 0$, show that C can be written as the Cumulant expansion $C = \sum_{n=1}^{\infty} \frac{c_n}{c!}$ with the first cumulants given by

$$c_1 = 0 \tag{2}$$

$$c_2 = \langle A^2 \rangle \tag{3}$$

$$c_3 = \langle A^3 \rangle \tag{4}$$

$$c_4 = \langle A^4 \rangle - 3 \langle A^2 \rangle^2. \tag{5}$$

What does this imply if $p(x_1, x_2, \ldots)$ is close to a (multivariate) gaussian probability distribution?