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Sheet 00: Differentiation and Integration

Posted: So 01.09.23 Due: never

(b)[2](E/M/A) means: problem (b) counts 2 points and is easy/medium hard/advanced

Example Problem 1: Differentiation of polynomials [1]
Points: (a)[0,5](E); (b)[0,5](E).

Compute the first and second derivatives of the following polynomials. [Check your results against those in square brackets, where $[a; b, c]$ stands for $f'(a) = b$, $f''(a) = c$.]

(a) $f(x) = 3x^3 + 2x - 1$ $\quad [2; 38, 36]$ (b) $f(x) = x^4 - 2x^2 + 2$ $\quad [2; 24, 44]$

Example Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [1]
Points: (a)[1](E); (b)[1](E)

Compute the first derivative of the following functions.
[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = x \sin x$ $\quad \left[\frac{\pi}{4}, \frac{1}{\sqrt{2}}\left(1 + \frac{\pi}{4}\right)\right]$ (b) $f(x) = \cos\left[\pi(x^2 + x)\right]$ $\quad \left[\frac{1}{2}, -\pi\sqrt{2}\right]$

(c) $f(x) = \dfrac{1}{7 - x^2}$ $\quad \left[3, \frac{3}{2}\right]$ (d) $f(x) = \dfrac{x-1}{x+1}$ $\quad \left[3, \frac{1}{8}\right]$

Example Problem 3: Differentiation of powers, exponentials, logarithms [2]
Points: [3](E).

Compute the first derivative of the following functions.
[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = -\dfrac{1}{\sqrt{2x}}$ $\quad \left[2, \frac{1}{8}\right]$ (b) $f(x) = \dfrac{x^{1/2}}{(x+1)^{1/2}}$ $\quad \left[3, \frac{1}{16\sqrt{3}}\right]$

(c) $f(x) = \mathrm{e}^x(2x - 3)$ $\quad [1, \mathrm{e}]$ (d) $f(x) = 3^x$ $\quad \left[-1, \frac{\ln 3}{3}\right]$

(e) $f(x) = x \ln x$ $\quad [1, 1]$ (f) $f(x) = x \ln(9x^2)$ $\quad \left[\frac{1}{3}, 2\right]$

Example Problem 4: Elementary integrals [1]
Points: (a)[0,5](E); (b)[0,5](E)

Compute the following integrals. [Check your results: (a) $I(2) = \frac{15}{2}$; (b) $I(\ln 2) = \frac{7}{3}$.]

(a) $I(x) = \displaystyle\int_1^x \mathrm{d}y(2y^3 - 2y + 3)$, (b) $I(x) = \displaystyle\int_0^x \mathrm{d}y\, \mathrm{e}^{3y}$.

[Total Points for Example Problems: 5]

Homework Problem 1: Differentiation of polynomials [1]
Points: (a)[0,5](E); (b)[0,5](E).

Compute the first and second derivatives of the following polynomials. [Check your results against those in square brackets, where $[a; b, c]$ stands for $f'(a) = b$, $f''(a) = c$.]

(a) $f(x) = 4x^5 - x^3 + 2$ $\quad \left[\frac{1}{2}; \frac{1}{2}, 7\right]$

(b) $f(x) = x^3 - 2x^2 - x + 9$ $\quad [3; 14, 14]$

Homework Problem 2: Derivatives involving powers, sine and cosine: product rule and chain rule [2]
Points: (a)[1](E); (b)[1](E)

Compute the first derivative of the following functions.
[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = (x+\frac{1}{\pi}) \sin\left[\pi\left(x+\frac{1}{4}\right)\right]$ $\quad [0, \sqrt{2}]$

(b) $f(x) = -x^2 \cos(\pi x)$ $\quad \left[\frac{1}{3}, -\frac{1}{3} + \frac{\pi}{6\sqrt{3}}\right]$

(c) $f(x) = \cos\left[\pi \sin(x)\right]$ $\quad \left[\frac{\pi}{6}, -\frac{\sqrt{3}}{2}\pi\right]$

(d) $f(x) = -\cos^4\left(\frac{3}{\pi}x^2 - x\right)$ $\quad \left[\frac{\pi}{2}, 2\right]$

(e) $f(x) = \dfrac{1}{x^3 - 2x^2}$ $\quad \left[3, -\frac{5}{27}\right]$

(f) $f(x) = \dfrac{x^2 - 2}{x^2 + 1}$ $\quad \left[2, \frac{12}{25}\right]$

Homework Problem 3: Differentiation of powers, exponentials, logarithms [2]
Points: [2](E) (Solve any 4 subproblems; beyond that: 0.25 bonus per subproblem.)

Compute the first derivative of the following functions.
[Check your results against those in square brackets, where $[a, b]$ stands for $f'(a) = b$.]

(a) $f(x) = \sqrt[3]{x^2}$ $\quad \left[8, \frac{1}{3}\right]$

(b) $f(x) = \dfrac{x}{(x^2+1)^{1/2}}$ $\quad \left[1, \frac{1}{\sqrt{8}}\right]$

(c) $f(x) = -\mathrm{e}^{(1-x^2)}$ $\quad [1, 2]$

(d) $f(x) = 2^{x^2}$ $\quad [1, 4\ln 2]$

(e) $f(x) = 2\dfrac{\sqrt{\ln x}}{x}$ $\quad \left[\mathrm{e}, -\frac{1}{\mathrm{e}^2}\right]$

(f) $f(x) = \ln\sqrt{x^2+1}$ $\quad \left[1, \frac{1}{2}\right]$

Homework Problem 4: Elementary integrals [1]
Points: (a)[0,5](E); (b)[0,5](E)

Compute the following integrals. [Check your results: (a) $I(6) = \ln 2$; (b) $I(\ln 9) = \frac{4}{3}$.]

(a) $I(x) = \displaystyle\int_0^x \mathrm{d}y \frac{1}{2y+4}$,

(b) $I(x) = \displaystyle\int_0^x \mathrm{d}y \, \sinh\left(\tfrac{1}{2}y\right)$.

[Total Points for Homework Problems: 6]