W0 zero gauge

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- We saw on problem sheet 12 that the instanton solution of winding 1 is given by

$$W_\mu = \frac{2}{g} \sigma_a \eta_{a\mu\nu} \frac{x^\nu}{r^2+g^2} \qquad (1)$$

- Now let us rotate the solution such that $W_4 = 0$:

$$W_4 \longmapsto U^\dagger W_4 U + i U^\dagger \partial_4 U \overset{!}{=} 0$$

With $\quad W_4 = \dfrac{i x_i \sigma_i}{r^2+g^2}$

$$\frac{i x_i \sigma_i}{r^2+g^2} U + \partial_4 U = 0$$

- sketchy derivation of solution:

$$\int_{-\infty}^{t} - \frac{i x_i \sigma_i}{r^2+g^2}\, dx_4 = \int_{U(t=-\infty)}^{U(t)} \frac{1}{U}\, dU$$

$$\rightarrow \ln\left(\frac{U(t)}{U(t=-\infty)}\right) = \int_{-\infty}^{t} - \frac{i x_i \sigma_i}{r^2+g^2}\, dx_4$$

$$\rightarrow U(t) = \exp\left(\int_{-\infty}^{t} - \frac{i x_i \sigma_i}{r^2+g^2}\, dx_4\right) U(t=-\infty)$$

- This is the gauge transformation that needs to be done at time t, if we want to keep $W_4 = 0$. Applying this transformation to W_i:

$$W_i' = U^\dagger(t)\, \underset{t\,(1)}{W_i}\, U(t) + i\, U^\dagger(t)\, \partial_t\, U(t)$$

!

and inserting this into $n(t) = \frac{g^2}{8\pi^2} \int d^3x \; \mathrm{Tr}\,(G_{\mu\nu} \tilde{G}^{\mu\nu})$

will reveal that the winding changes continously from n to $n+1$.

- Let us take $t \to +\infty$:

$$U(t=+\infty) = \exp\left(\int_{-\infty}^{\infty} - \frac{i x_i \sigma_i}{\underset{x_i x_i + x_4^2}{r^2} + S^2}\, dx_4 \right) U(t=-\infty)$$

$$= \underbrace{\exp\left(- i\pi \frac{x_i \sigma_i}{x_i x_i + S^2} \right)}_{=:\,S} U(t=-\infty)$$

$$n\left[U(t=+\infty)\right] = \underbrace{n\,[S]}_{=1} + n\left[U(t=-\infty)\right]$$

$\rightarrow$ The winding number changes by 1 in the time interval $t \in \,]-\infty, \infty[$