LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WiSe 2023/24
TMP - TA4: Condensed Matter Many-Body-Physics and Field Theory II
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_23_24/TMP-TA4/index.html

Sheet 9:

Hand-out: Tuesday, Dec. 19, 2023; Solutions: Tuesday, Jan. 09, 2024

Problem 1 Calculation of pseudopotentials

Calculate haldane pseudopotentials for the following real space interactions.

(1.a) $V(\boldsymbol{r}) = 4\pi V_0 \delta^{(2)}(\boldsymbol{r})$.

(1.b) $V(\boldsymbol{r}) = 2\pi V_1 \nabla^2 \, \delta^{(2)}(\boldsymbol{r})$.

(1.c) $V(\boldsymbol{r}) = \alpha \, e^{-r^2}$.
Hint: for the second potential, use the Laplacian operator in the cylindical coordinate $\nabla^2 = 1/r \, \partial_r \big(r \partial_r\big)$*, and do integration by parts.*

Problem 2 2D one component plasma analogy

(2.a) For the Laughlin $1/m$ state, show that

$$|\psi_{1/m}(\{z_i\})|^2 = e^{-\beta \, \Phi(\{z_i\})} \,, \tag{1}$$

where Φ is in the form of a thermodynamic potential. Obtain an expression for Φ.

(2.b) Interpret the individual terms in Φ.
Hint: the photon-mediated Coulomb interaction in $(2+1)D$ *is of the form* $V_C^{2D}(r) \propto e^2 \, log(r)$.

(2.c) Use charge neutrality to show that the overall density of the Laughlin state is $\rho = \frac{1}{2\pi l_{\mathrm{B}}^2 m}$.

(2.d) Generalize the above construction to the quasihole of a $1/m$ Laughlin state. Use the prefect screening property of plasma to derive the $e^* = e/m$ charge of the Laughlin quasihole.