LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fakultät für Physik im WiSe 2023/24
TMP - TA4: Condensed Matter Many-Body-Physics and Field Theory II
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https://www2.physik.uni-muenchen.de/lehre/vorlesungen/wise_23_24/TMP-TA4/index.html

# Sheet 4:

Hand-out: Tuesday, Nov. 14, 2023; Solutions: Tuesday, Nov. 21, 2023

**Problem 1** Hubbard-Stratonovich for anisotropic pairing

In the lecture, we have discussed the Hubbard-Stratonovich approach for isotropic $s$-wave pairing, where we have introduced the isotropic pairing field

$$\hat{A} = \sum_{\mathbf{k}} \hat{c}_{-\mathbf{k},\downarrow}\hat{c}_{\mathbf{k},\uparrow}. \tag{1}$$

By condensing the field $\Delta = \langle \hat{A} \rangle$, we have found the BCS mean-field solution – fluctuation of the order parameters can be included systematically.

Now, we consider a case of unconventional superconductivity with momentum-dependent, anisotropic pairing. In Problem 2, we find that such anisotropic pairing channels can arise in strongly-correlated electrons. Here, we start from the empirical Hamiltonian given by

$$\hat{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}\sigma}\hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}^\dagger_{-\mathbf{k}\downarrow}\hat{c}_{-\mathbf{k}'\downarrow}\hat{c}_{\mathbf{k}'\uparrow} \tag{2}$$

with a factorizable pairing potential $V_{\mathbf{k},\mathbf{k}'} = \frac{-g}{L^d}\gamma_{\mathbf{k}}\gamma_{\mathbf{k}'}$; $\epsilon(\mathbf{k})$ is the dispersion relation, $L^d$ is the volume of the system in $d$ dimensions, $g_0$ is the interaction strength.

(1.a) Write down the Hamiltonian in terms of an anisotropic pairing field operator $\hat{A}$.

(1.b) Perform a Hubbard-Stratonovich transformation by coupling the pairing field to a bosonic field $\Delta$. Show that we obtain the same expression as for $s$-wave pairing but with momentum dependent field $\Delta \to \Delta_{\mathbf{k}} = \Delta\gamma_{\mathbf{k}}$.

(1.c) Derive that gap equation for a general $\gamma_{\mathbf{k}}$. Now, assume $d = 2$ and $\gamma_{\mathbf{k}} = \cos(k_x) - \cos(k_y)$. Sketch the pairing gap.

**Problem 2** RPA derivation of d-wave pairing near a spin-density-wave instability

In this problem, we derive the effective electron interaction due to paramagnon exchange using RPA for the Fermi-Hubbard model. The effective interactions in the singlet and triplet channels, denoted by $V_s$ and $V_t$ are given in Fig. 1. The first set of diagrams for $V_s$ describes electron-hole scattering, while the second set describes interaction mediated by even number of bubbles. In $V_t$, the diagrams include an odd number of bubbles.
*Note: This exercise is based on Scalapino, et al., 1986, "D-wave pairing near a spin-density-wave instability". Physical Review B, 34(11), p.8190.*

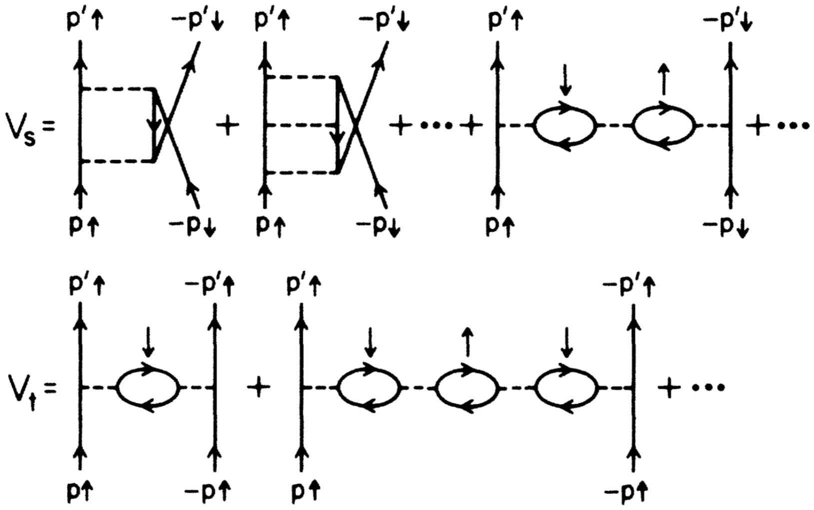


Abbildung 1: Diagrams included in the effective electron-electron interaction for the Hubbard model.

(2.a) From the diagrams in Fig. 1, obtain expressions for $V_s(\boldsymbol{p},\boldsymbol{p}')$ and $V_t(\boldsymbol{p},\boldsymbol{p}')$ in terms of $U$ and $\chi_0(\boldsymbol{q},\omega=0)$.

(2.b) Show that a generic interaction $V(\boldsymbol{p},\boldsymbol{p}') = V(p,p',\hat{\Omega},\hat{\Omega}')$ over a spherically symmetric Fermi surface ($p = p' = p_F$) can be decoupled into s, p, d, $\cdots$ wave as

$$\frac{1}{\int d^2\Omega\, g_\alpha^2(\hat{\Omega})} \int d^2\Omega \int d^2\Omega'\, g_\alpha(\hat{\Omega}) V(p_F,p_F,\hat{\Omega},\hat{\Omega}') g_\alpha(\hat{\Omega}') \tag{3}$$

where $g_\alpha(\hat{\Omega})$ is a suitable $l-$wave function defined on the unit sphere, for $l = s,p,d,\cdots$. For instance, some $d-$wave functions are $g_{x^2-y^2} = \cos(\Omega_x) - \cos(\Omega_y)$, $g_{xy} = \sin(\Omega_x)\sin(\Omega_y)$. *Hint: do not take any integrals!*

(2.c) What is the condition to have $d-$wave pairing in the $d_{x^2-y^2}$ channel?