Quantum Field Theory (Quantum Electrodynamics)

Problem Set 13 29 & 31 January 2024

1. Scalar QED

The action of scalar QED reads

$$S = \int \mathrm{d}^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - ieA^\mu \left(\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi \right) + e^2 A_\mu A^\mu \phi^* \phi \right) ,$$

where ϕ is a complex scalar field with mass m, A_μ is the photon and e the electric charge of ϕ.

1. Is the theory invariant under the $U(1)$ gauge transformation ?
2. Derive the Feynman rules for the theory.
3. Find the diagram(s) contributing to the $\phi\phi \to \phi\phi$ process, at the lowest order.
4. Assuming that m is much smaller than the energy, compute the differential cross-section in the center-of-mass frame.