

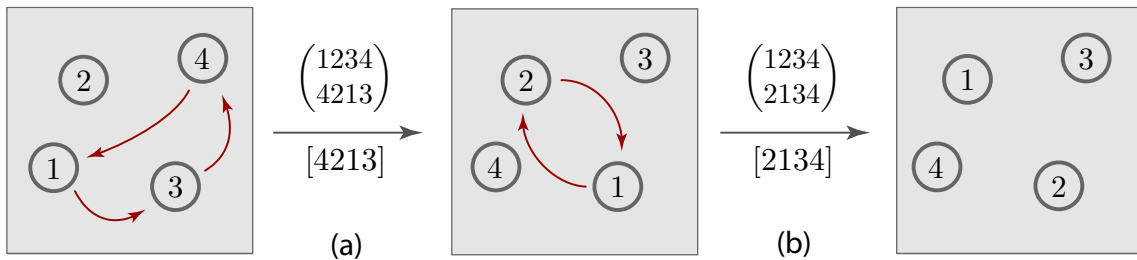
<https://moodle.lmu.de> → Kurse suchen: 'Rechenmethoden'

Sheet 00: Permutations

(b)[2](E/M/A) means: problem (b) counts 2 points and is easy/medium hard/advanced

Optional Problem 1: Permutations [4]

Points: (a)[1](E); (b)[3](E).



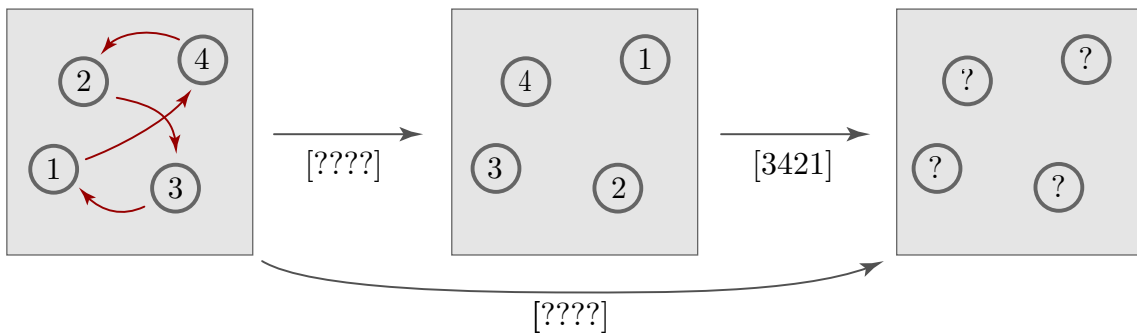
Consider four arbitrary but distinguishable objects, for example four billiard balls, labeled 1 to 4 (see figure). A permutation is a rearrangement of these objects into a different order. For example, the reordering of four objects indicated in the left panel of the figure leads to the new arrangement shown in the middle.

We need a compact notation for specifying permutations. One approach denotes the permutation (a) shown on the left, which replaces 1 by 4, 2 by 2, 3 by 1 and 4 by 3, by $\begin{pmatrix} 1234 \\ 4213 \end{pmatrix}$: the top row lists the numbers to be permuted in increasing order, and the permutation is obtained by replacing each number in the top row by the number just below it in the bottom row. Similarly, the permutation (b) shown on the right can be represented by $\begin{pmatrix} 1234 \\ 2134 \end{pmatrix}$, and the combined effect both permutations (first the left one, then the middle one) is given by $\begin{pmatrix} 1234 \\ 4123 \end{pmatrix}$.

Having agreed to write the top row in standard order, it is not necessary to write it down at all – each permutation is fully specified by the bottom row. Writing it in angular brackets, we arrive at the more compact notation $[4213]$ for permutation (a), which replaces 1 by 4, 2 by 2, 3 by 1 and 4 by 3. Similarly, permutation (b) is denoted as $[2134]$, and their combination (first (a), then (b)) by $[2134] \circ [4213] = [4123]$.

Fill in the blanks in the following figures and equations (using compact notation):

(a)



(b)

$$123 \xrightarrow{[321]} ??? \xrightarrow{[132]} ??? \Rightarrow [???] = [132] \circ [321].$$

$$1234 \xrightarrow{[2134]} \text{????} \xrightarrow{[3214]} \text{????} \Rightarrow [????] = [3214] \circ [2134].$$

$$1234 \xrightarrow{[2431]} \text{????} \xrightarrow{[1243]} \text{????} \xrightarrow{[4231]} \text{????} \Rightarrow [????] = [4231] \circ [1243] \circ [2431].$$

[Total Points for Optional Problems: 4]
