Inverse problems and machine learning in medical physics

Tomographic image reconstruction for ion imaging

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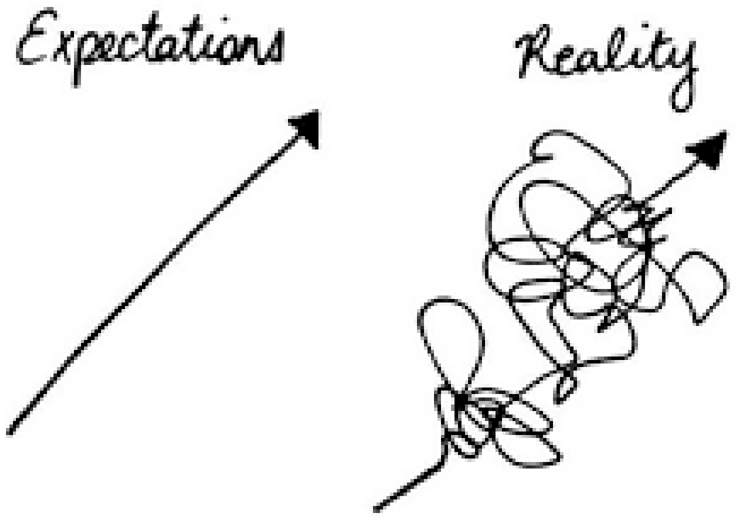
Detector configuration and ion trajectories

- The concept of ion trajectory for different detector configuration plays a crucial role in the forward-projection model, which is a foundation in ion imaging

$$\overrightarrow{WET} \neq A * \overrightarrow{RSP_t} \quad (t \text{ for "ground truth"})$$

- However, in clinical scenarios the intrinsic inconsistencies of the forward-projection model are in the same order of magnitude of the inaccuracies of the semi-empirical calibration of the X-ray CT

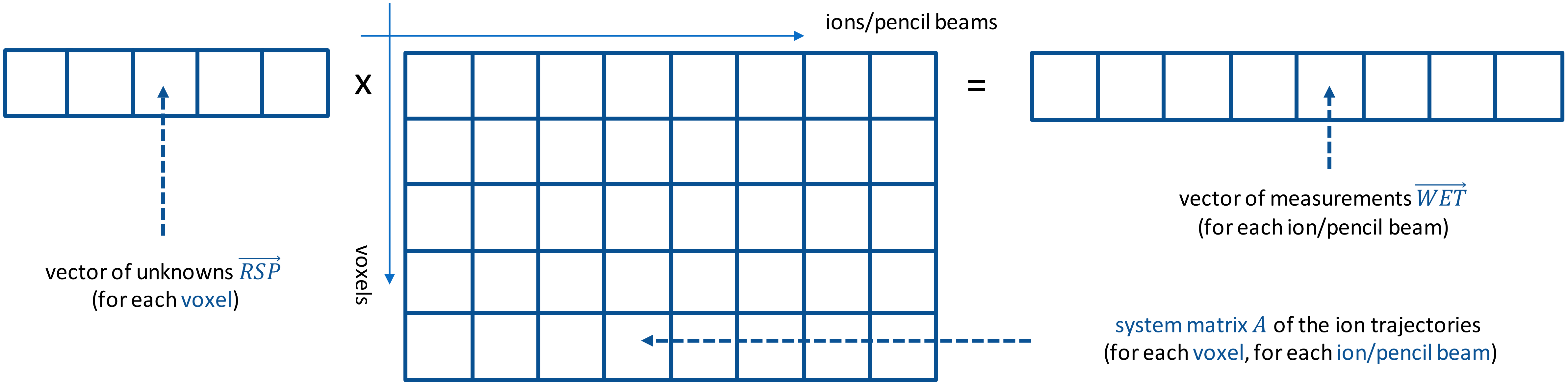
  - Relying on Monte Carlo simulations, the normalized root mean square error between the ion radiography and the forward-projection of the ground truth ion CT image is 1-2.5% for list-mode detector configuration and up to 2.5-5% for integration-mode detector configuration



Gianoli et al. *2020 Phys. Med. Biol.*

Tomographic image reconstruction

- Tomographic image reconstruction is applied to several ion radiographies, with projection angles covering 180°
  - The ordered subsets simultaneous algebraic reconstruction technique (OS-SART) coupled with total variation superiorization currently represents the state-of-the-art in ion imaging[1,2,3]
  - Information redundancy mitigates the intrinsic inaccuracies of the forward-projection model[4]



[1]Penfold et al. 2010 Med. Phys. [2]Meyer et al. 2019 *Phys. Med. Biol.* [3]Meyer et al. 2021 *Phys. Med. Biol.*
[4]Gianoli et al. 2019 *Phys. Med.*

Ordered Subsets

- The ordered subsets (OS) approach is introduced to accelerate numerical image reconstruction

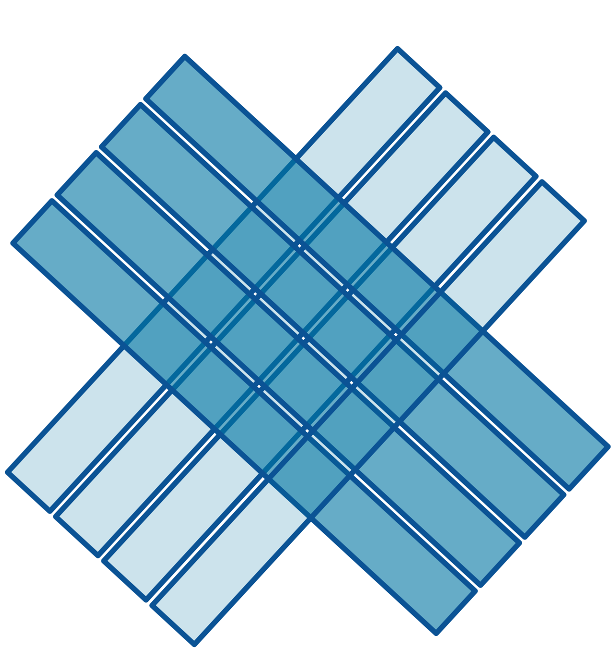
- In OS approach, instead of accessing all projections simultaneously for updating the image, the image is updated relaying on a subset of projections

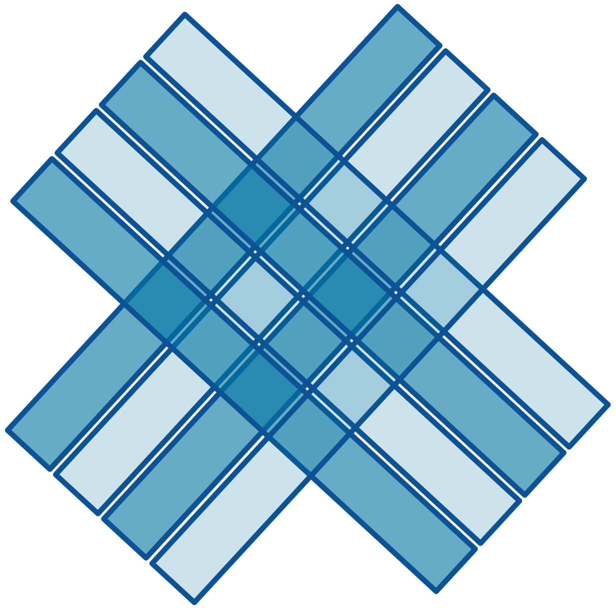
- ART can be interpreted as OS-SART with only one projection per subset and SART can be interpreted as OS-SART with only one subset

  - An update performed using a single subset is called a sub-iteration

  - An iteration is completed when all subsets have been processed once

  - The convergence acceleration is expressed in terms of number of iterations (not sub-iterations)





Ordered Subsets

- The idea of was originally proposed for emission tomography and then transferred to transmission tomography (the SART and the ML-EM produce the maximum likelihood estimate in the Gaussian and Poisson data, respectively)

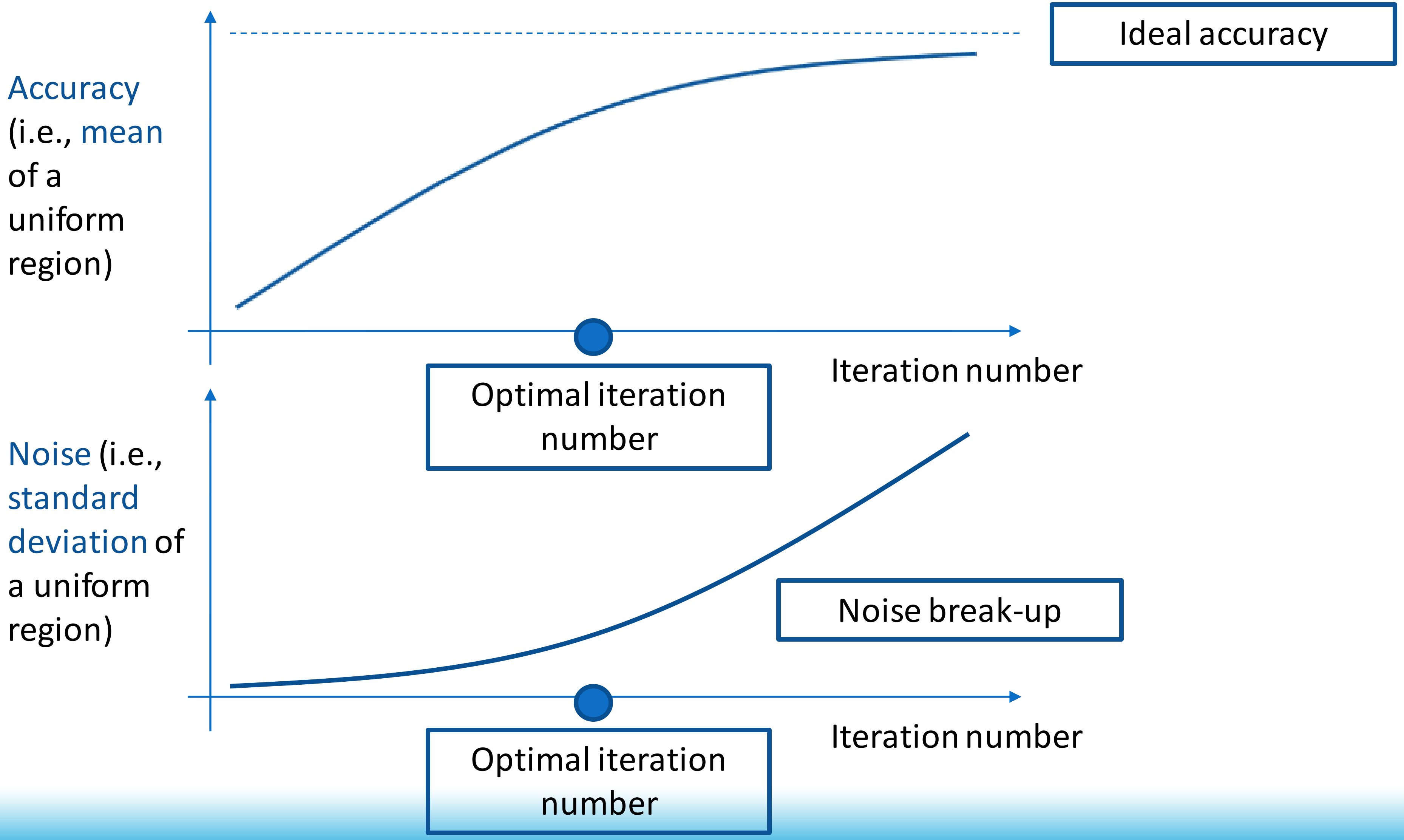
- The subset of projections $S(s)$ is employed for updating of the image, and this update, together with a different subset of projections $S(s+1)$, is then used for calculating the next update

$$\overrightarrow{f^{(s+1)}} = \overrightarrow{f^{(s)}} - \frac{\sum_{i \in S(s)} \left(a_{ij} \cdot \left(\frac{\overrightarrow{f^{(s)}} \cdot \overrightarrow{a_{ij}}^T - \vec{g}}{\sum_j a_{ij}} \right) \right)}{\sum_i a_{ij}}$$

- The best ordering of the subsets is defined according to the maximum angular distance ("as orthogonal as possible") from the previously used projections
    - This ordering further accelerates convergence as compared to sequential or random orderings
    - The increased convergence speed (in function of the number of iterations) and the reduced memory requirement (due to a reduced dimension of the system matrix) comes at the cost of an increased noise of the reconstructed image

Hudson, H. M., & Larkin, R. S. (1994). Accelerated image reconstruction using ordered subsets of projection data. IEEE transactions on medical imaging, 13(4), 601-609.

Outlook


Accuracy (i.e., mean of a uniform region)
Ideal accuracy
Optimal iteration number
Iteration number
Noise (i.e., standard deviation of a uniform region)
Noise break-up
Optimal iteration number
Iteration number

Tomographic image reconstruction

- Because of the intrinsic inconsistencies of the forward-projection model, the optimal solution minimizing the objective function of the tomographic image reconstruction algorithm is not necessarily the solution that best reconstructs the ground truth image

- The fundamental approach for the mitigation of the intrinsic inconsistencies of the forward-projection model is to stop the tomographic image reconstruction algorithm before the solution diverges (albeit with a lower objective function), thus being referred to as semi-convergence

- Another approach is to use superiorization techniques to shift the solution at each iteration to one that is superior to the current solution

    - A superior solution is defined in terms of a certain merit function φ

algorithm	$x^{k+1} = f(x^k)$
superiorized algorithm	$x^{k+1} = f\left(x^k + \beta_k v^k\right)$ so that $\varphi(x^k + \beta_k v^k) \leq \varphi(x^k)$

Tomographic image reconstruction

- In transmission imaging (i.e., ion imaging), the merit function φ is typically the total variation
- For a two-dimensional (2D) image representation in i and j of the image vector x^k is defined as:

$x^1_{i-1,j-1}$	$x^4_{i,j-1}$	$x^7_{i+1,j-1}$
$x^2_{i-1,j}$	$x^5_{i,j}$	$x^8_{i+1,j}$
$x^3_{i-1,j+1}$	$x^6_{i,j+1}$	$x^9_{i+1,j+1}$

$$\varphi(x^k) = \sum_i \sum_j \sqrt{(x^k_{i+1,j} - x^k_{i,j})^2 + (x^k_{i,j+1} - x^k_{i,j})^2}$$

- $\beta_k v^k$ is the perturbation term, with β_k real non-negative numbers (typically $0 < \beta_k < 1$) and v^k the perturbation vector, typically defined as non-ascending direction of the merit function φ at x^k

$$v^k = -\frac{\nabla\varphi(x^k)}{\|\nabla\varphi(x^k)\|_2} = -\frac{s^k}{\|s^k\|}$$

  - The perturbation vector is calculated as the negative of the normalized sub-gradient s^k (generalized concept of derivative for convex functions which are not necessarily differentiable) of the total variation $\varphi(x^k)$ for the image vector x^k

Penfold, S. N., Schulte, R. W., Censor, Y., & Rosenfeld, A. B. (2010). Total variation superiorization schemes in proton computed tomography image reconstruction. Medical physics, 37(11), 5887-5895.

Tomographic image reconstruction

- Another approach is to use regularization technique to add a penalty function to the objective function
  - The penalty function enforces a desired feature on the reconstructed image

$$x_{\min} = \mathrm{argmin}_x \, F(x) + \lambda\phi(x)$$

  - $\phi(x)$ is the penalty function (i.e., total variation or smoothness-related functions) and λ is a parameter controlling its weighting
  - The regularization changes the problem!
  - The optimal value of λ depends on the level of noise in the data
    - λ too small under-regularizes (i.e., not substantially improve image quality)
    - λ too large over-regularizes, resulting typically in an oversmoothed image

Defrise, M., Vanhove, C., & Liu, X. (2011). An algorithm for total variation regularization in high-dimensional linear problems. Inverse Problems, 27(6), 065002.

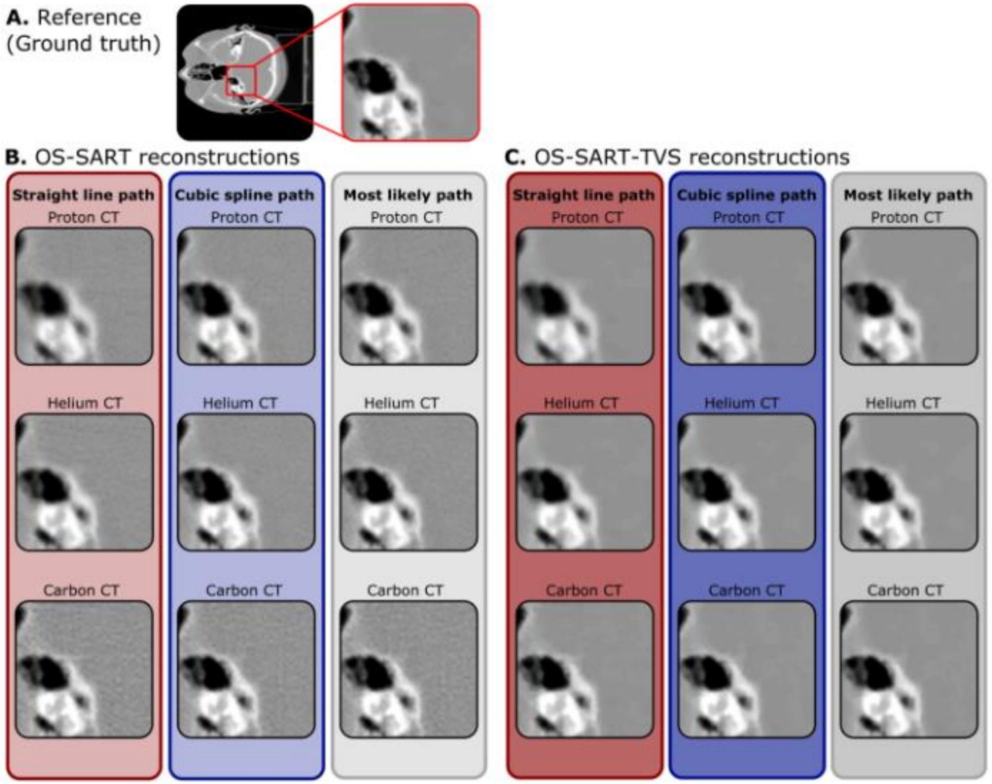
Tomographic image reconstruction for list-mode detector configuration

- The superiorization of the algorithm for tomographic image reconstruction introduces a "perturbation" of the solution in tomographic domain in order to reduce, and not necessarily minimize, a merit function φ (i.e., the total variation)

- The superiorization changes the algorithm by adding a shifting step but not the problem!

set $k = 0$, $l = 0$, $\beta_k = 1$;
while *repeat for each subset over the requested number of cycles* **do**
  if $\|s^k\| \geq 0$ **then**
    $v^k = -\frac{s^k}{\|s^k\|}$;
  else
    $v^k = s^k$;
  end
  while *loop* **do**
    set $y^k = x^k + \beta_k v^k$;
    if $\Phi(y^k) \leq \Phi(x^k)$ **then**
      $x^k = y^k$;
      set $loop = false$;
    end
    $\beta_k = \beta_k / 2$;
  end
  $x^{k+1} = \mathbf{P}_Q x^k$;
  $k = k + 1$;
end

P_Q is the operator that updates the OS-SART algorithm

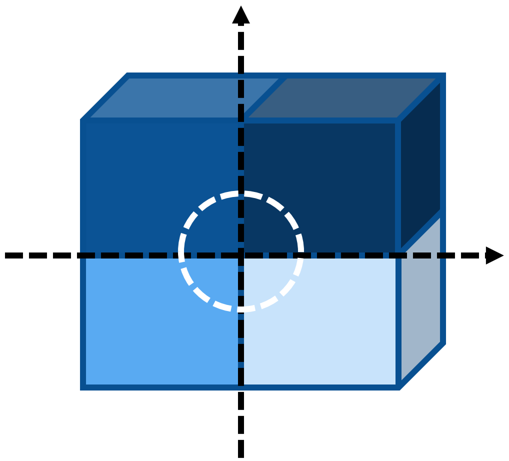


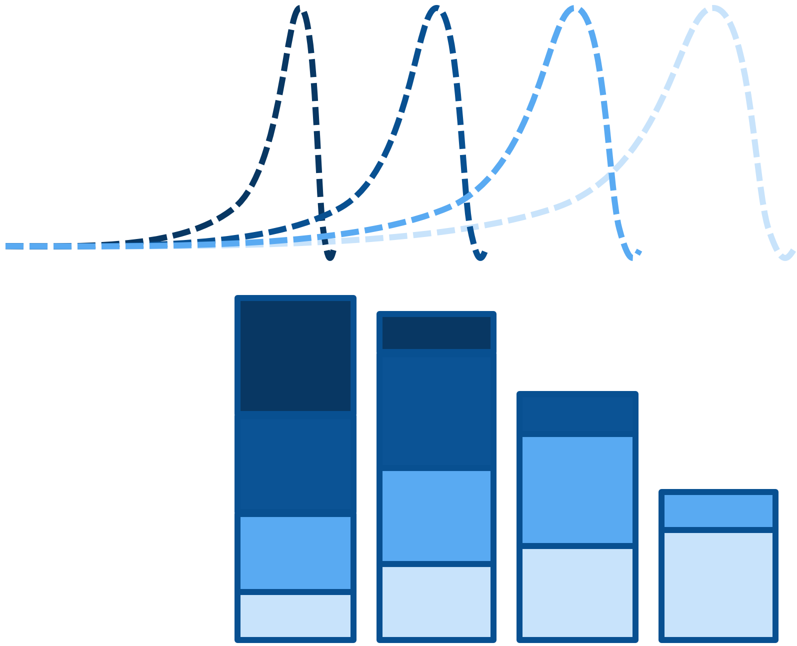
Meyer, S., Pinto, M., Parodi, K., & Gianoli, C. (2021). The impact of path estimates in iterative ion CT reconstructions for clinical-like cases. Physics in Medicine & Biology, 66(9), 095007.

Integration-mode detector configuration for pencil beams

- In integration-mode detector configuration, the Bragg peak signal for each pencil is discretized according to the multiple layers (i.e., channels) or according to the multiple initial energies in a single layer

- Due to lateral inhomogeneity traversed by the pencil beam, the Bragg peak signal results in a linear combination of elementary Bragg peak signals





- The Bragg peak of the component with the larger WET (i.e., the shorter range) takes advantages from the Bragg peaks of the components with smaller WET

Integration-mode detector configuration for pencil beams

- Linear decomposition[1,2] (inverse problem) is applied to retrieve the WET histogram as WET occurrence for each WET component by solving the system of linear equations $\overrightarrow{BP} = LUT * \overrightarrow{WET}$

  - $\overrightarrow{BP}$ is the discretized Bragg peak signal

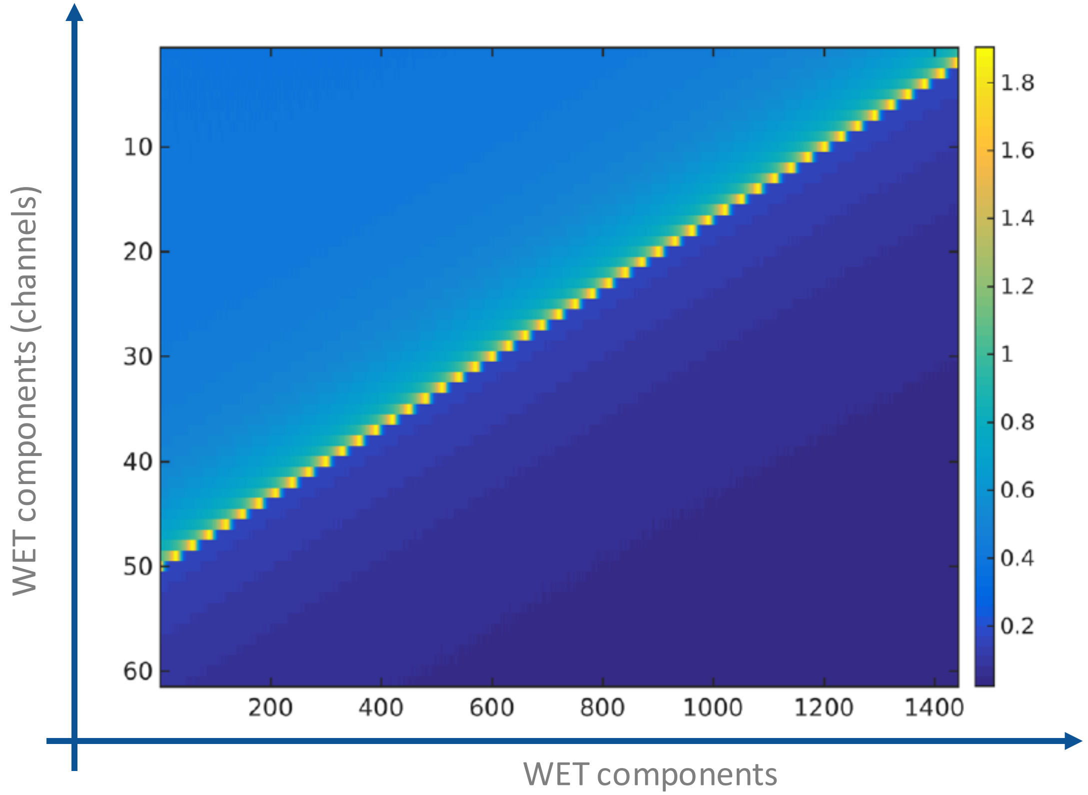
    - $\overrightarrow{WET}$ is the unknown vector of WET occurrences

    - LUT is the look-up-table of individual Bragg peak signals for each WET component

  - The least square optimization is based on Euclidean distance minimization

$$argmin_{\overrightarrow{WET}} \frac{1}{2} \left\| LUT * \overrightarrow{WET} - \overrightarrow{BP} \right\|_2^2$$

  - An histogram of WET occurrences for each WET component is obtained



[1]Krah et al. 2015 *Phys. Med. Biol.*
[2]Meyer et al. 2017 *Phys. Med. Biol.*

Tomographic image reconstruction for integration-mode detector configuration

- Defining p_{ik} the WET components and w_{ik} the WET occurrences, the tomographic image reconstruction for integration-mode detector configuration deals with the additional channel dimension of the WET histogram (WET_{hist})

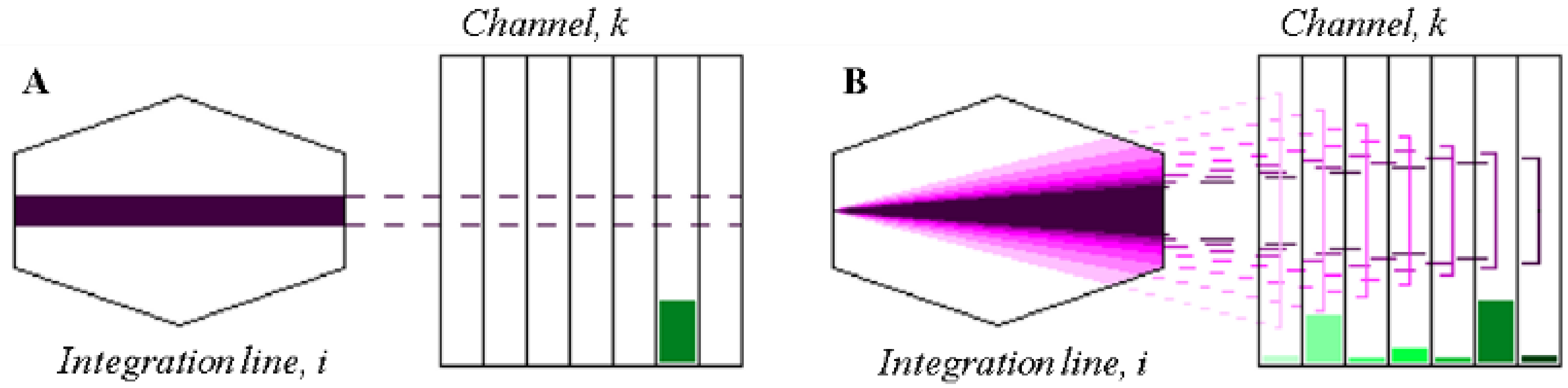
- The typical approach, the WET histogram is reduced to a single WET component
  - $WET_i = \max\{p_{ik}\}$ the WET component with maximum WET occurrence (WET_{mode} or WET_{max})
  - $WET_i = mean(p_{ik} w_{ik})$ the weighted averaged WET components (WET_{mean})

$$WET_i = \sum_j a_{ij} RSP_j$$

- Therefore, the integration line is assumed as straight or coinciding to the mean ion trajectory of the pencil beam
  - Analytical or numerical algorithms for tomographic image reconstruction are applied

Tomographic image reconstruction for integration-mode detector configuration

- Alternatively, WET_{hist} is handled within numerical tomographic image reconstruction (ART&SART)
  - WET components and WET occurrents are entirely exploited
  - The integration line is defined according to the scattering model (conical Gaussian) for each WET component
    - The WET components are spatially assigned (the *mean ion trajectory* is valid only for WET_{max} and WET_{mean})



Tomographic image reconstruction for integration-mode detector configuration

- The SART algorithm is considered for numerical tomographic image reconstruction

$$f_j^{n+1} = f_j^n + \frac{\sum_i a_{ij} \cdot \frac{g_i - \sum_j a_{ij} \cdot f_j^n}{\sum_j a_{ij}}}{\sum_i a_{ij}}$$

- The SART algorithm is modified to handle the additional channel dimension

$$m_{ik} = \sum_k w_{ik} a_{(ik)j} \qquad g_i = \sum_k w_{ik} g_{ik}$$

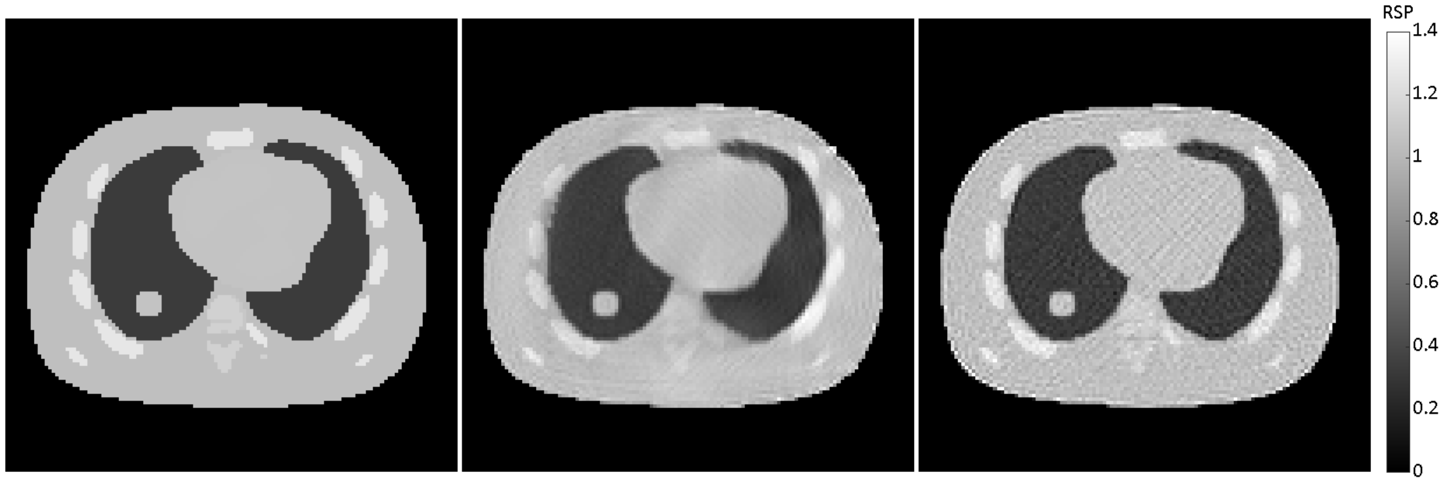
where $a_{(ik)j}$ describe the conical Gaussian for each channel
(indexing j the pixel/voxel, k the channel and i the measurement)

$$f_j^{n+1} = f_j^n + \frac{\sum_i m_{ij} \cdot \frac{g_i - \sum_j m_{ij} \cdot f_j^n}{\sum_j m_{ij}}}{\sum_i m_{ij}}$$

Seller Oria et al. 2018 *IEEE NSS-MIC*

Tomographic image reconstruction for integration-mode detector configuration

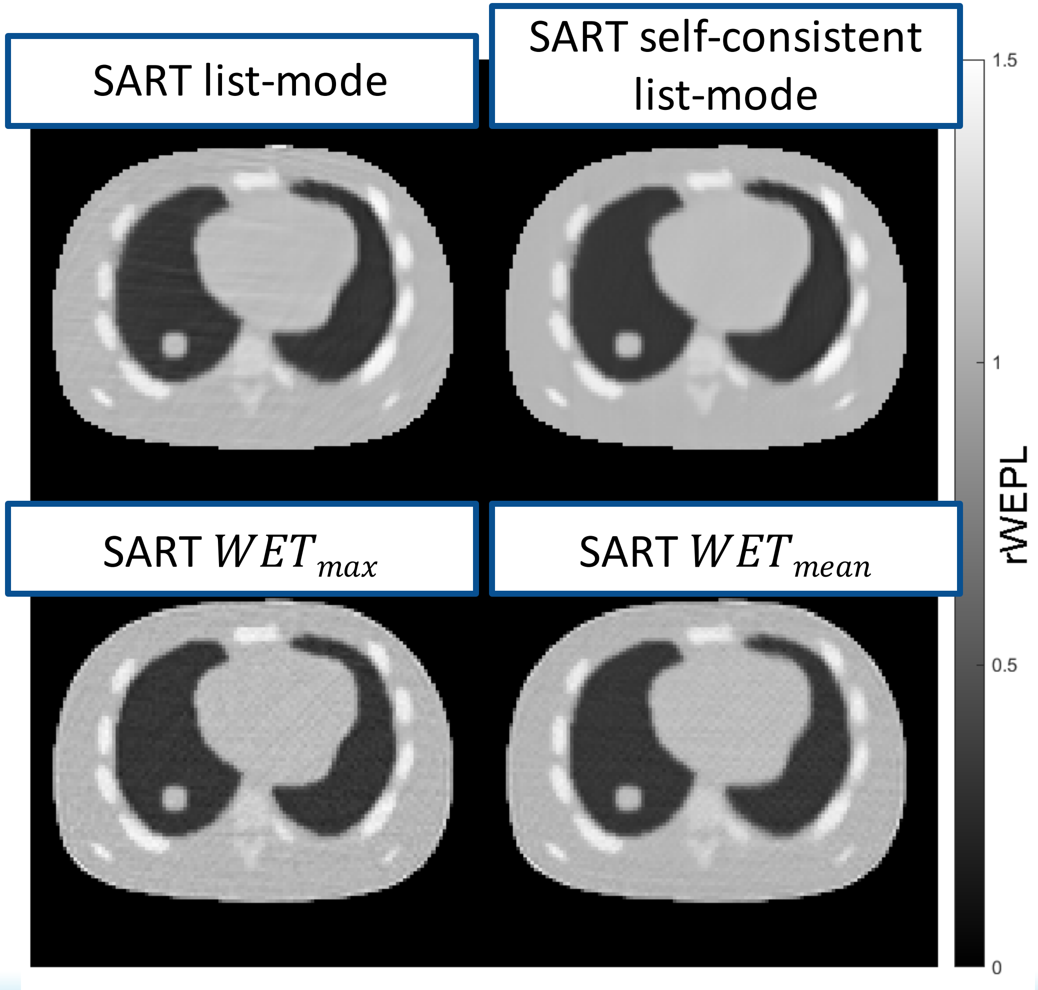
Ground truth	SART WET_{hist}	SART WET_{max}



Tomographic image reconstruction for integration-mode detector configuration

- Comparison between list-mode and integration-mode detector configurations

- The self-consistent tomographic image reconstruction pretends to know the exact trajectories of the protons (to overcome the ill-posed nature of the inverse problem in ion imaging)

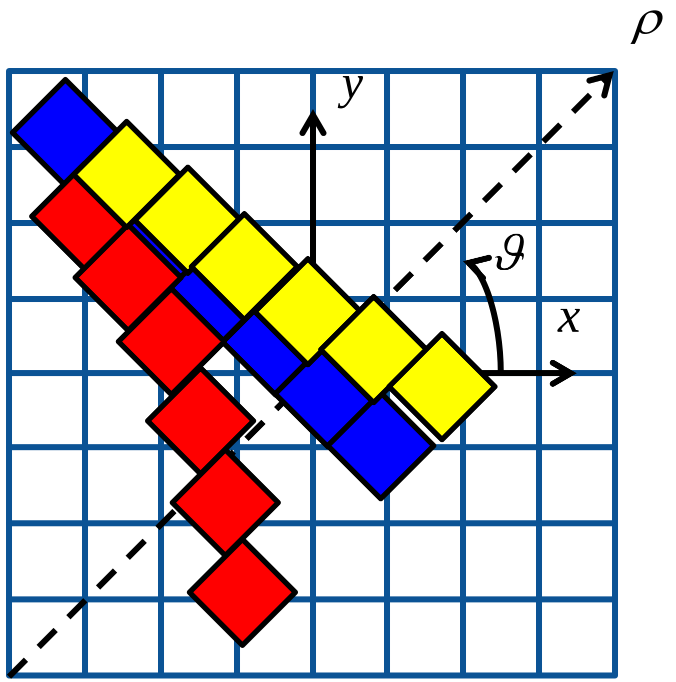


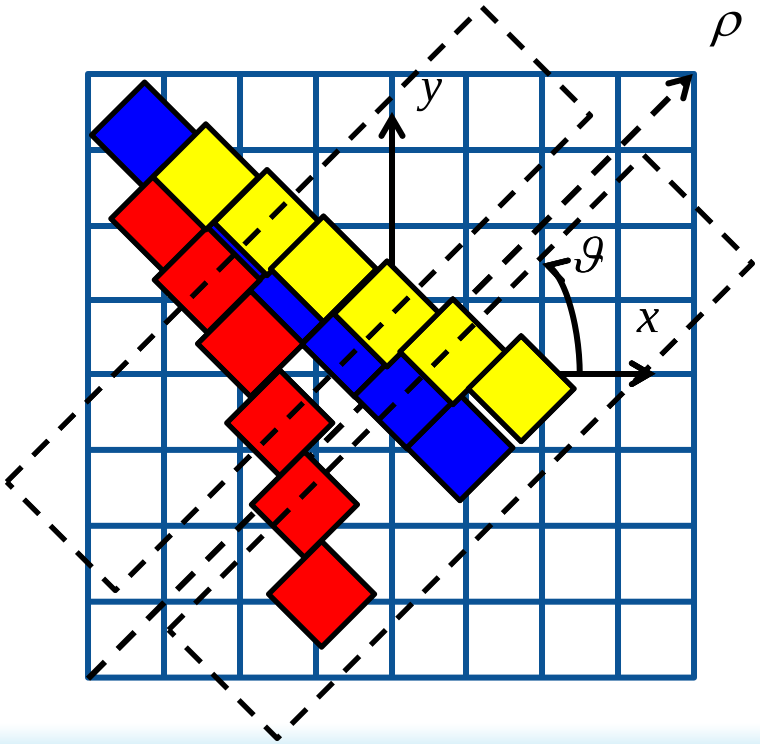
Gianoli et al. 2019 *Phys. Med.*

Tomographic image reconstruction (list-mode detector configuration)

- The MLP is the estimation of the proton trajectory that can be easily adopted in the system matrix of numerical reconstruction

- Alternatively, the estimation of the proton trajectory can be used to implement a “modified” FBP, based on a distance-driven binning of projections for individual source positions

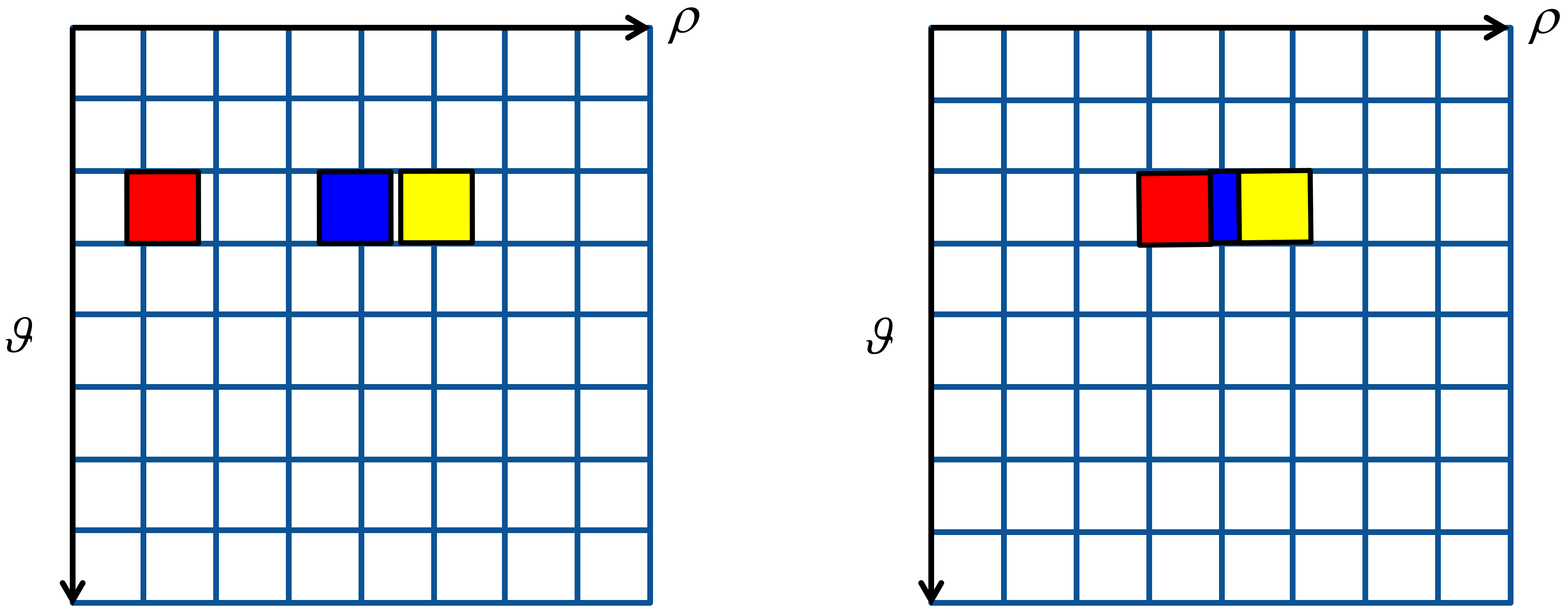




Rit, S., Dedes, G., Freud, N., Sarrut, D., & Létang, J. M. (2013). Filtered backprojection proton CT reconstruction along most likely paths. Medical physics, 40(3), 031103.

Tomographic image reconstruction (list-mode detector configuration)

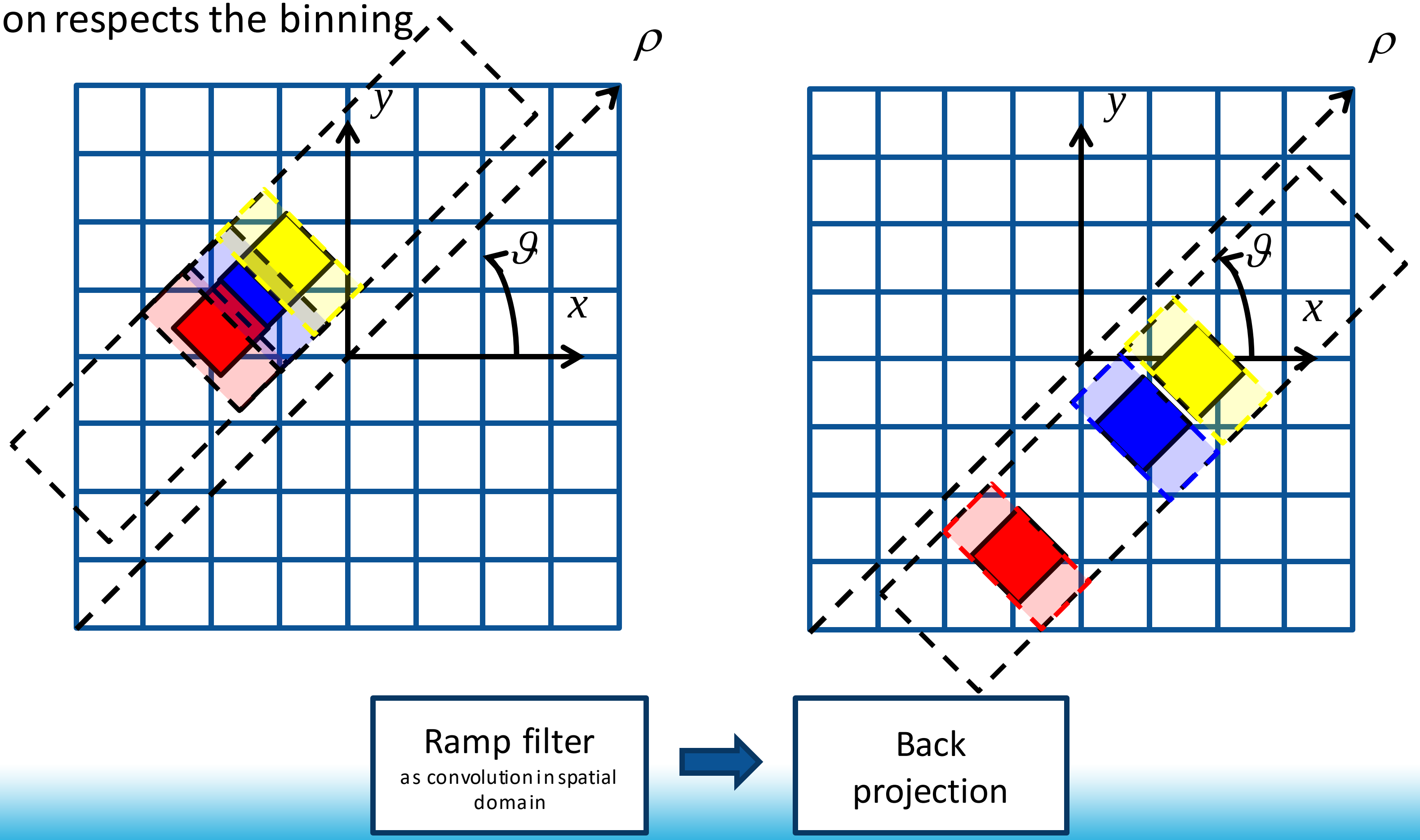
- The projection is binned (subdivided) according to the source to detector distance
- The binning provides the sinogram an additional dimension, whose size is defined by the number of bins



Tomographic image reconstruction (list-mode detector configuration)

- The filtering, enabled by the distance-driven binning, is applied to each binned projection of the sinogram (without filtering the method is simply a back-projection along the MLP)

- The back-projection respects the binning



Exercise #1

- Analytical calculation of the sinogram or Radon Transform where each projection is calculated as line integrals of the image intensity

  - Choose the image (phantom.png)

  - Choose the number of projection lines ($n\rho = 128$) and the number of projection angles ($n\vartheta = 180$ with spacing $\Delta\vartheta = 1$ degree)

  - For loop over projection angles

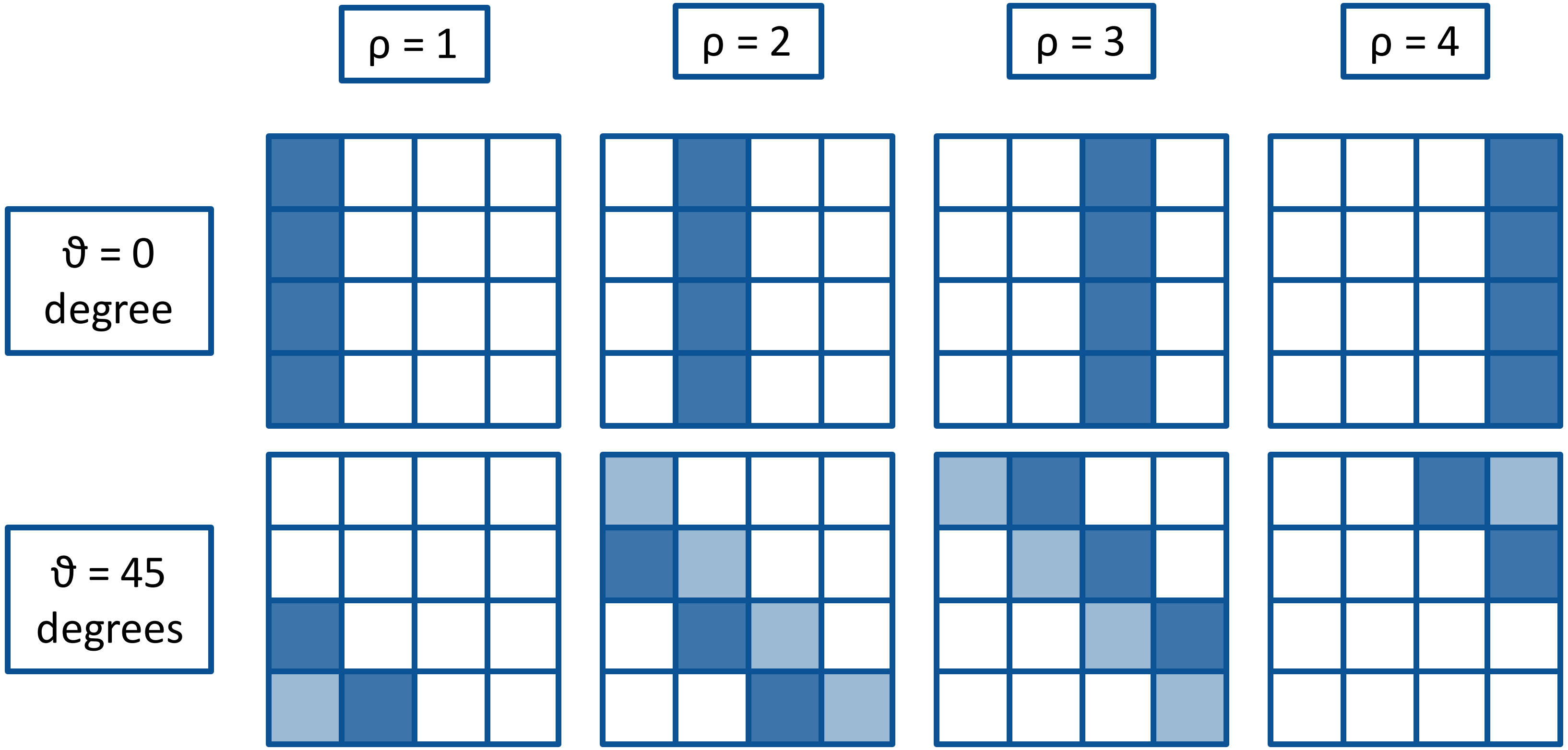
    - Rotate the image matrix according to the projection angle and integrate the image matrix along the straight integration lines (instead of rotating the integration lines)

    - Store the resulting vector as a column in the sinogram matrix

Exercise #1

- Analytical calculation of the system matrix
  - Choose the number of projection lines ($n\rho$ = 128) and the number of projection angles ($n\vartheta$ = 180 with spacing $\Delta\vartheta$ = 1 degree)
  - For loop over projection angles
    - For loop over projections
      - Create image matrix made of a column in correspondence of the projection
      - Rotate the image matrix according to the projection angle
      - Store the resulting vector as a column in the system matrix
- Analytical calculation of the sinogram or Radon Transform as forward-projection of the image to be compared to the previous sinogram
  - Are they different? Why?

Exercise #1



Exercise #1

- Implementation of the numerical tomographic image reconstruction algorithm SART
  - Take a sinogram and the system matrix
    - Initialize the vectorized image as vector of zero
    - For loop over iterations
    - Updating formula of the SART

$$\overrightarrow{f^{(n+1)}} = \overrightarrow{f^{(n)}} - \frac{\sum_i a_{ij} \cdot \left(\frac{\overrightarrow{f^{(n)}} \cdot \overrightarrow{a_{ij}}^T - \vec{g}}{\sum_j a_{ij}} \right)}{\sum_i a_{ij}}$$

Forward projection (vector of length I)

Back projection (vector of length J)