GUT Course 22/23

Lecture XVIII

LMU

Winter 2023

Unification of gauge couplings

$$G \xrightarrow[M_{GUT}]{} G_{SM} \xrightarrow[M_W]{} U(1)_{em} \times SU(3)_c$$

$$\alpha_1 = \alpha_2 = \alpha_3$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ U(1) & SU(2) & SU(3) \end{array}$$

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln \frac{E_2}{E_1}$$

particles: $m_p \le E_1$

QED

$$e(E) = e_0 + c\,e_0^3 \ln \frac{\Lambda}{E}$$

(electron)

gauge bosons

scalars

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S$$

fermions

$$b > 0 \Rightarrow \alpha(E_2) < \alpha(E_1)$$

AF: Asymptotic Freedom

$$T\,\delta_{ab} = \mathrm{Tr}\; T_a T_b$$

gauge bosons: <u>adjoint</u>

$$A \to U A U^{+} + \text{gauge dependence}$$

<u>SU(N)</u>

$$\boxed{T_{GB} = N}$$

<u>Proof:</u>

$$A_N = N \times \bar{N} \qquad SU(N)$$

$$\longrightarrow \text{fund.}$$

$$A_{N+1} = (N+1) \times (\overline{N+1})$$

$$= N \times \bar{N} + N + \bar{N} + 1$$

$$T(A_N) = T(N \times \bar{N}) = N \quad \text{assumption}$$

$$T(A_{N+1}) = T(N \times \bar{N}) + T(N) + T(\bar{N})$$
$$= N + \frac{1}{2} + \frac{1}{2} = N+1$$

Q.E.D.

$$G_{GUT} = SU(5)$$

- $$A = \underbrace{A_{SM}}_{\text{glue} + W, Z, A} + \underbrace{(X, Y)}_{M_{GUT}}$$

gauge bosons

do not see

- fermions $= \underbrace{\bar{5}_F + 10_F}_{\text{SM fermions}}$

- scalars:

$$5_H = \begin{pmatrix} T \\ \phi \end{pmatrix}$$

heavy

no run

$$24_H = \underbrace{8_c + 3_w + 1_s}_{} + \text{“eaten”}$$

no run

$$\left\{ \begin{array}{c} m_8 \propto m_3 \propto m_1 \propto M_{GUT} \\ \text{“naively”}: \ \underline{\text{no}} \text{ run} \end{array} \right\}$$

$\Downarrow$

$\boxed{\text{only SM particles run}}$

$\boxed{\text{"naive"}}$

SM: $m_e \propto M_W$ <u>but</u>

$(m_e \propto y_e M_W)$, $y_e \ll 1$

electron runs at $E \ll M_W$

$$\boxed{\begin{gathered}\Rightarrow \quad m_p \propto M_{GUT} \text{ does } \underline{\text{not}} \\ \text{imply } m_p = M_{GUT}\end{gathered}}$$

f: $\quad q = \begin{pmatrix} u \\ d \end{pmatrix}_L , \quad l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

$u_R , \quad d_R , \quad e_R$

s: $\quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ = doublet

$\Downarrow$

SU(3)

$$T_{GB} = 3$$

$$T_f = \underbrace{\underset{\substack{\uparrow \\ u_L}}{\frac{1}{2}} + \underset{\substack{\uparrow \\ d_L}}{\frac{1}{2}} + \underset{\substack{\uparrow \\ u_R}}{\frac{1}{2}} + \underset{\substack{\uparrow \\ d_R}}{\frac{1}{2}}}_{1 \text{ generation}} = 2$$

$$\Rightarrow \boxed{T_f = 2\, n_g}$$

$$b_3 = \frac{11}{3}\cdot 3 - \frac{2}{3}\, 2 n_g = \frac{33-4n_g}{3}$$

$$\boxed{b_3^{SM} = 7}$$

<u>SU(2)</u>

$$T_f = \underbrace{\underset{\substack{\uparrow \\ q(\alpha)}}{\frac{3}{2}} + \underset{\substack{\uparrow \\ \ell}}{\frac{1}{2}}}_{1\ \text{gen}} = 2$$

$$\Rightarrow \boxed{T_f = 2 n_g}$$

$$b_2 = \frac{11}{3} \cdot 2 - \frac{4}{3} n_g - \frac{1}{3} \cdot \underbrace{\frac{1}{2}}_{\phi}$$

$$\boxed{b_2 = \frac{22 - 4 n_g}{3} - \frac{1}{6}}$$

$$b_2 = \frac{44 - 8 n_g - 1}{6}$$

$$\boxed{b_2^{SM} = \frac{19}{6}}$$

<u>$U(1)$</u>

$$G_{SM} = \underbrace{SU(2)}_{\alpha_2} \times \underbrace{U(1)}_{\alpha'} \times \underbrace{SU(3)_c}_{\alpha_3}$$

Normalization of gauge couplings

$$5 = \begin{pmatrix} T_c \\ \\ D_W \end{pmatrix}$$

$$\left.\begin{aligned} T_{3c}\,(5) &= \frac{1}{2} \\ T_{3w}\,(5) &= \frac{1}{2} \end{aligned}\right\}$$

$$D_\mu = \partial_\mu - i g\, T_3^{\,w} A_{3\mu}^{w} - i g\, T_3^{\,c} A_{3\mu}^{c}$$

$$- i g' \frac{Y}{2} B_\mu$$

$$\Downarrow$$

$$\sqrt{\; g' \frac{Y}{2} = g_1 I_1}$$

$$\therefore \quad \underbrace{Tr\, I_1^2 = Tr\, T_{3c}^2 = T_{3W}^2}_{\text{complete } SU(5) \text{ repr.}}$$

$$\alpha = \frac{g^2}{4\pi} \quad \therefore \quad \underbrace{\alpha_1 = \alpha_2 = \alpha_3}_{M_{GUT}}$$

Q. Is $g' = g_1$? Is g' well renormalized?

A. 5 of $SU(5)$

$$Tr\left(\frac{Y}{2}\right)^2 (5) = ?$$

$$\bar{5}_F = \begin{pmatrix} d^c \\ \hline \ell \end{pmatrix}_L \leftarrow \text{anti down}$$

$$\frac{Y}{2}(d^c) = Q_{em}(d^c) = \frac{1}{3}$$

$$\frac{Y}{2}(\ell) = (Q - T_3)\ell = -\frac{1}{2}$$

$$\Downarrow$$

$$\mathrm{Tr}\left(\frac{Y}{2}\right)^2(5) = \underbrace{\frac{1}{9}\cdot 3}_{d^c} + \underbrace{\frac{1}{4} + \frac{1}{4}}_{\ell}$$

$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

$$\Downarrow$$

but: $\mathrm{Tr}\, I_1^2 = \mathrm{Tr}\, T_{3c}^2 = \mathrm{Tr}\, T_{3w}^2 = \frac{1}{2}$

$$\mathrm{Tr}\left(\frac{Y}{2}\right)^2 = \frac{5}{3} \cdot \frac{1}{2} = \frac{5}{3}\, \mathrm{Tr}\, I_1^2$$

$$\Downarrow$$

$$\boxed{\frac{Y}{2} = \sqrt{\frac{5}{3}} \cdot I_1}$$ with

$$g' \frac{Y}{2} = g_1 I_1$$

$$\Rightarrow \boxed{g' = \sqrt{\frac{3}{5}}\, g_1}$$

Q. Is $\alpha_1 = \alpha_{em}$?

A. NO

Proof $\qquad \bar{5}_F = \begin{pmatrix} d^c \\ \begin{pmatrix} \nu \\ e \end{pmatrix} \end{pmatrix}$

$$\mathrm{Tr}\, Q_{em}^2 = \frac{1}{9} \cdot 3 + 0 + 1 = \frac{4}{3}$$

$$\neq \frac{1}{2}$$

$$\mathrm{Tr}\, Q_{em}^2 \neq \mathrm{Tr}\, T_{3w}^2 = \mathrm{Tr}\, T_{3c}^2$$

Q_{em} $\neq$ properly normalized

$\boxed{\text{Q.E.D.}}$

SU(5)

$$D_\mu = \partial_\mu - i g_5 T_i A_\mu^i$$

$$i = 1, \dots, 24$$

$$\therefore \boxed{\mathrm{Tr}\, T_i^2 = \frac{1}{2}}$$

$$\boxed{I_1 = T_{24}} \propto \frac{Y}{2}$$

$$\Downarrow$$

$$\frac{1}{\alpha_1(M_{GUT})} = \frac{1}{\alpha_1(M_W)} + \frac{b_1}{2\pi} \ln \frac{M_{GUT}}{M_W}$$

$$\parallel$$

$$\frac{1}{\alpha_2(M_{GUT})} = \frac{1}{\alpha_3(M_{GUT})} = \frac{1}{\alpha_5}$$

$$b_1 \neq b'$$

$$I_1 = \sqrt{\frac{3}{5}}\ \frac{Y}{2}$$

$$\Rightarrow \boxed{b_1 = \frac{3}{5}\, b'}$$

$$b' \longleftrightarrow \boxed{Tr\left(\frac{Y}{2}\right)^2}$$

f: $Tr\left(\frac{Y}{2}\right)^2 =$

$$= \frac{1}{6^2}\cdot 2\cdot 3 + \frac{1}{4}\cdot 2 + \frac{4}{9}\cdot 3 + \frac{1}{9}\cdot 3 + 1$$

$$\underbrace{}_{\ell} \qquad \underbrace{}_{\ell} \quad \underbrace{}_{u_R} \quad \underbrace{}_{d_R} \quad \underbrace{}_{e_R}$$

$$= \underbrace{\frac{1}{6} + \frac{1}{2}}_{} + \underbrace{4/3 + 1/3 + 1}_{8/3}$$

$$= \quad 2/3 \quad + \qquad\qquad = \frac{10}{3}$$

$$\Rightarrow \boxed{T_1^f = \frac{\cancel{3}}{5} \cdot \frac{10}{\cancel{3}} u_g = 2 u_g}$$

$S:$ $\frac{Y}{2}(\phi) = \frac{1}{2} \Rightarrow$

$$T_1^s = \frac{3}{5} \cdot \frac{1}{2} = 3/10$$

$$\Downarrow$$

$$b_1 = -\frac{2}{3}\cdot 2n_g - \frac{1}{3}\cdot 3/10$$

$$\boxed{b_1 = -\frac{4}{3}n_g - \frac{1}{10}}$$

$$\Rightarrow \boxed{b_1^{SM} = -\frac{41}{10}}$$

$$\Downarrow$$

$$\frac{1}{\alpha_1(M_{GUT})} = \frac{1}{\alpha_i(M_W)} + \frac{b_i}{2\pi}\ln\frac{M_{GUT}}{M_W}$$

$$b_3 = \frac{33 - 4n_g}{3} = 7$$

$$b_2 = \frac{22 - 4n_g}{3} - 1/6 = \frac{19}{6}$$

$$b_1 = -\frac{4n_g}{3} - 1/10 = -\frac{41}{10}$$

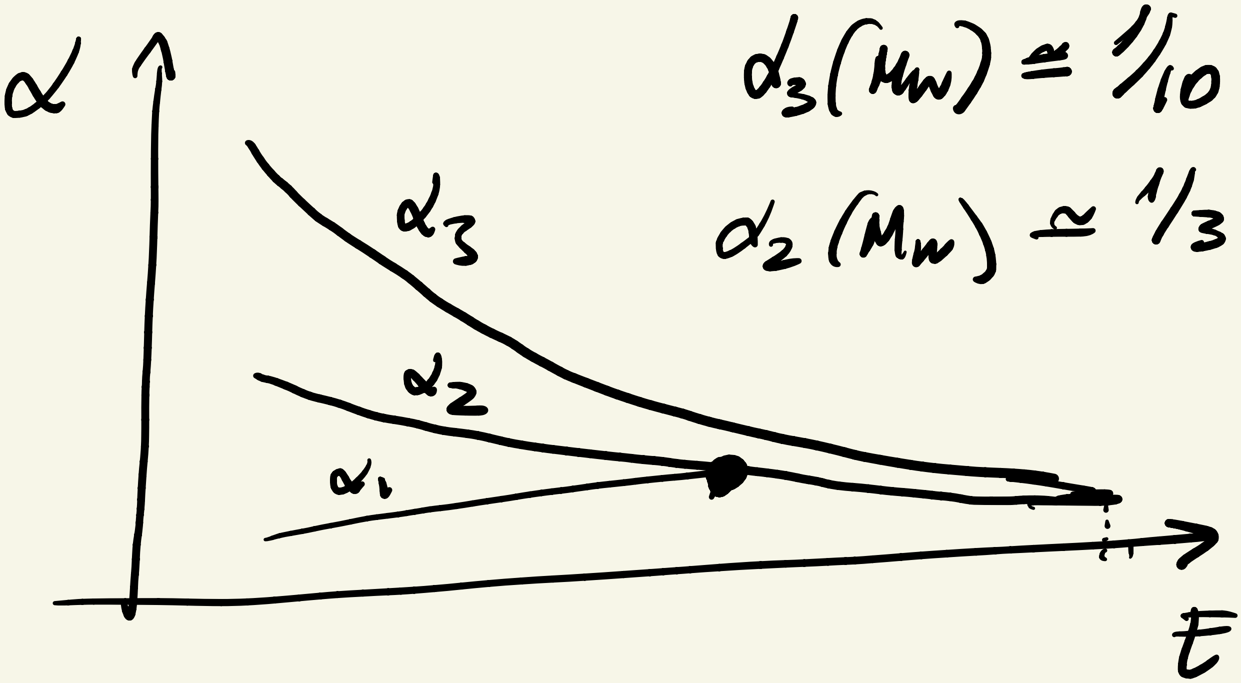
$$\Downarrow \quad \boxed{(\alpha_1 = \alpha_2 = \alpha_3)\Big|_{M_{GUT}}}$$

2-3

$$\frac{1}{\alpha_2(M_W)} - \frac{1}{\alpha_3(M_W)} = \frac{b_3 - b_2}{2\pi} \ln \frac{M_{GUT}}{M_W}$$

$$\alpha_3(M_W) \simeq 1/10$$

$$\alpha_2(M_W) \simeq 1/30$$



$$\Rightarrow \boxed{M_{GUT}^{23} \simeq (10^{16} - 10^{17})\,\mathrm{GeV}}$$

1-2

$$\frac{1}{\alpha_1(M_W)} - \frac{1}{\alpha_2(M_W)} = \frac{b_1 - b_2}{2\pi} \ln \frac{M_{GUT}}{M_W}$$

$$\Rightarrow \boxed{M_{GUT}^{12} \simeq (10^{12} - 10^{13})\,\mathrm{GeV}}$$

$$\Updownarrow$$

$$\boxed{\text{FAILURE!}}$$

$SU(5)$: couplings do <u>not</u> unify

$\boxed{\text{Naive}}$

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but: $m_T \gtrsim 10^{12}$ GeV (ignore)

$m_3 \propto m_8 \propto b\, v_{GUT}$

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carry <u>no</u> Y

$Y(3_w) = Y(8_c) = 0$

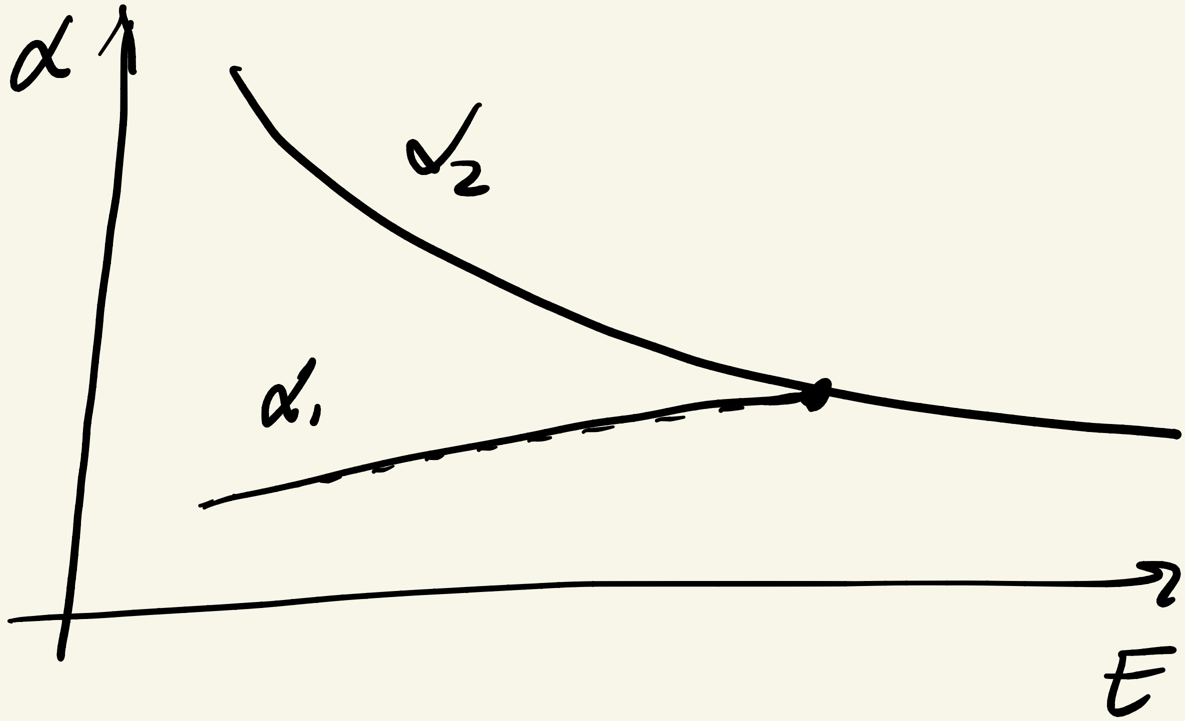
$$\frac{1}{\alpha_2(M_{GUT})} = \frac{1}{\alpha_2(M_W)} + \frac{b_2}{2\pi} \ln \frac{m_3}{M_W}$$

$$+ \frac{b_2'}{2\pi} \ln \frac{M_{GUT}}{m_3}$$

$$\frac{1}{\alpha_3(M_{GUT})} = \frac{1}{\alpha_3(M_W)} + \frac{b_3}{2\pi} \ln \frac{m_8}{M_W}$$

$$+ \frac{b_3'}{2\pi} \ln \frac{M_{GUT}}{m_3}$$

$$\frac{1}{\alpha_1(M_{GUT})} = \text{old one}$$



Q. Is it possible to slow down α_1?

A. NO! (needs g.l.)

$\Downarrow$

$\boxed{\text{slow down } \alpha_2 \text{!}}$

$\Downarrow$

3_w should be as light as possible!

$$b_2' = b_2 - \underset{t}{\frac{1}{3}\cdot\cancel{\frac{1}{2}\cdot 2}} = \overset{SM}{\frac{19}{6}} - \frac{2}{6}$$

3_w = real repr.

$\Rightarrow$ **NOT enough**

$\therefore \quad \overset{12}{M_{GUT}} \leq 10^{14}\ \text{GeV}$

~~too~~ low!

weak mixing angle

$$\sin^2\theta_W = \frac{g'^2}{g^2+g'^2} \Leftarrow \tan\theta_W = \frac{g'}{g}$$

$$= \frac{\frac{3}{5}g_1^2}{g^2+\frac{3}{5}g_1^2} = \frac{\frac{3}{5}\alpha_1}{\alpha_2+\frac{3}{5}\alpha_1}$$

$$\Downarrow$$

$$\boxed{\sin^2\theta_W\Big|_{M_{GUT}} = \frac{\frac{3}{5}}{8/5} = \frac{3}{8}}$$

$$\Downarrow$$

$$\sin^2\theta_W(M_W) = \sin^2\theta_W(M_{GUT})$$

$$+ \underbrace{c \ln \frac{M_{GUT}}{M_W}}$$

$$\boxed{c < 0}$$

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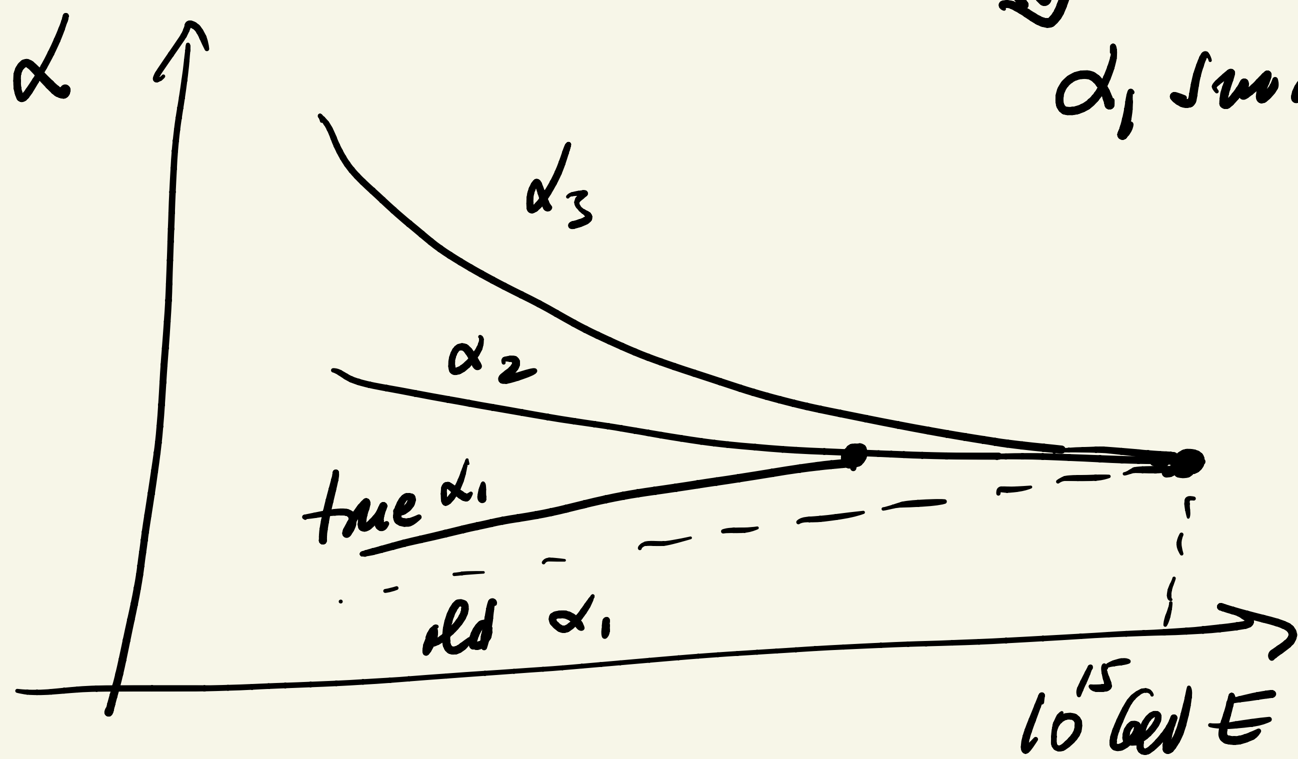
late '70's – '80's

$$\sin^2\theta_W^{\exp}(M_W) \simeq 0.2$$

$\Rightarrow$ α_1 smaller!



$$\Rightarrow \boxed{\begin{array}{c}\text{unification} + \\ \tau_p \leq 10^{32}\,\text{yr}\end{array}}$$

$$\frac{1}{\alpha_i(M_{GUT})} = \frac{1}{\alpha_i(M_W)} + \frac{b_i}{2\pi} \ln \frac{M_{GUT}}{M_W}$$

$$\Downarrow$$

$$\boxed{f(\alpha_1, \alpha_2, \alpha_3)\Big|_{M_W} = 0}$$

$$\Downarrow$$

$$\sin^2\theta_W(M_W) = f(\alpha_1, \alpha_2, \alpha_3)$$

* DERIVE *