Neutrino Mass and Grand Unification

Lecture VI

5/11/2021

LMU
Fall 2021

QCD: $SU(3)_c$ gauge theory of strong interactions

q^α $\qquad \alpha = 1, 2, 3$

(red, yellow, blue)

$$\Delta^{++} = u^\alpha u^\beta u^\gamma \underbrace{\epsilon_{\alpha\beta\gamma}}_{\text{anti-symmetric}}$$

$\boxed{\text{1966 Nambu}}$

$\Downarrow$

$$q \longrightarrow \underset{\substack{\shortparallel \\ 3\times 3}}{U_c}\, q \qquad \left[\begin{array}{l} U_c U_c^\dagger = U_c^\dagger U_c = 1 \\ \det U = 1 \end{array}\right]$$

$\Leftrightarrow$ SU(3) symmetry

$$U = e^{iH} \qquad H = H^\dagger, \ \mathrm{Tr}\, H = 0$$

$$H = \theta_a \underbrace{T_a}_{\text{generators}} \ , \quad a = 1, \cdots, 8$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$T_a \equiv \frac{\lambda_a}{2} \qquad \text{Gell-Mann}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}$$

generates $SU(2) \subseteq SU(3)$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\boxed{\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}}$$

$$\boxed{\operatorname{Tr} T_a T_b = \frac{1}{2} \delta_{ab}}$$

<u>Cartan</u> $\{T_i,\ [T_i, T_j] = 0\}$

# of gen in Cartan = rank

$$r(SU(2)) = 1$$

$$r(SU(3)) = 2$$

$$\boxed{SU(2) \times U(1) \subseteq SU(3)}$$

$\Updownarrow$

$$[T_8, T_{1,2,3}] = 0$$

SU(3) gauge:

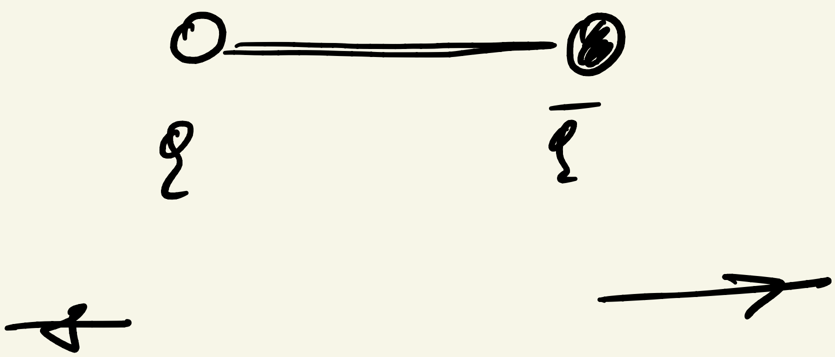
$$\theta_a = \theta_a(x)$$

$$\boxed{\mathcal{L}_{QCD} = i\sum_q \bar{q}\,\gamma^\mu D_\mu q - m_q \bar{q} q \qquad (m_q = y_q \langle \Phi \rangle) \qquad -\frac{1}{4} G_{\mu\nu}^{a} G^{\mu\nu a}}$$

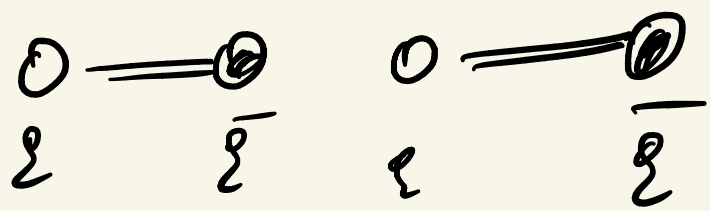
$$G_{\mu\nu}^{a} = \partial_\mu A_\nu^{a} - \partial_\nu A_\mu^{a} + \boxed{g_s f_{abc} A_\mu^{b} A_\nu^{c}}$$

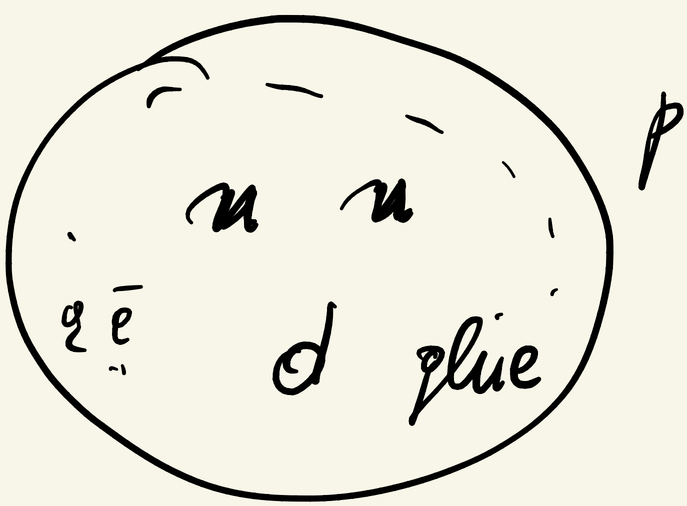
$$D_\mu = \partial_\mu - i g_s T_a A_\mu^{a}$$

quarks = confined in hadrons



(meson)





$V(r) \propto r$

QFT

gauge coupling $\alpha = \frac{g^2}{4\pi}$

$\neq$ constant

$$\boxed{\alpha = \alpha(E)}$$

't Hooft 1972

Politzer
Gross, Wilczek
1973

QCD

$\alpha(E_2) < \alpha(E_1)$

$E_2 > E_1$

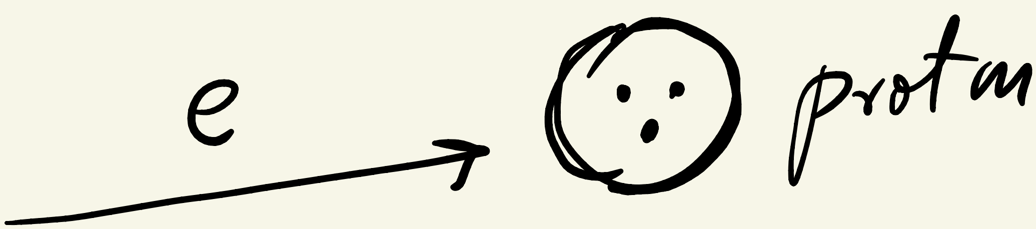
Asymptotic Freedom

"discovery" of quarks

("seeing" quarks)

Feynman '60s

Bjorken '60s



$E_e \gg m_p \simeq \text{GeV}$

Deep Inelastic Scattering

$r \ll m_p^{-1}$

experiment $\Rightarrow$ proton is made of "free" quarks

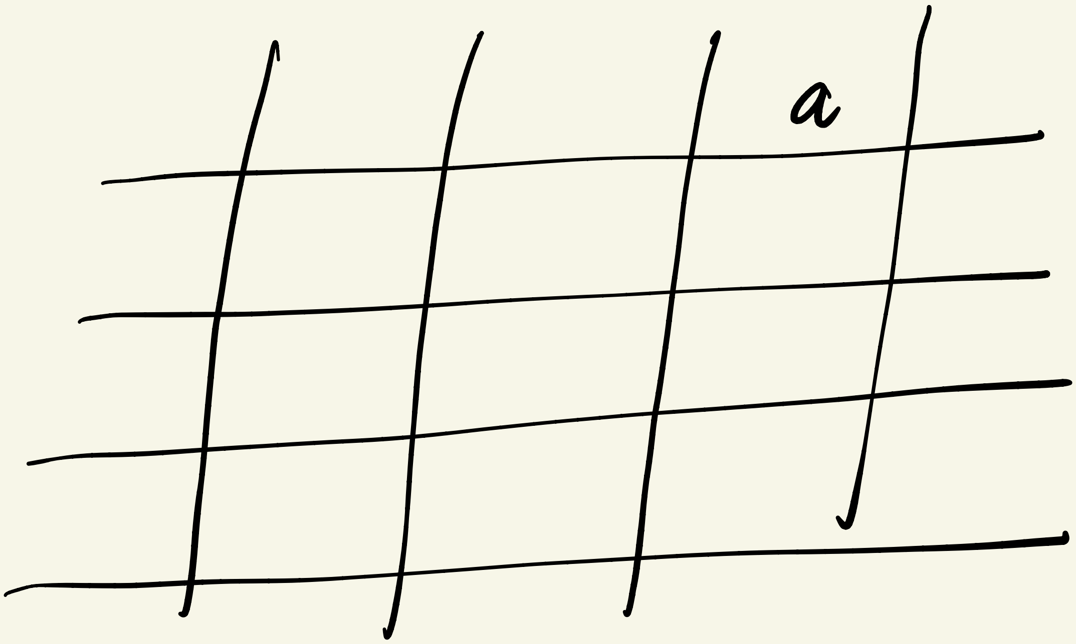
QCD at high E (LHC)

= beautiful, perturbative theory

$$\alpha_s(M_W) \simeq 1/10 \qquad (\alpha_3)$$

$$\alpha_W(M_W) \simeq 1/30 \qquad (\alpha_2)$$

$$\alpha_{em}(M_W) \simeq 1/120 \qquad (\alpha_1)$$

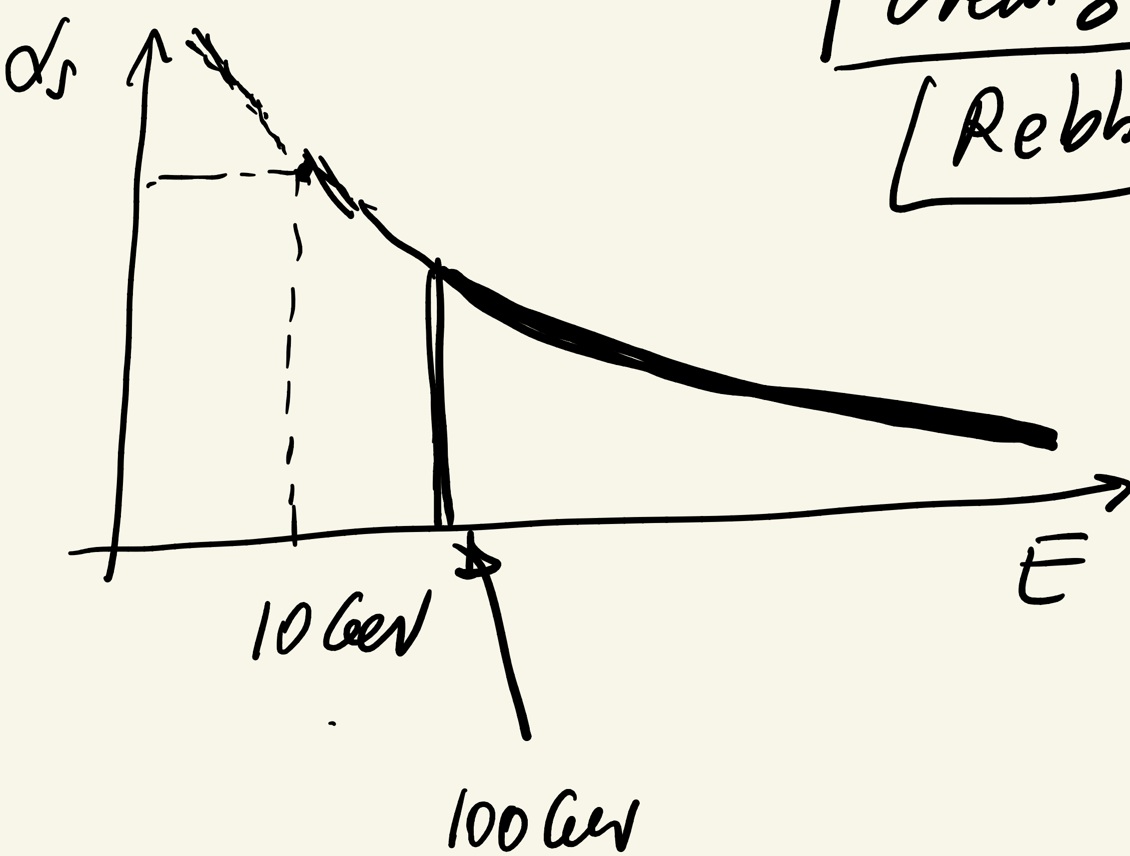


lattice

a = regulator

Creutz 1979

Rebbi



$$\underbrace{\frac{1}{\alpha(E_2)} = \frac{1}{\alpha(E_1)} + \frac{b}{2\pi} \ln E_2/E_1}_{1\text{ - loop}}$$

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S$$

gauge boson   fermion   scalar

$$\boxed{T \delta_{ab} = \mathrm{Tr}\, T_a T_b}$$

<u>fundamental</u> $T_{fund} = 1/2$

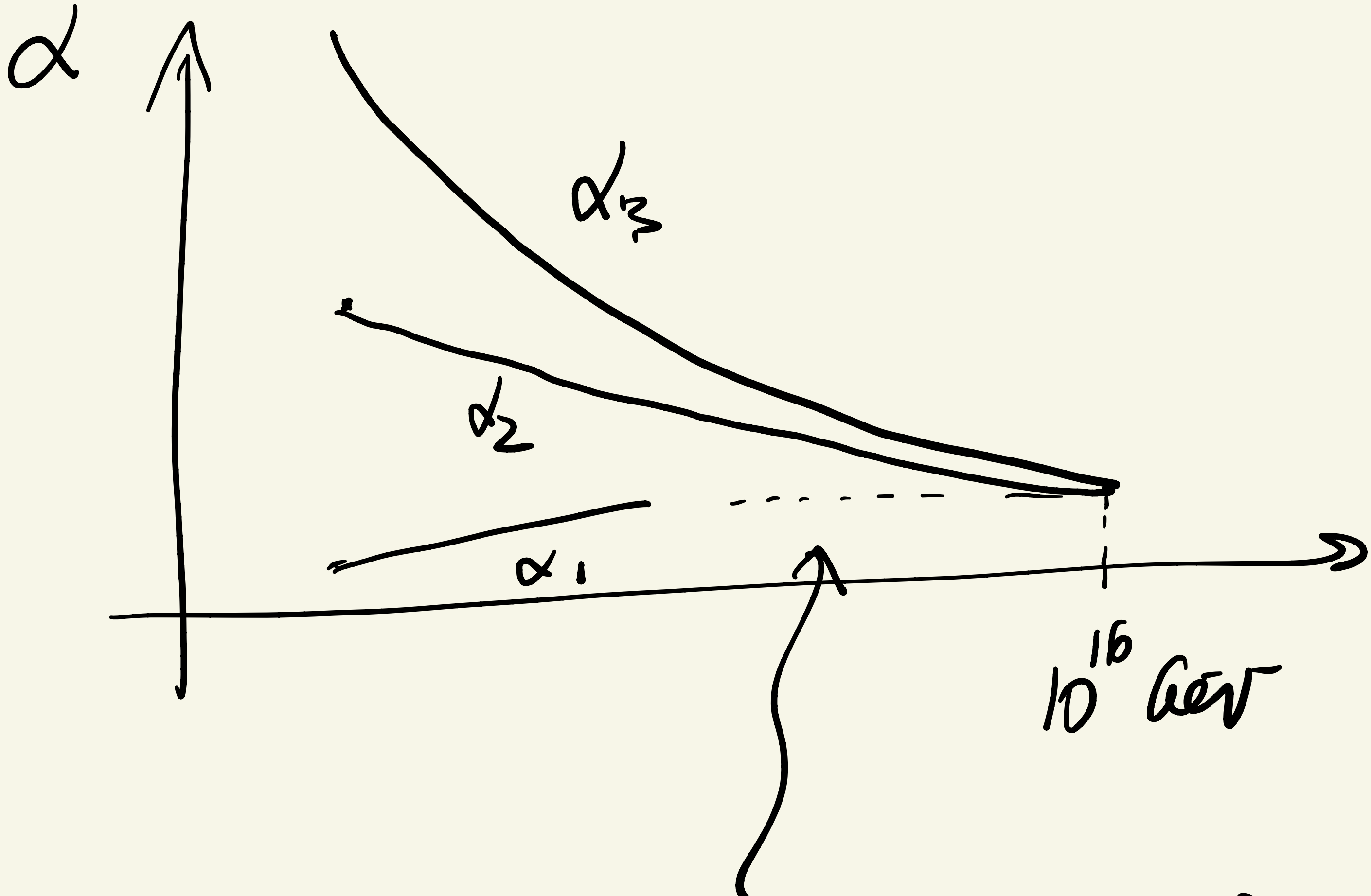
• quarks (u, d) = light

$$m_q \ll m_p$$

$m_p = ?$ source?

$$\boxed{\text{kinetic energy of quarks and gluons}}$$

$$\Downarrow$$



Grand Unification ?

why unity?

(i) why not?

(ii) $SU(2)_{ew}$ theory & stat

$\Downarrow$

- $g = e$

- $\mathrm{Tr}\, T_a = 0 \Rightarrow \mathrm{Tr}\, Q_{em} = 0$

  $(Q_{em} = c_a T_a)$

$\Rightarrow$ miracle of charge quantization

<u>alas</u> $SU(2) \times U(1)$ ugly, arbitrary

$\Rightarrow$ <u>no</u> Q_{em} quant.

but $QCD = SU(3)$?

$$\underbrace{SU(2) \times SU(3)}$$

minimal group $= SU(5)$

$\underline{SU(2)}$: $\begin{pmatrix} u \\ d \end{pmatrix}$ & $\begin{pmatrix} \nu \\ e \end{pmatrix}$

$SU(3)$: $\begin{pmatrix} q^r \\ q^y \\ q^b \end{pmatrix}$

$$\left.\begin{pmatrix} q^r \\ q^y \\ q^b \\ \hdashline \nu \\ e \end{pmatrix}\right\} \underline{5 \text{ dim}}$$

$r(SU(5)) = 4$ A

$$U_5 = e^{i\theta_a T_a} \qquad a = 1, \dots, 24$$

$\Rightarrow$ 4 diagonal gen.

$$r(SM) = r(SU(2)) + r(SU(3)) + r(U(1))$$
$$= \underbrace{1 + 2 + 1}_{4}$$

$$\boxed{Q_{em} = \sum_{SU(5)} c_i T_i}$$

$\Downarrow$

$$\mathrm{Tr}\, Q_{em} = 0 \qquad (\mathrm{Tr}\, T_i = 0)$$

$\Downarrow$

$$\underbrace{GUT} \Rightarrow \mathrm{Tr}\, Q_{em} = 0$$

Grand Unified Theory

$$\boxed{Q_{em} \text{ is quantized}}$$

$\underline{SU(5)}$ = minimal theory

Georgi, Glashow
1974

$$T_a = \frac{\lambda_a}{2} \qquad a = 1, \dots, 24$$

$$\Downarrow$$

$$\begin{cases} \lambda_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & & & \\ \vdots & & & 0 & \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ \vdots & & 0 \end{pmatrix} \\ \lambda_3 = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \end{cases}$$

λ_4 (σ_1 in 1-3), λ_5 (σ_2 in 1-3)

$$\boxed{\sigma_1, \sigma_2 \longrightarrow 20 \text{ matrices}}$$

$$\left.\begin{matrix} 1-2; & 1-3; & 1-4; & 1-5 \\ & 2-3; & 2-4; & 2-5 \\ & & 3-4; & 3-5 \\ & & & 4-5 \end{matrix}\right\} \boxed{10 \times 2}$$

20

$$\boxed{\begin{array}{c}\text{2D off-diagonal gen} \\ (\sigma_1, \sigma_2)\end{array}}$$

$$+$$

$$\boxed{\text{Cartan}} \quad \boxed{r = 4}$$

$$\lambda_3 = \left(\begin{array}{ccc|cc} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ \hline & & & 0 & \\ & & & & 0 \end{array}\right) \leftarrow \text{color space}$$

$$\lambda_? = \left(\begin{array}{ccc|cc} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ \hline & & & 1 & \\ & & & & -1 \end{array}\right) \leftarrow \text{flavor space}$$

$$\lambda_8 = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -2 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \frac{1}{\sqrt{3}}$$

$$\lambda_{24} = \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ \hline & & & -3/2 & \\ & & & & -3/2 \end{array}\right) \quad N$$

$$\boxed{\text{4 gen. of Cartan}}$$

$$\lambda_{??} = \cancel{\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix}}$$

$SU(N)$

$\{\sigma_1, \sigma_2\}$ ← how many? $\left.\right\}$ $\underbrace{N^2-1}_{\text{gen}}$

Cartan : $N-1$

$\boxed{SU(5)}$ $\boxed{\text{Fermions} = \text{"matter"}}$

$$\begin{pmatrix} \cdot \\ ? \\ \hline \nu \\ e \end{pmatrix}_L \quad \begin{array}{l} \} \; SU(3) \text{ (no weak)} \\ \} \; SU(2)_L \text{ (no color)} \end{array}$$

quark or $\boxed{\underbrace{\text{anti-quark?}}_{\text{singlet under weak}}}$

$\boxed{SM}$ $\begin{pmatrix} u \\ d \end{pmatrix}_L$ $(u^c)_L, (d^c)_L$

u_R, d_R

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $e_R, (e^c)_L$

$$f \longrightarrow f^c \equiv C\bar{f}^T$$

fermion anti-fermion

$$C\bar{f_R}^T = i\gamma_2\gamma_0\left(f_R^\dagger\gamma^0\right)^T$$

$$= i\gamma_2 f_R^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$\Downarrow \quad = \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix} \leftarrow \boxed{LH}$$

$$\boxed{C\,\bar{f}_R^{\,T} = (f^c)_L}$$

$$\boxed{\begin{array}{lcl} d_R & \longleftrightarrow & (d^c)_L \\ u_R & \longleftrightarrow & (u^c)_L \end{array}}$$

$$\Downarrow$$

$(d^c)_L$ = weak singlet

$(u^c)_L$ = weak singlet

$$\Downarrow$$

$SU(5)$

$$\left(\begin{array}{c} d^c, u^c \\ \hline \nu \\ e \end{array}\right)_L \Big\} ?????$$

d^c or u^c

$$\Rightarrow\!\!\!\!/ \quad (d^c)_L \;!!!$$

$$\boxed{\begin{array}{c} \mathrm{Tr}\, Q_{em} = 0 \iff \\ \sum\limits_{repr} q_{em} = 0 \end{array}}$$

$$\Downarrow$$

$$\bar{5}_F = \left(\begin{array}{c} (d^c)^r \\ (d^c)^y \\ (d^c)^b \\ \hdashline \nu \\ e \end{array}\right)_L$$

$$\boxed{Q_e + 3\, Q_{d^c} = 0}$$