Fate of the false vacuum: Semiclassical theory*

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It is possible for a classical field theory to have two homogeneous stable equilibrium states with different energy densities. In the quantum version of the theory, the state of higher energy density becomes unstable through barrier penetration; it is a false vacuum. This is the first of two papers developing the qualitative and quantitative semiclassical theory of the decay of such a false vacuum for theories of a single scalar field with nonderivative interactions. In the limit of vanishing energy density between the two ground states, it is possible to obtain explicit expressions for the relevant quantities to leading order in $\hbar$; in the more general case, the problem can be reduced to solving a single nonlinear ordinary differential equation.

I. INTRODUCTION

Consider the quantum field theory of a single scalar field in four-dimensional space-time with nonderivative interactions[1]

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi)\ . \tag{1.1}$$

Let U possess two relative minima, $\phi_\pm$, only one of which, ϕ_-, is an absolute minimum (see Fig. 1). The state of the classical field theory for which $\phi = \phi_-$ is the unique classical state of lowest energy, and, at least in perturbation theory, corresponds to the unique vacuum state of the quantum theory. The state of the classical field theory for which $\phi = \phi_+$ is a stable classical equilibrium state. However, it is rendered unstable by quantum effects, in particular, by barrier penetration. It is a false vacuum. This is the first of two papers developing the quantitative theory of the decay of such false vacuums; the second will be written with Curtis Callan.[2]

(To my knowledge, the first attempt to develop such a quantitative theory is the beautiful paper of Voloshin, Kobzarev, and Okun.[3] At many points the theory developed here duplicates their conclusions; however, there are significant disagreements. I have chosen to write a self-contained description of the theory in this paper, and to discuss the similarities and differences with the work of Voloshin *et al.* in an appendix.)

The qualitative features of such decay processes have long been understood.[4] They closely parallel the nucleation processes of statistical physics, the crystallization of a supersaturated solution or the boiling of a superheated fluid. Imagine Fig. 1 to be a plot of the free energy of a fluid as a function of density. The false vacuum corresponds to the superheated fluid phase and the true vacuum to the vapor phase. Thermodynamic fluctuations are continually causing bubbles of the vapor phase to materialize in the fluid phase. If the bubble is too small, the gain in volume energy caused by the materialization of the bubble is more than compensated for by the loss in surface energy, and the bubble shrinks to nothing. However, once in a while, a bubble is formed large enough so that it is energetically favorable for the bubble to grow. Once this occurs, the bubble expands until it converts the available fluid to vapor.

An identical picture describes the decay of the false vacuum, with quantum fluctuations replacing thermodynamic ones. Once in a while, a bubble of true vacuum will form large enough so that it is classically energetically favorable for the bubble to grow. Once this happens, the bubble spreads throughout the universe converting false vacuum to true.

Thus the thing to compute is the probability of decay of the false vacuum per unit time per unit volume, Γ/V. Of course, such a computation would be bootless were it not for cosmology. An infinitely old universe must be in a true vacuum, no matter how slowly the false vacuum decays. However, the universe is not infinitely old. At the time of the big bang, the energy per unit volume was very high, and the state of the universe was very far from any vacuum, true or false. As the universe expanded and cooled down, it might well have settled into a false vacuum instead of a true one. The relevant parameter for describing future events is that cosmic time for which the product of Γ/V and the four-volume of the past light cone becomes of order unity. If this time is on the order of milliseconds, the universe is still

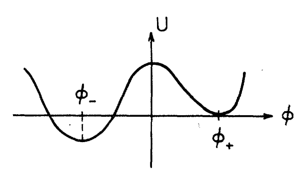

FIG. 1. The nonderivative part of the Lagrangian, $U(\phi)$, for a theory with a false vacuum.

hot when the false vacuum decays, even on the scale of high-energy physics, and the zero-temperature computation of Γ/V is inapplicable. If this time is on the order of years, the decay of the false vacuum will lead to a sort of secondary big bang with interesting cosmological consequences. If this time is on the order of 10^9 yr, we have occasion for anxiety.

As we shall see, the expression for Γ/V is of the form

$$\Gamma/V = A e^{-B/\hbar}[1 + O(\hbar)], \tag{1.2}$$

where A and B depend on the theory under study. This paper deals exclusively with the theory of the coefficient B; the theory of A is somewhat more complicated and will be dealt with in the second paper.

Section II discusses the general formalism for studying barrier-penetration effects in field theory. The discussion is on a heuristic level; the formalism will be more carefully founded in the second paper. Section III applies this formalism to a scalar field theory with nonderivative interactions. Even though we are dealing with barrier penetration in a system with an infinite number of degrees of freedom, the coefficient B can be expressed in terms of the solution of a single ordinary differential equation. Section IV solves this equation in the limit of a small energy-density difference between the two vacuums. Section V discusses the evolution of the bubble of true vacuum after it materializes.

II. BARRIER PENETRATION IN MANY DIMENSIONS

Consider a particle of unit mass moving in one dimension,

$$L = \tfrac{1}{2}\dot{q}^2 - V(q), \tag{2.1}$$

where the potential, V, is as sketched in Fig. 2. (Note that I have chosen the zero of energy such that the point of classical stable equilibrium, q_0, is a zero of V.) As we all know, there is no quantum-mechanical stable equilibrium state corresponding to the classical stable equilibrium. In

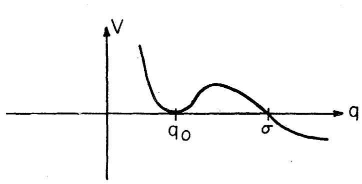

FIG. 2. $V(q)$, the potential for a particle theory with a false ground state.

semiclassical language (a good approximate description for small $\hbar$), the particle penetrates the potential barrier and materializes at the escape point, σ, with zero kinetic energy, after which it propagates classically. The width associated with this process is given by an expression of the form

$$\Gamma = A e^{-B/\hbar}[1 + O(\hbar)], \tag{2.2}$$

where

$$B = 2\int_{q_0}^{\sigma} dq\,(2V)^{1/2}. \tag{2.3}$$

To my knowledge, the generalization of this description to a particle moving in many dimensions was first given by Banks, Bender, and Wu.[5] We assemble the coordinates of the particle into a vector, $\vec{q}$. The Lagrangian is

$$L = \tfrac{1}{2}\dot{\vec{q}}\cdot\dot{\vec{q}} - V(\vec{q}). \tag{2.4}$$

Just as in one dimension, V is assumed to have a local minimum at some point, $\vec{q}_0$, and the zero of the energy is chosen such that $V(\vec{q}_0)$ vanishes. Of course, σ, the single other zero of V in the one-dimensional case, is replaced by a surface of zeros, Σ. According to Banks, Bender, and Wu,

$$B = 2\int_{\vec{q}_0}^{\vec{\sigma}} ds\,(2V)^{1/2}, \tag{2.5}$$

where

$$(ds)^2 = d\vec{q}\cdot d\vec{q}, \tag{2.6}$$

$\vec{\sigma}$ is some point on Σ, and the integral is over that path for which B is a minimum,

$$\delta\int_{\vec{q}_0}^{\vec{\sigma}} ds\,(2V)^{1/2} = 0. \tag{2.7}$$

That is to say, the particle penetrates the barrier along the path of least resistance. After penetrating the barrier, the particle emerges at $\vec{\sigma}$ with zero kinetic energy and propagates classically.

It will be convenient to cast these formulas in another form more convenient for our purposes. It is well known that the solutions to the variational problem

$$\delta\int ds\,[2(E - V)]^{1/2} = 0, \tag{2.8}$$

with fixed end points, are the paths in configuration space traced out by solutions to the Euler-Lagrange equations

$$\frac{d^2\vec{q}}{dt^2} = -\frac{\partial V}{\partial \vec{q}}, \tag{2.9}$$

with

$$\frac{1}{2}\frac{d\vec{q}}{dt}\cdot\frac{d\vec{q}}{dt} + V = E. \tag{2.10}$$

The variational problem (2.7) is of precisely this form except that E is zero, the sign of V is reversed, and the end point $\vec{\sigma}$ is not fixed but is free to vary along the surface Σ. For the moment, let us ignore this last point and fix $\vec{\sigma}$. Then the solutions to (2.7) are the paths in configuration space traced out by the solutions to the differential equation

$$\frac{d^2\vec{q}}{d\tau^2}=\frac{\partial V}{\partial \vec{q}}\ , \tag{2.11}$$

with

$$\frac{1}{2}\frac{d\vec{q}}{d\tau}\cdot\frac{d\vec{q}}{d\tau}-V=0\ . \tag{2.12}$$

Note that Eq. (2.11) is the imaginary-time version of Eq. (2.9); it is obtained by the formal substitution $\tau=it$. It is the Euler-Lagrange equation for the imaginary-time version of Hamilton's principle,

$$\delta\int d\tau\, L_E=0\ , \tag{2.13}$$

where

$$L_E=\frac{1}{2}\frac{d\vec{q}}{d\tau}\cdot\frac{d\vec{q}}{d\tau}+V\ . \tag{2.14}$$

(The subscript E stands for "Euclidean." In field theory, the passage from real to imaginary time takes us from Minkowski space to Euclidean space.)

By Eq. (2.12) the classical equilibrium point, q_0, can only be reached asymptotically, as τ goes to minus infinity,

$$\lim_{\tau\to-\infty}\ \vec{q}=\vec{q}_0\ . \tag{2.15}$$

By time translation invariance, we might as well choose the imaginary time at which the particle reaches $\vec{\sigma}$ to be $\tau=0$. At this time, again by Eq. (2.12), $dq/d\tau$ vanishes:

$$\left.\frac{d\vec{q}}{dt}\right|_0=\vec{0}\ . \tag{2.16}$$

Yet again by Eq. (2.12),

$$\int_{\vec{q}_0}^{\vec{\sigma}} ds\,(2V)^{1/2}=\int_{-\infty}^{0} d\tau\, L_E\ . \tag{2.17}$$

By Eq. (2.16), the variation of this expression with respect to changes in the end point $\vec{\sigma}$ vanishes; thus the condition imposed at the beginning of the preceding paragraph, that $\vec{\sigma}$ is fixed, can be dropped.

Equation (2.16) also tells us that the motion of the particle for positive τ is just the time reversal of its motion for negative τ; the particle simply bounces off Σ at $\tau=0$ and returns to $\vec{q}_0$ at $\tau=+\infty$. Thus I call this motion "the bounce." The coefficient B is the total Euclidean action for the bounce,

$$B=\int_{-\infty}^{\infty} d\tau\, L_E\equiv S_E\ . \tag{2.18}$$

Thus, to find the coefficient B, we need only find the bounce, the solution of the imaginary-time equations of motion obeying the boundary conditions (2.15) and (2.16). [Equation (2.12) is a consequence of the equations of motion and Eq. (2.15).] This simple prescription needs three qualifying comments: (1) The bounce must really reach the surface Σ at $\tau=0$, that is to say, a position from which a classical particle, released at rest, can escape to infinity. Thus the trivial solution to the equations, constant $\vec{q}$, is not allowed. (2) There may be several bounces; in this case, the preferred bounce is the one of minimum Euclidean action. (3) There may even be many bounces with the same Euclidean action. (Typically, we expect this to occur as a consequence of some symmetry of the theory.) In this case, we must sum the contributions to Γ of all these bounces (integrate over the symmetry group); of course, this only affects the coefficient A.

III. BARRIER PENETRATION IN FIELD THEORY

It is the work of a moment to translate the prescription of the preceding section to the field-theory problem described in the Introduction: The Euclidean (imaginary-time) equation of motion is

$$\left(\frac{\partial^2}{\partial\tau^2}+\nabla^2\right)\phi=U'(\phi), \tag{3.1}$$

where the prime denotes differentiation with respect to ϕ, the boundary conditions for the bounce are

$$\lim_{\tau\to\pm\infty}\ \phi(\tau,\vec{x})=\phi_+, \tag{3.2}$$

and

$$\frac{\partial\phi}{\partial\tau}(0,\vec{x})=0\ . \tag{3.3}$$

The coefficient B is given by

$$B=S_E=\int d\tau\, d^3x\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2+\frac{1}{2}(\vec{\nabla}\phi)^2+U\right]. \tag{3.4}$$

Finally, it is easy to see that for B to be finite it is necessary that

$$\lim_{|\vec{x}|\to\infty}\ \phi(\tau,x)=\phi_+\ . \tag{3.5}$$

This last condition is also consistent with the qualitative description of vacuum decay given in the Introduction: Quantum fluctuations make a bubble appear someplace; far from this place, the false vacuum persists undisturbed.

No nontrivial solution of these equations is invariant under spatial translations. Thus, any spatial translation of a solution is also a solution, with the same Euclidean action, and to obtain the total width of the false vacuum we must integrate over the group of spatial translations, as explained at the end of Sec. II. This is the formal reason why a factor of the volume of space necessarily appears in the expression for the total width, Eq. (1.2); the physical reason was explained in the Introduction.

It is not difficult to guess the form of the solution of these equations, for they are all consistent with the assumption that ϕ is invariant under four-dimensional Euclidean rotations. To be precise, if we define ρ by

$$\rho=(\tau^2+|\vec{x}|^2)^{1/2}\ , \tag{3.6}$$

then the assumption is that ϕ is a function of ρ only.

Under this assumption, Eq. (3.1) becomes

$$\frac{d^2\phi}{d\rho^2}+\frac{3}{\rho}\frac{d\phi}{d\rho}=U'(\phi)\ , \tag{3.7}$$

Eqs. (3.2) and (3.5) become a single equation,

$$\lim_{\rho\to\infty}\phi(\rho)=\phi_+\ , \tag{3.8}$$

and Eq. (3.4) becomes

$$B=S_E=2\pi^2\int_0^\infty \rho^3 d\rho\left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2+U\right]\ . \tag{3.9}$$

Also,

$$\left.\frac{d\phi}{d\rho}\right|_0=0\ . \tag{3.10}$$

Otherwise, ϕ would be singular at the origin of coordinates.

I will shortly analyze Eqs. (3.7)–(3.10) and show that they always have a solution, that is to say, that the system always admits an O(4)-invariant bounce. I will assume that if there are any O(4)-noninvariant bounces, they have higher Euclidean action than the O(4)-invariant bounce, and thus can be safely ignored. (*Note added in proof.* I recently proved this assumption; the proof will appear in a Physical Review Comment.)

Now for the analysis: If we interpret ϕ as a particle position and ρ as time, Eq. (3.9) is the mechanical equation for a particle moving in a potential *minus U* and subject to a somewhat peculiar viscous damping force with Stokes's-law coefficient inversely proportional to the time. The particle is released at rest at time zero [Eq. (3.10)]; we wish to show that if the initial position is properly chosen, the particle will come to rest at time infinity at ϕ_+, that is to say, on top of the right-hand hill in Fig. 3.

I shall demonstrate this by showing that if the particle is released to the right of ϕ_-, and is sufficiently close to ϕ_-, it will overshoot and pass ϕ_+ at some finite time; on the other hand, if it is released sufficiently far to the right of ϕ_-, it will undershoot and never reach ϕ_+; thus, by continuity, there must be an intermediate initial position for which it just comes to rest at ϕ_+.

To demonstrate undershoot is trivial. If the particle is released to the right of ϕ_1, it does not have enough energy to climb the hill to ϕ_+. The viscous damping force does not affect this argument, because viscous damping always diminishes the energy:

$$\frac{d}{d\rho}\left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2-U\right]=-\frac{3}{\rho}\left(\frac{d\phi}{d\rho}\right)^2\leqslant 0\ . \tag{3.11}$$

To demonstrate overshoot requires a little more work. For ϕ very close to ϕ_- we may safely linearize Eq. (3.7):

$$\left(\frac{d^2}{d\rho^2}+\frac{3}{\rho}-\mu^2\right)(\phi-\phi_-)=0\ , \tag{3.12}$$

where

$$\mu^2\equiv U''(\phi_-)\ . \tag{3.13}$$

The solution to Eq. (3.12) is

$$\phi(\rho)-\phi_-=2[\phi(0)-\phi_-]I_1(\mu\rho)/\mu\rho\ . \tag{3.14}$$

Thus, if we choose ϕ to be initially sufficiently close to ϕ_-, we can arrange for it to stay arbitrarily close to ϕ_- for arbitrarily large ρ. But for sufficiently large ρ, the viscous damping force can be neglected, since its coefficient is inversely proportional to ρ. But if we neglect the viscous damping, the particle overshoots. Q.E.D.

IV. THE THIN-WALL APPROXIMATION

Let us consider a symmetric function of ϕ, $U_+(\phi)$,

$$U_+(\phi)=U_+(-\phi), \tag{4.1}$$

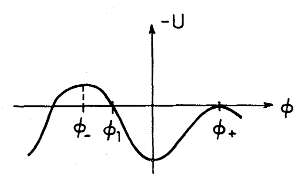

FIG. 3. The potential energy for the mechanical analogy to Eq. (3.7).

with minima at some points $\pm a$,

$$U_+'(\pm a)=0\ . \tag{4.2}$$

Also, let us define

$$\mu^2=U_+''(\pm a)\ . \tag{4.3}$$

For example, a function of this form is

$$U_+=\frac{\lambda}{8}\left(\phi^2-\frac{\mu^2}{\lambda}\right)^2\ . \tag{4.4}$$

For this example, $a^2=\mu^2/\lambda$.

Now let us add to U_+ a small term that breaks the symmetry,

$$U=U_++\frac{\epsilon}{2a}(\phi-a)\ , \tag{4.5}$$

where ϵ is a positive number. This now defines a theory of the class we have been discussing; to lowest nontrivial order in ϵ,

$$\phi_\pm=\pm a, \tag{4.6}$$

and ϵ is the energy-density difference between the true and false vacuums.

I shall now show that in the limit of small ϵ it is possible to compute the coefficient B in closed form. From the arguments of Sec. III, it is easy to see the qualitative form of the bounce for small ϵ. In order not to lose too much energy, we must choose $\phi(0)$, the initial position of the particle in our mechanical analogy, very close to ϕ_-. The particle then stays close to ϕ_- until some very large time, $\rho=R$. Near time R, the particle moves quickly through the valley in Fig. 3, and slowly comes to rest at ϕ_+ at time infinity. Translating from mechanical language back into field-theoretic language, the bounce looks like a large four-dimensional spherical bubble of radius R, with a thin wall separating the false vacuum without from the true vacuum within.

To go on, we need more information about the wall of the bubble. For ρ near R, we can neglect the viscous damping term in Eq. (3.7), and we can also neglect the ϵ-dependent term in U. We thus obtain the familiar equation for what is sometimes called a soliton in a one-dimensional field theory,

$$\frac{d^2\phi}{dx^2}=U_+'(\phi)\ , \tag{4.7}$$

where x is the spatial variable in the one-dimensional theory.

The properties of this equation have been extensively discussed in the recent literature,[6] and I will simply summarize them here. The fundamental solution is an odd function of x, $\phi_1(x)$, defined by

$$x=\int_0^{\phi_1}\frac{d\phi}{[2U_+(\phi)]^{1/2}}\ . \tag{4.8}$$

For this solution, the one-dimensional action is given by

$$S_1=\int dx\left[\frac{1}{2}\left(\frac{d\phi_1}{dx}\right)^2+U_+\right]$$

$$=\int_{-a}^{a}d\phi[2U(\phi)]^{1/2}\ . \tag{4.9}$$

For $\mu|x|\gg 1$,

$$\phi_1=\pm(a-Ke^{-\mu|x|})\ , \tag{4.10}$$

where K is a constant which depends on the detailed form of U_+. For example, for the theory defined by Eq. (4.4),

$$\phi_1=a\tanh(\tfrac{1}{2}\mu x)\ , \tag{4.11}$$

$$S_1=\mu^3/3\lambda\ , \tag{4.12}$$

and

$$K=2a. \tag{4.13}$$

In terms of ϕ_1, we can express analytically our approximate description of the bounce,

$$\begin{aligned}\phi&=-a,\quad \rho\ll R\\ &=\phi_1(\rho-R),\quad \rho\approx R\\ &=a,\quad \rho\gg R\ .\end{aligned} \tag{4.14}$$

The only thing missing from this description is the value of R. This is easily obtained by a variational computation:

$$S_E=2\pi^2\int_0^\infty\rho^3d\rho\left[\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2+U\right]$$

$$=-\tfrac{1}{2}\pi^2R^4\epsilon+\pi^2R^3S_1\ . \tag{4.15}$$

(The first term comes from the interior of the bubble, the second term from the wall.) Varying with respect to R, we obtain

$$\frac{dS_E}{dR}=0=-2\pi^2R^3\epsilon+6\pi^2R^2S_1\ . \tag{4.16}$$

Hence,

$$R=3S_1/\epsilon\ . \tag{4.17}$$

Note that, consistent with our qualitative picture, R does indeed go to infinity as ϵ goes to zero. We can use Eq. (4.17) to give a more precise condition for the validity of our approximation,

$$\mu R=3S_1\mu/\epsilon\gg 1\ . \tag{4.18}$$

We can also use it to compute

$$B=S_E=27\pi^2S_1^4/2\epsilon^3\ . \tag{4.19}$$

This is the desired closed-form expression for the coefficient B in the limit of small ϵ.

For the example of Eq. (4.4), the condition for the validity of the approximation is

$$\mu^4/\epsilon\lambda \gg 1\,, \tag{4.20}$$

and the approximate expression for B is

$$B = \frac{\pi^2\mu^{12}}{6\epsilon^3\lambda^4}\,. \tag{4.21}$$

V. THE FATE OF THE FALSE VACUUM

In Sec. II, I gave the semiclassical description of the decay of a false ground state in particle mechanics: The classical particle makes a quantum jump from the local minimum of the potential to the escape point, $\vec{q}(\tau=0)$. When it appears at the escape point, its momentum, $\dot{\vec{q}}$, is zero. Afterwards, it propagates classically.

Mutatis mutandem, this description applies to field theory. The classical field makes a quantum jump (say at $t=0$) to the state defined by

$$\phi(t=0,\vec{x}) = \phi(\tau=0,\vec{x})\,, \tag{5.1}$$

$$\frac{\partial}{\partial t}\phi(t=0,\vec{x}) = 0\,. \tag{5.2}$$

Afterwards, it evolves according to the classical field equation,

$$-\frac{\partial^2\phi}{\partial t^2} + \nabla^2\phi = U'(\phi)\,. \tag{5.3}$$

The first of these equations implies that the same function, $\phi(\rho)$, that gives the shape of the bounce in four-dimensional Euclidean space also gives the shape of the bubble at the moment of its materialization in ordinary three-space. Indeed, it does more; because the Minkowskian field equation, (5.3), is simply the analytic continuation of the Euclidean field equation, (3.1), back to real time, the desired solution of Eq. (5.3) is simply the analytic continuation of the bounce:

$$\phi(t,\vec{x}) = \phi(\rho = (|\vec{x}|^2 - t^2)^{1/2})\,. \tag{5.4}$$

[As a consequence of Eq. (3.3), ϕ is an even function of ρ, so we need not worry about which branch of the square root to take.]

We can immediately draw some very interesting consequences from Eq. (4.4):

(1) O(4) invariance of the bounce becomes O(3, 1) invariance of the solution of the classical field equations. In other words, the growth of the bubble, after its materialization, looks the same to any Lorentz observer.

(2) In the case of small ϵ, discussed in Sec. IV, there is a thin wall, localized at $\rho = R$, separating false vacuum from true vacuum. As the bubble expands, this wall traces out the hyperboloid

$$|\vec{x}|^2 - t^2 = R^2\,. \tag{5.5}$$

(This situation is shown graphically in Fig. 4.) Typically, we would expect R to be a microphysical number, on the order of a fermi, give or take a few orders of magnitude. This means that by macrophysical standards, once the bubble materializes it begins to expand almost instantly with almost the velocity of light.

(3) As a consequence of this rapid expansion, if a bubble were expanding toward us at this moment, we would have essentially no warning of its approach until its arrival. This is also shown in Fig. 4. The stationary observer, O, cannot tell a bubble has formed until he intercepts the future light cone, W, projected from the wall at the time of its formation. A time R later, that is to say, on the order of 10^{-21} sec later, he is inside the bubble.

(4) The rapidity expanding bubble wall obviously carries a lot of energy. How much? A section of bubble wall at rest carries energy S_1 per unit area. Because any part of the bubble wall at any time is obtained from any other part by a Lorentz transformation, a section of wall expanding with velocity v carries energy $S_1(1-v^2)^{-1/2}$ per unit area. Thus, at a time when the radius of the bubble is $|\vec{x}|$, the energy of the wall is given by

$$E_{\text{wall}} = 4\pi|\vec{x}|^2 S_1(1-v^2)^{-1/2}\,. \tag{5.6}$$

By Eq. (5.5),

$$v = \frac{d|\vec{x}|}{dt} = \frac{(|\vec{x}|^2 - R^2)^{1/2}}{|\vec{x}|}\,. \tag{5.7}$$

Thus,

$$E_{\text{wall}} = 4\pi|\vec{x}|^3 S_1/R = 4\pi\epsilon|\vec{x}|^3/3\,, \tag{5.8}$$

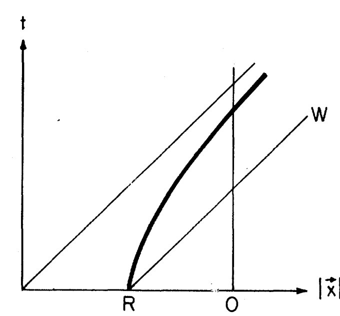

FIG. 4. A space-time diagram of the classical growth of the bubble of true vacuum after its materialization. The hyperbola is the path traced out by the bubble wall. The observer O only receives warning that the bubble is expanding toward him when he crosses the light cone W.

where I have used Eq. (4.17) at the last step. Thus, in the thin-wall approximation, *all* the energy released by converting true vacuum to false vacuum goes to accelerate the bubble wall. This refutes the naive expectation that the decay of the false vacuum would leave behind it a roiling sea of mesons. In fact, the expansion of the bubble leaves behind only the true vacuum.

This concludes what I know about the fate of the false vacuum. There remain many interesting unanswered questions:

(1) I have discussed the expansion of a bubble of true vacuum into false vacuum. What if the initial state of the world is not the false vacuum, but some state of nonzero particle density built on the false vacuum? What happens when a bubble wall encounters a particle?

(2) I have discussed spontaneous decay of the false vacuum. However, there is also the possibility of induced decay. In particular, in a collision of two particles of very high energy, there might be a non-negligible cross section for the production of a bubble. How can one estimate this cross section?

(3) If we assume that the universe starts out in a false vacuum, at some time in its expansion bubbles begin to form. Because the formation of bubbles is totally Lorentz invariant, the average distance between bubbles at their time of formation must be of the same order of magnitude as the time at which bubbles begin to appear. Because bubble walls expand with the speed of light, after a time interval of the same order of magnitude, bubble walls begin to collide. What happens then? Can such events be accommodated in the history of the early universe?

ACKNOWLEDGMENTS

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APPENDIX: COMPARISON WITH EARLIER WORK

Voloshin, Kobzarev, and Okun[3] consider the case when the energy-density difference between the two vacuums is small. They assume that the only relevant field configurations are those corresponding to a spherical bubble of true vacuum separated from false vacuum by a thin wall whose shape is given by the function I call ϕ_1. They insert this ansatz into the Lagrangian of the theory and obtain an effective Lagrangian depending only on a single dynamical variable, the radius of the bubble as a function of time. This they quantize by the standard canonical method, giving them a one-dimensional Hamiltonian which they analyze by the usual methods of one-dimensional quantum mechanics to obtain the coefficient I call B. Since all of their assumptions appear as conclusions in this paper, it is no surprise that their formula for B agrees with my Eq. (4.19).

The only advantages I claim for the method presented here are these: (1) I believe I work with fewer assumptions and thus that my derivation is more convincing. The reader may well disagree. (2) The method presented here is more suited to the computation of the next quantum corrections. The forthcoming second paper on this subject will attempt to justify this claim. (3) The method of Voloshin *et al.* obscures the Lorentz structure of the theory and this causes them to go astray at one important point. (Or so I believe; it is possible that it is *my* methods that have caused *me* to go astray. At any rate, we disagree.) They argue that they have computed only the probability for the production of a bubble at rest. By Lorentz invariance, the probability for the production of a bubble in motion must be the same, and thus to obtain the total decay width of the false vacuum it is necessary to integrate over the Lorentz group. Such an integration is, of course, divergent; in order to eliminate the divergence, Voloshin *et al.* have to introduce an *ad hoc* cutoff, related to the radius of the universe. I hold otherwise. As I argued in Sec. V, an expanding bubble looks the same to all Lorentz observers, and to integrate over the Lorentz group is to erroneously count the same final state many times.

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[1]Notation: Units are chosen such that $c=1$. The signature of the metric tensor is $(+---)$. Note Planck's constant is *not* set equal to one.

[2]These papers are those cited in Ref. 2 of P. H. Frampton [Phys. Rev. Lett. 37, 1378 (1976)]. Frampton attempts to apply the methods developed here to the problem of vacuum instability in a Weinberg-Salam model.

[3]M. B. Voloshin, I. Yu. Kobzarev, and L. B. Okun', Yad. Fiz. 20, 1229 (1974) [Sov. J. Nucl. Phys. 20, 644 (1975)].

[4]See Ref. 3 and references cited therein. I first heard about the qualitative description many years ago from K. Wilson (private communication).

Added in proof. A close parallel to the analysis presented here is the work of Langer on the droplet model in statistical physics [J. S. Langer, Ann. Phys. (N.Y.) 41, 108 (1967)]. I thank E. Brézin and A. Linde for calling this paper to my attention.

[5]T. Banks, C. Bender, and T. T. Wu, Phys. Rev. D 8, 3346 (1973); 8, 3366 (1973).

[6]See, for example, J. Goldstone and R. Jackiw, Phys. Rev. D 11, 1486 (1975), or N. Christ and T. D. Lee, *ibid.* 12, 1606 (1975).