[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

[12] M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters 17, 888 (1966).

[13] The predicted ratio [eq. (12)] from the current algebra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta\rightarrow\pi^+\pi^-\gamma)/\Gamma(\gamma\gamma)$ calculated in Refs. 12 and 14.

[14] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

A MODEL OF LEPTONS*

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Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons.[2] This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing the photon and the intermediate-boson fields as gauge fields.[3] The model may be renormalizable.

We will restrict our attention to symmetry groups that connect the observed electron-type leptons only with each other, i.e., not with muon-type leptons or other unobserved leptons or hadrons. The symmetries then act on a left-handed doublet

$$L \equiv [\tfrac12(1+\gamma_5)]\begin{pmatrix}\nu_e\\ e\end{pmatrix} \qquad (1)$$

and on a right-handed singlet

$$R \equiv [\tfrac12(1-\gamma_5)]e. \qquad (2)$$

The largest group that leaves invariant the kinematic terms $-\bar{L}\gamma^\mu\partial_\mu L - \bar{R}\gamma^\mu\partial_\mu R$ of the Lagrangian consists of the electronic isospin $\vec{T}$ acting on L, plus the numbers N_L, N_R of left- and right-handed electron-type leptons. As far as we know, two of these symmetries are entirely unbroken: the charge $Q = T_3 - N_R - \frac12 N_L$, and the electron number $N = N_R + N_L$. But the gauge field corresponding to an unbroken symmetry will have zero mass,[4] and there is no massless particle coupled to N,[5] so we must form our gauge group out of the electronic isospin $\vec{T}$ and the electronic hyperchange $Y \equiv N_R + \frac12 N_L$.

Therefore, we shall construct our Lagrangian out of L and R, plus gauge fields $\vec{A}_\mu$ and B_μ coupled to $\vec{T}$ and Y, plus a spin-zero doublet

$$\varphi = \begin{pmatrix}\varphi^0\\ \varphi^-\end{pmatrix} \qquad (3)$$

whose vacuum expectation value will break $\vec{T}$ and Y and give the electron its mass. The only renormalizable Lagrangian which is invariant under $\vec{T}$ and Y gauge transformations is

$$\mathcal{L} = -\tfrac14(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu\times\vec{A}_\nu)^2 - \tfrac14(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \bar{R}\gamma^\mu(\partial_\mu - ig'B_\mu)R - L\gamma^\mu(\partial_\mu ig\vec{t}\cdot\vec{A}_\mu - i\tfrac12 g'B_\mu)L$$
$$-\tfrac12|\partial_\mu\varphi - ig\vec{A}_\mu\cdot\vec{t}\varphi + i\tfrac12 g'B_\mu\varphi|^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L) - M_1^{\,2}\varphi^\dagger\varphi + h(\varphi^\dagger\varphi)^2. \qquad (4)$$

We have chosen the phase of the R field to make G_e real, and can also adjust the phase of the L and Q fields to make the vacuum expectation value $\lambda \equiv \langle\varphi^0\rangle$ real. The "physical" φ fields are then φ^-

and

$$\varphi_1 \equiv (\varphi^0 + \varphi^{0\dagger} - 2\lambda)/\sqrt{2} \quad \varphi_2 \equiv (\varphi^0 - \varphi^{0\dagger})/i\sqrt{2}. \quad (5)$$

The condition that φ_1 have zero vacuum expectation value to all orders of perturbation theory tells us that $\lambda^2 \cong M_1^2/2h$, and therefore the field φ_1 has mass M_1 while φ_2 and φ^- have mass zero. But we can easily see that the Goldstone bosons represented by φ_2 and φ^- have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates φ^- and φ_2 everywhere[6] without changing anything else. We will see that G_e is very small, and in any case M_1 might be very large,[7] so the φ_1 couplings will also be disregarded in the following.

The effect of all this is just to replace φ everywhere by its vacuum expectation value

$$\langle\varphi\rangle = \lambda\binom{1}{0}. \quad (6)$$

The first four terms in $\mathcal{L}$ remain intact, while the rest of the Lagrangian becomes

$$-\tfrac{1}{8}\lambda^2 g^2[(A_\mu^{\ 1})^2 + (A_\mu^{\ 2})^2] - \tfrac{1}{8}\lambda^2(gA_\mu^{\ 3} + g'B_\mu)^2 - \lambda G_e \bar{e}e. \quad (7)$$

We see immediately that the electron mass is λG_e. The charged spin-1 field is

$$W_\mu \equiv 2^{-1/2}(A_\mu^{\ 1} + iA_\mu^{\ 2}) \quad (8)$$

and has mass

$$M_W = \tfrac{1}{2}\lambda g. \quad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu^{\ 3} + g'B_\mu), \quad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu^{\ 3} + gB_\mu). \quad (11)$$

Their masses are

$$M_Z = \tfrac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \quad (12)$$

$$M_A = 0, \quad (13)$$

so A_μ is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

$$\frac{ig}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu W_\mu + \text{H.c.} + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{i(g^2+g'^2)^{1/2}}{4}\left[\left(\frac{3g'^2-g^2}{g'^2+g^2}\right)\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e + \bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]Z_\mu. \quad (14)$$

We see that the rationalized electric charge is

$$e = gg'/(g^2 + g'^2)^{1/2} \quad (15)$$

and, assuming that W_μ couples as usual to hadrons and muons, the usual coupling constant of weak interactions is given by

$$G_W/\sqrt{2} = g^2/8M_W^{\ 2} = 1/2\lambda^2. \quad (16)$$

Note that then the e-φ coupling constant is

$$G_e = M_e/\lambda = 2^{1/4}M_e G_W^{\ 1/2} = 2.07\times10^{-6}.$$

The coupling of φ_1 to muons is stronger by a factor M_μ/M_e, but still very weak. Note also that (14) gives g and g' larger than e, so (16) tells us that $M_W > 40$ BeV, while (12) gives $M_Z > M_W$ and $M_Z > 80$ BeV.

The only unequivocal new predictions made by this model have to do with the couplings of the neutral intermediate meson Z_μ. If Z_μ does not couple to hadrons then the best place to look for effects of Z_μ is in electron-neutron scattering. Applying a Fierz transformation to the W-exchange terms, the total effective e-ν interaction is

$$\frac{G_W}{\sqrt{2}}\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\left\{\frac{(3g^2-g'^2)}{2(g^2+g'^2)}\bar{e}\gamma^\mu e + \tfrac{3}{2}\bar{e}\gamma^\mu\gamma_5 e\right\}.$$

If $g \gg e$ then $g \gg g'$, and this is just the usual e-ν scattering matrix element times an extra factor $\frac{3}{2}$. If $g \simeq e$ then $g \ll g'$, and the vector interaction is multiplied by a factor $-\frac{1}{2}$ rather than $\frac{3}{2}$. Of course our model has too many arbitrary features for these predictions to be taken very seriously, but it is worth keeping in mind that the standard calculation[8] of the electron-neutrino cross section may well be wrong.

Is this model renormalizable? We usually do not expect non-Abelian gauge theories to be renormalizable if the vector-meson mass is not zero, but our Z_μ and W_μ mesons get their mass from the spontaneous breaking of the symmetry, not from a mass term put in at the beginning. Indeed, the model Lagrangian we start from is probably renormalizable, so the question is whether this renormalizability is lost in the reordering of the perturbation theory implied by our redefinition of the fields. And if this model is renormalizable, then what happens when we extend it to include the couplings of $\vec{A}_\mu$ and B_μ to the hadrons?

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[1]The history of attempts to unify weak and electromagnetic interactions is very long, and will not be reviewed here. Possibly the earliest reference is E. Fermi, Z. Physik 88, 161 (1934). A model similar to ours was discussed by S. Glashow, Nucl. Phys. 22, 579 (1961); the chief difference is that Glashow introduces symmetry-breaking terms into the Lagrangian, and therefore gets less definite predictions.

[2]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[3]P. W. Higgs, Phys. Letters 12, 132 (1964), Phys. Rev. Letters 13, 508 (1964), and Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Letters 13, 585 (1964).

[4]See particularly T. W. B. Kibble, Phys. Rev. 155, 1554 (1967). A similar phenomenon occurs in the strong interactions; the ρ-meson mass in zeroth-order perturbation theory is just the bare mass, while the A_1 meson picks up an extra contribution from the spontaneous breaking of chiral symmetry. See S. Weinberg, Phys. Rev. Letters 18, 507 (1967), especially footnote 7; J. Schwinger, Phys. Letters 24B, 473 (1967); S. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967), Eq. (13) et seq.

[5]T. D. Lee and C. N. Yang, Phys. Rev. 98, 101 (1955).

[6]This is the same sort of transformation as that which eliminates the nonderivative $\vec{\pi}$ couplings in the σ model; see S. Weinberg, Phys. Rev. Letters 18, 188 (1967). The $\vec{\pi}$ reappears with derivative coupling because the strong-interaction Lagrangian is not invariant under chiral gauge transformation.

[7]For a similar argument applied to the σ meson, see Weinberg, Ref. 6.

[8]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1957).

SPECTRAL-FUNCTION SUM RULES, ω-φ MIXING, AND LEPTON-PAIR DECAYS OF VECTOR MESONS*

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Within the framework of vector-meson dominance, the current-mixing model is shown to be the only theory of ω-φ mixing consistent with Weinberg's first sum rule as applied to the vector-current spectral functions. Relations among the leptonic decay rates of ρ^0, ω, and φ are derived, and other related processes are discussed.

We begin by considering Weinberg's first sum rule[1] extended to the $(1+8)$ vector currents of the eightfold way[2]:

$$\int dm^2[m^{-2}\rho_{\alpha\beta}^{(1)}(m^2)+\rho_{\alpha\beta}^{(0)}(m^2)]=S\delta_{\alpha\beta}+S'\delta_{\alpha 0}\delta_{\beta 0}, \tag{1}$$