Neutrino Mass and Grand Unification

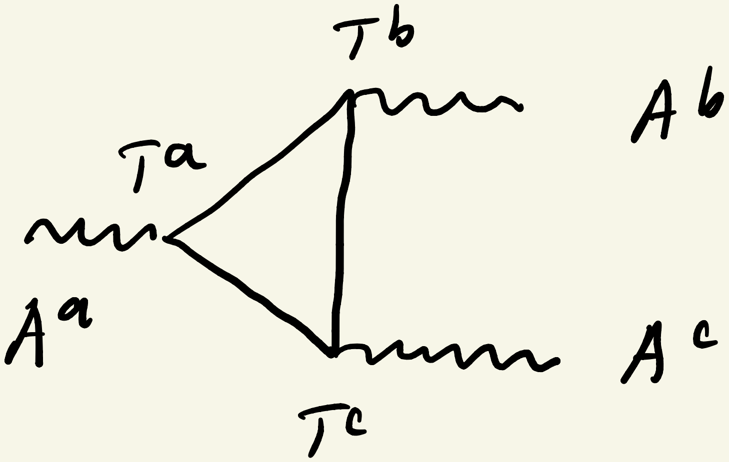
Lecture XXI

14/11/2022

LMU

Winter 2022

Anomalies (gauge) II



$$\boxed{\mathcal{A}_{abc}^{(R)} \propto \mathrm{Tr}\,\{T_a, T_b\}\,T_c = d_{abc}\,\mathcal{A}(R)}$$

$$\mathcal{A}_{abc}(F) = d_{abc}\,1$$

$$\Longleftrightarrow \boxed{\mathcal{A}(F) = 1}$$

$F \to UF \quad (T_a{}^F)$

$$\boxed{\mathcal{A}(\bar{F}) = -1}$$

$$\bar{F} \to U^* \bar{F}$$

$$T_a^{\bar{F}} = -T_a^T$$

$$(F_1)_L + (\bar{F}_2)_L \Rightarrow \mathcal{A}_{TOT} = 1 - 1 = 0$$

$$\Leftrightarrow \quad F_{1L} + F_{2R} \Rightarrow \mathcal{A}_{TOT} = 0$$

(no γ_5 in int.)

- $SU(2) \Rightarrow \mathcal{A} = 0$

$$\mathcal{A}(F) = \mathrm{Tr}\left\{\frac{\sigma_a}{2}, \frac{\sigma_b}{2}\right\}\frac{\sigma_c}{2} = 0$$

$\mathrm{Tr}\, T_3 = 0 \Rightarrow$ charges are quantized

- $U(1)$ $\quad Q_1$ = arbitrary

$$\mathrm{Tr}\, Q_1 \overset{\downarrow}{=} 0 \quad \leftarrow \text{demand}$$

- <u>SM anomaly</u>

$$(Y_q)\ q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad l \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L\ (Y_\ell)$$

$$\begin{matrix} u_R & d_R & e_R & , & \Phi \\ (Y_U) & (Y_D) & (Y_E) & & (Y_\Phi) \end{matrix}$$

. QED ≠ chiral $\Leftrightarrow$ $Q^L_{em} = Q^R_{em}$

$$\mathcal{L}_Y = y_d\ \overline{q_L}\ \Phi\ d_R + y_u\ \overline{q_L}\ i\sigma_2\ \Phi^* u_R$$

(1)

$$+ y_e \bar{\ell}_L \Phi e_R + h.c.$$

$$Q_{em} = T_3 + \frac{Y}{2}$$

$$\boxed{Y_\Phi = +1} \quad (2)$$

convention =

= normalization

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \Rightarrow \langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\mathcal{L}_Y \rightarrow y_f \bar{f}_L \phi_0 (\phi_0^*) f_R + h.c.$$

$$f = u, d, e$$

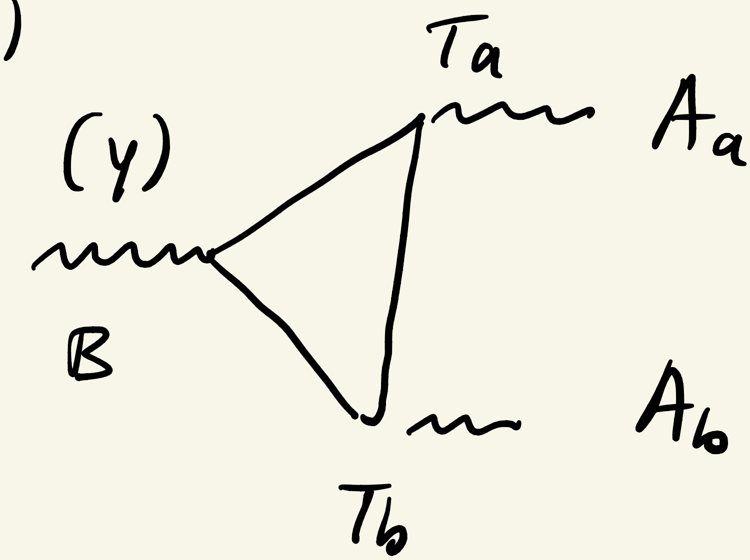
$$\Rightarrow \boxed{Q_L^{em} = Q_R^{em}} \quad (3)$$

$$(1) \Rightarrow \quad Y_D = -1 + Y_q$$

$$Y_U = +1 + Y_q \qquad (4)$$

$$Y_E = -1 + Y_l$$

(ii)



$$T_a = T_3 = T_b \Rightarrow (u, d, e, \nu)_L$$

$$\Rightarrow \quad \mathrm{Tr}\, Y = Y_q\left[\left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2\right] \cdot \underbrace{3}_{\text{color}}$$

$$\|$$

$$0 = \mathrm{Tr}\, Y_L \qquad + Y_e\left[\left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2\right]$$

$$\Rightarrow \boxed{3Y_q + Y_l = 0}$$

$$\Downarrow$$

$$\mathrm{Tr}\, Q_L = 0 \Rightarrow \mathrm{Tr}\, Q_R = 0$$

$$\Rightarrow \mathrm{Tr}\, Y_R = 0$$

$$(Y_U + Y_D)3 + Y_E = 0$$

$$+ (4)$$

$$\Downarrow$$

$$\boxed{Y_q = \frac{1}{3}} \quad \boxed{Y_l = -1}$$

$$\Downarrow$$

$$\boxed{Q_\nu^{em} = 0}$$

$$Q_e = +3\, Q_d$$

$$Q_u = -2\, Q_d$$

SM consistency $\Rightarrow$ charge quantisation

for SM q and l

new $\underbrace{f_L' + f_R'}$

if $T_a'^L = {}'T_a^R$

$Y_L' = Y_R'$

$\Rightarrow$ anomaly cancelled

$\boxed{\cdot \text{ all true for } m_\nu = 0}$

$\boxed{\cdot\ m_\nu \neq 0} \Rightarrow \exists\, \nu_R \longleftrightarrow \exists\, e_R$

(Y_N)

$\Downarrow$

by def. $\Delta \mathcal{L}_Y = y_\nu\, \bar{l}_L\, i\sigma_2\, \Phi^* \nu_R$ $\quad$ (5)

$+ h.c.$

$\Rightarrow \quad T_a\, \nu_R = 0 \Leftrightarrow SU(2)$ singlet

(5) $\Rightarrow \boxed{Y_N = +1 + Y_\ell}$

$\Downarrow$

$$\mathrm{Tr}\, Y_R \overset{\downarrow}{=} 0 \qquad (\text{demand})$$

$$(Y_U + Y_D)\,3 + Y_E + Y_N = 0$$

$$3\left[(\cancel{+1} + Y_q) + (\cancel{-1} + Y_q)\right] + (\cancel{-1} + Y_e) + (\cancel{+1} + Y_e) = 0$$

$$\Rightarrow (3\,Y_q + Y_e)\,2 = 0$$

$$\boxed{3\,Y_q + Y_e = 0}$$

arbitrary

$$\Rightarrow \boxed{Q_\nu^{em} = 0}$$

- If $\exists$ Majorana mass for ν_R

$$\Delta \mathcal{L}_Y = M_R \, \nu_R^T C \nu_R + h.c.$$

$$\Leftrightarrow Y_N = 0$$

$$Y(\Delta \mathcal{L}_Y) = 2 Y_N \overset{!}{=} 0$$

$$\Rightarrow \boxed{Q_N = \frac{Y_N}{2} = 0}$$

$$\Rightarrow \boxed{Y_\ell = -1 \Rightarrow Y_q = 1/3}$$

$$\boxed{\text{Equivalent} \Rightarrow \underline{\text{Principle}} \text{ of } \underline{\text{most general}} \text{ gauge invariant SM interactions}}$$

SM $\longrightarrow$ LR $\therefore$ ν_R a must

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$B_q = 1/3, \quad L_q = 0$$

$$B_\ell = 0, \quad L_\ell = 1$$

$$\boxed{T, (B-L)_L = 0} \Rightarrow \boxed{3(B-L)_q + (B-L)_\ell = 0}$$

$$+ \quad \Delta\mathcal{L}_\Delta = y_\Delta \, \ell_R^T C \, i\sigma_2 \, \Delta_R \, \ell_R + h.c.$$

$$\Delta_R \to V_R \Delta_R V_R^\dagger$$

$$\Downarrow$$

$$y_\Delta \langle\Delta_R\rangle \nu_R^T C \nu_R$$

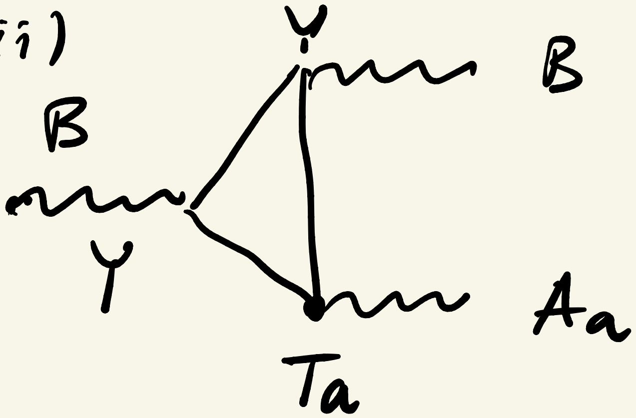
$$\parallel$$

$$M_R$$

$$\Rightarrow \boxed{Q_{\nu_R} = 0 = Q_{\nu_L}}$$

$\Leftrightarrow$ charge quantization

(ii)



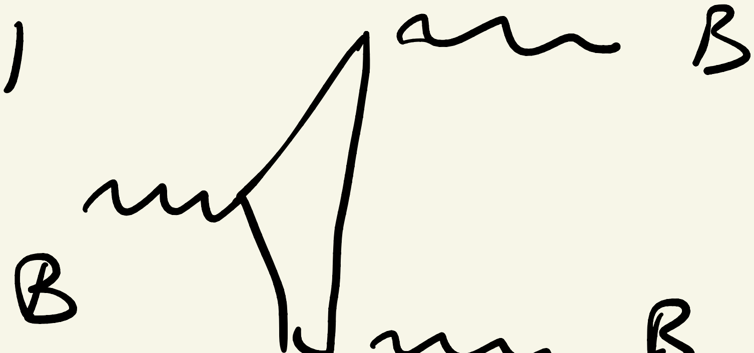
$$[Y, T_a] = 0 \quad \Leftrightarrow \quad SU(2) \times U(1)$$

$$\Rightarrow \quad Y(\text{multiplet}) = \text{fixed}$$

$$Y_q \quad (Y_{u_L} = Y_{d_L} \equiv Y_L) \quad ---$$

$$\mathrm{Tr}\, T_a \Rightarrow \mathcal{A}(ii) = 0$$

(iii)



$$\Rightarrow \boxed{\mathrm{Tr}\, Y_L^3 \overset{\downarrow}{=} \mathrm{Tr}\, Y_R^3} \ \boxed{(\text{check})}$$

$$\boxed{\text{Comment on LR}}$$

$$SU(2)_L \times SU(2)_R \times \bar{U}(1)$$
$$(\bar{Y})$$

$$L \longleftrightarrow R$$

$$Q_{em} = T_{3L} + \underbrace{T_{3R} + \frac{\bar{Y}}{2}}_{\parallel}$$
$$\equiv T_{3L} + \frac{Y}{2}$$

$$\mathrm{Tr}\, Y_L = 0 \iff \mathrm{Tr}\, \bar{Y}_L = 0$$
$$\Updownarrow$$
$$\mathrm{Tr}\, \bar{Y}_R = 0$$

$$\boxed{3\bar{Y}_q + \bar{Y}_e = 0}$$

I wrote : $\boxed{\bar{Y} = B-L}$

not true in general

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to be true $\Longleftrightarrow$ $\bar{Y}_e = -1$

$\Longleftrightarrow$ assume charge quantization

$\Longleftrightarrow$ $\nu_R =$ Majorana

---

but $\longrightarrow$ back to SM $\boxed{\text{with } \nu_R}$

$\mathrm{Tr}\, Y_L = 0 \Rightarrow 3Y_q + Y_e = 0$

$\Rightarrow \mathrm{Tr}\, Y_R = 0 \Rightarrow$ same

$$\mathrm{Tr}\, Y_L^3 \overset{\downarrow}{=} \mathrm{Tr}\, Y_R^3 \quad \underline{\text{a must}}$$

$$\mathrm{Tr}\, Y_L^3 = \left(Y_q^3 + Y_q^3\right)3 + Y_e^3 + Y_e^3$$

$$(u_L) \quad (d_L) \quad (\nu_L) \quad (e_L)$$

$$= 2\left[3 Y_q^3 + 2 Y_e^3\right]$$

$$\mathrm{Tr}\, Y_R^3 = \left[Y_U^3 + Y_D^3\right]3 + Y_E^3 + Y_N^3$$

$$= \left[(1 + Y_q)^3 + (-1 + Y_q)^3\right]3 +$$

$$+ (-1 + Y_e)^3 + (1 + Y_e)^3$$

$$= \left[Y_q^3 + Y_q^3 + Y_q (1)^2 + Y_q (-1)^2\right]3$$

$$+ \left[Y_e^3 + Y_e^3 + Y_e (1)^2 + Y_e (-1)^2\right]$$

$$= 2\left[Y_q^3\, 3 + Y_e^3\right] + 2\,\underbrace{\left[3\, Y_q + Y_e\right]}_{=0}$$

$$= \mathrm{Tr}\, Y_L^3$$

$$\boxed{\text{Q.E.D.}}$$

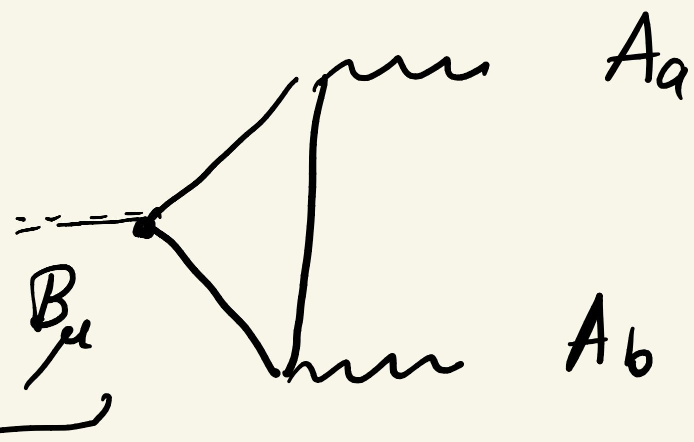
$\boxed{\text{GLOBAL ANOMALY}}$

$$\left[\ \underbrace{B,\ L}_{\text{conserved at the tree level}} \qquad \Delta B = \Delta L = 0 \ \right]$$

$$\Uparrow$$

$$\left\{ \begin{array}{ll} W[\bar{u}d + \bar{\nu}e] & \\ A[\bar{f}f] & Z[\bar{f}f] \end{array} \right\}$$

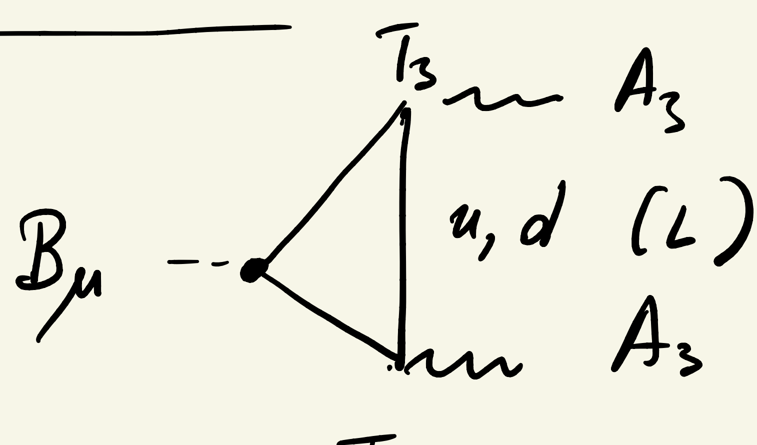
$\left[\quad h\bar{f}f \quad \right]$



B current

$$\left[\begin{array}{l} B_\mu = \sum_q \frac{1}{3}\, \bar{q}\, \gamma_\mu q \\ \\ L_\mu = \sum_\ell \bar{\ell}\, \gamma_\mu \ell \end{array}\right]$$

↑ Lepton current



$T_3 \longrightarrow$ only $L = \frac{1+\gamma_5}{2}$

$$\partial^\mu B_\mu = 3G\,\frac{1}{3}\Big[\underbrace{\left(\tfrac{1}{2}\right)^2}_{u_L} + \underbrace{\left(-\tfrac{1}{2}\right)^2}_{d_L}\Big]\underbrace{F F^d}_{SU(2)}$$

color!

$$F F^d = \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu}_a F^{\alpha\beta}_a$$

$$\boxed{\partial^\mu B_\mu = \frac{3}{6}\, G\, F F^d}$$

$\Rightarrow$ $\boxed{\text{proton} \neq \text{stable}}$

$$\boxed{\begin{aligned} F F^d &= \partial_\mu k^\mu \Leftarrow \\ k^\mu &= A^a \partial_\mu A_a + \cdots \end{aligned}}$$

- Why not write $F\tilde{F}$ in $\mathcal{L}_{QED}$?

$\Rightarrow$ Because it is a total divergence $\rightarrow$ boundary term

$\Rightarrow$ $F\tilde{F} = 0$!?

$\boxed{\text{true only in } U(1)}$

$\boxed{\text{However, in Y.M.} \neq 0}$

$\downarrow$

$\boxed{\text{NON perturbative}}$

't Hooft '1975

$\frac{1}{\alpha} \simeq 30$

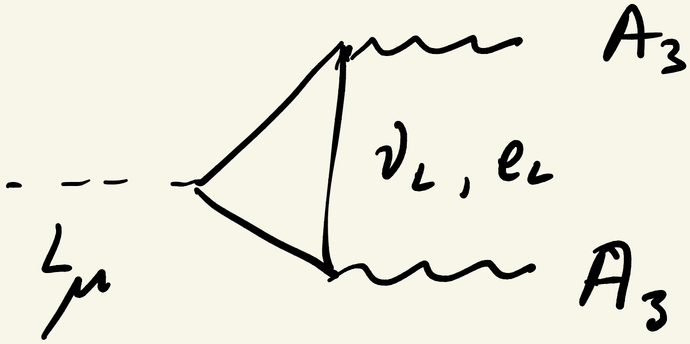
$$\boxed{\Gamma_p \propto e^{-\frac{4\pi}{\alpha}} M_W}$$

essential sing. in $\lambda \to 0$

$$\propto e^{-400} M_W \longrightarrow 0$$

$$\Rightarrow \boxed{\tau_p \simeq 10^{120}\ yr\ ?}$$

$$\infty$$



$$\partial_\mu L^\mu = C(+1)\left[\left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2\right]$$

$\times F F^d$

$$= \frac{1}{2} C F F^d$$

$$\Downarrow$$

$$\boxed{\partial_\mu B^\mu = \partial^\mu L_\mu}$$

$$\Rightarrow \boxed{\partial_\mu (B^\mu - L^\mu) = 0}$$

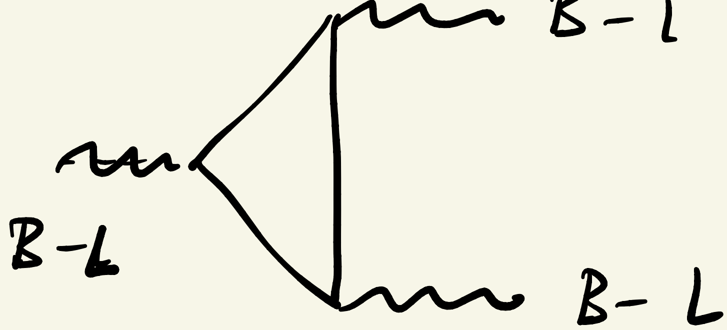
B-L = anomaly free

$$\Downarrow$$

gauge it ?

<u>example</u> $\boxed{LR \Rightarrow \bar{Y} = B - L}$

$\Rightarrow$ a new anomaly?



$$\stackrel{!}{=} 0$$

$$\Rightarrow \quad \exists \, \nu_R$$

$\boxed{\text{PROVE}}$