


Neutrino mass

Oscillations

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

Pontecovo

//

$$\nu_e = \cos \theta \nu_1 + \sin \theta \nu_2$$

$$\nu_\mu = -\sin \theta \nu_1 + \cos \theta \nu_2$$

} lepton
mixing
↕

$$\nu_i : m_i \neq 0$$

$$g_{\text{weak}} = \\ = \text{calibho}$$

↘

$$P(\nu_e \rightarrow \nu_\mu) = ?$$

- Maximal $\theta = \theta_{\max} = 45^\circ$

- $\Delta m^2 \neq 0$

a) bigger $\Delta m^2 \Rightarrow$ bigger P

b) $E \rightarrow \infty \Rightarrow P \rightarrow 0$

c) bigger $L \Rightarrow$ bigger P

$m/E \Rightarrow E \rightarrow \infty \Leftrightarrow m \rightarrow 0$

⇓

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

⇓ oscillations

$$\Delta m^2 \neq 0 \Rightarrow m_\nu \neq 0$$

$$\bullet \Delta m_A^2 \simeq 10^{-3} \text{ eV}^2 \quad (\text{atmospheric})$$

$$\theta_A \simeq 45^\circ$$

$$\bullet \Delta m_\odot^2 \simeq 10^{-5} \text{ eV}^2$$

$$\theta_\odot \simeq 30^\circ$$

$\Rightarrow$ 2 massive

$\Downarrow$
S.M. $\neq$ NOT complete

$\bullet$ if no fundamental theory



Effective theory

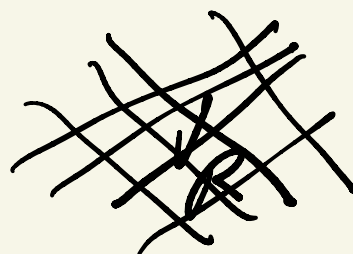
$$[\Delta L \neq 0]$$

Weinberg '79

S.M. states

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$e_R$$



$$\phi_0$$

$$\equiv (v+h)$$

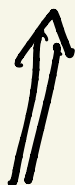
$\Downarrow$ Majorana

$$\nu_L^T C \nu_L$$

$$\frac{\phi_0 \phi_0}{\Lambda}$$



$$T_3 : \frac{1}{2} + \frac{1}{2} - \frac{1}{2}$$



$$l^T i \sigma_2 c l$$

Hint: / $\gamma_e \bar{e}_L e_R \phi_0$

$$T_3 : \underbrace{\frac{1}{2} + 0 - \frac{1}{2}}_{\text{good}} = 0$$

$\Downarrow$

$$\begin{aligned} \mathcal{L}_{eff}^{(y)} &= v_L^T c v_L \frac{\phi_0^2}{\Lambda} \\ &= v_L^T c v_L \frac{(v+h)^2}{\Lambda} \end{aligned}$$

$$\simeq \nu_L^T C \nu_L \frac{\nu^2 + 2\nu h}{\Lambda}$$

$$\boxed{m_\nu^M = \frac{\nu^2}{\Lambda}} \quad \boxed{g_\nu = \frac{\nu}{\Lambda}}$$

$$m_\nu^M \nu_L^T C \nu_L + 2 \frac{\nu}{\Lambda} h \nu_L^T C \nu_L$$

(i) Neutrino = Majorana

$$\Leftrightarrow \Delta L = 2$$

$\Rightarrow$ neutrinoless double beta decay ($0\nu 2\beta$)

$$(ii) \quad g_\nu = \frac{v}{\Lambda} = \frac{m_\nu v}{v^2} =$$

$$M_W = \frac{g}{2} v \Rightarrow \quad = \frac{g}{2} \frac{m_\nu}{M_W}$$



$$g_e = \frac{g}{2} \frac{m_e}{M_W}$$

$$m_\nu < 1 \text{ eV} \quad (*)$$

KATRIN exp.

0ν2e: $m_\nu \simeq (1/4 - 3/4) \text{ eV} ?$

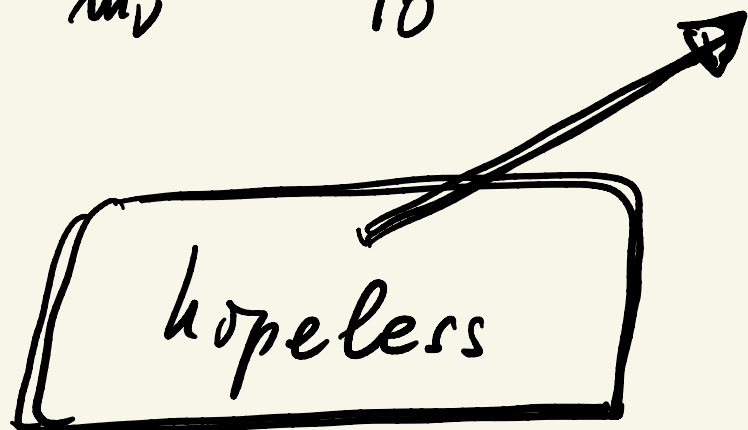
$$\Rightarrow g_\nu < \frac{10^{-9}}{100} = 10^{-11}$$

$\Downarrow$

$$h \rightarrow \nu \bar{\nu} \Rightarrow \boxed{B(h \rightarrow \nu \bar{\nu}) \leq 10^{-20}}$$

• Mass : $m_\nu = \frac{m^2}{\Lambda} \simeq \frac{M_W^2}{\Lambda}$

$$\Lambda = \frac{M_W^2}{m_\nu} \approx \frac{10^4}{10^{-9}} = 10^{13} \text{ GeV}$$

 hopeless



not $SO(2) \times U(1)$ inv.

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L; \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$$



$$\xrightarrow{\text{unitary}} \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}$$

$$\rightarrow \underbrace{(l_L^T i \sigma_2 l_L)}_{\Lambda} \underbrace{(\Phi^T i \sigma_2 \Phi)}_{\circ} \leftarrow$$

$\circ \quad \Lambda \quad \circ$



$$\Phi_i \Phi_j (i \sigma_2)_{ij} = 0$$

$$l_L^T i \sigma_2 C l_L = - l^T (i \sigma_2)^T C^T l$$

$$= - l^T (-i \sigma_2) (-C) l$$

$$= - l^T i \sigma_2 C l = 0$$

what to do?

$l = \text{doublet}, \bar{l} = \text{doublet}$



$$(1) \left(\underset{\uparrow}{l_L^T} i \sigma_2 \overset{\uparrow}{\bar{l}} \right) \overset{\uparrow}{C} \left(\overset{\uparrow}{\bar{l}^T} i \sigma_2 \underset{\uparrow}{l_L} \right) \frac{1}{\Lambda}$$

+ 2 new ways $\Leftrightarrow$ find them

$\Rightarrow$ show all three equivalent

• expand $\bar{\Phi} = \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}$

$\Downarrow$

$$l^T i \sigma_2 \bar{\Phi} = \bar{\nu}_L^T (\nu + h)$$

$\Downarrow$

$$(1) = (\nu_L^T C \nu_L) (\nu + h)^2 \frac{1}{\Lambda}$$

neutrino Yukawa

- $s=0 \quad | \uparrow \downarrow - \downarrow \uparrow \rangle$

$$\Leftrightarrow D_1^T i \sigma_2 D_2$$

- Lorentz

$$\left\{ \begin{array}{l} \psi_1^T C \psi_2 \\ C^T = -C \\ C \gamma_\mu C^{-1} = -\gamma_\mu^T \end{array} \right.$$

$$\psi^c = C(\bar{\psi})^T = \begin{pmatrix} C \\ C \end{pmatrix} \gamma_0 \psi^*$$

$$C = i \sigma_2 \gamma_0 = \begin{pmatrix} i \sigma_2 & 0 \\ 0 & -i \sigma_2 \end{pmatrix}$$

See saw mechanism

= (Pon man's) UV completion



Ultra Violet

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \parallel \quad e_R, \nu_R, \Phi$$

$\boxed{Y_{\nu_R} = 0}$

SM singlet

$$\mathcal{L}_Y = Y_e \bar{l}_L \Phi e_R +$$
$$+ Y_D \bar{l}_L i \sigma_2 \Phi^* \nu_R + h.c.$$



$$y_D \bar{\nu}_L \nu_R (v+h) + \\ + y_e \bar{e}_L e_R (v+h)$$

$$\Rightarrow m_\nu = m_0 = y_D v$$

$$m_e = y_e v$$

$$\Rightarrow y_D = \frac{m_\nu}{v} = \frac{g}{2} \frac{m_\nu}{M_W}$$

$$B(h \rightarrow \nu \bar{\nu}) \leq 10^{-20} : C$$

Forget

$\nu_R = \text{singlet}$

$$(a) \bar{\nu}_R \nu_R = 0$$

$$\nu_R = \frac{1 - \gamma_5}{2} \nu = R \nu$$

$$\bar{\nu}_R \nu_R = \nu_R^\dagger \gamma_0 \nu_L =$$

$$= (R \nu)^\dagger \gamma_0 R \nu =$$

$$= \nu^\dagger R \gamma_0 R \nu = \nu^\dagger \gamma_0 L R \nu$$

$$= 0$$

$$(ii) \underbrace{\nu_R^T C \nu_R}_{\text{Majorana term}}$$

$$\underbrace{\cancel{\nu_L^T C \nu_L}}_{SU(2) \times U(1)}$$

↓
Majorana term

⇓

$$\frac{1}{2} M_R \left(\nu_R^T C \nu_R + \nu_R^\dagger C^\dagger \nu_R^* \right)$$

$$\mathcal{L}(\nu_R) = i \bar{\nu}_R \gamma^\mu \partial_\mu \nu_R -$$

$$- \left(\frac{1}{2} \right) M_R (\nu_R^T C \nu_R + \text{h.c.})$$

$$N_H = \nu_R + C \bar{\nu}_R^T = \nu_R + i \gamma_2 \nu_R^*$$

$$C = i \gamma_2 \gamma_0$$

$$N_H = \begin{pmatrix} 0 \\ \nu_R \end{pmatrix} + \begin{pmatrix} 0 & i \gamma_2 \\ -i \gamma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \nu_R^* \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \nu_R \end{pmatrix} + \begin{pmatrix} i \gamma_2 \nu_R^* \\ 0 \end{pmatrix}$$

4 comp.

$$N_M = \begin{pmatrix} i\sigma_2 u_R^* \\ u_R \end{pmatrix} \quad \psi_0 = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$u_L \neq u_R$

$\Downarrow$

$$\bar{N} N = (\bar{v}_R + C \bar{v}_R^T) (v_R + C v_R^T)$$

$$= \bar{v}_R v_R + C \bar{v}_R^T v_R + \dots$$

$\equiv$
0

$$= v_R^T C v_R + h.c.$$

$$\bullet \quad \bar{v}_R \gamma^\mu \partial_\mu v_R = \frac{1}{2} \bar{N} \gamma^\mu \partial_\mu N \text{ (show)}$$

⇓

$$\mathcal{L}(\nu_R) = \frac{1}{2} \left[\bar{N} \gamma^\mu \partial_\mu N - M_R \bar{N} N \right]$$

same as Dirac

$$N = \nu_R + C \bar{\nu}_R^T = \nu_R + (\nu^c)_L$$

⇓

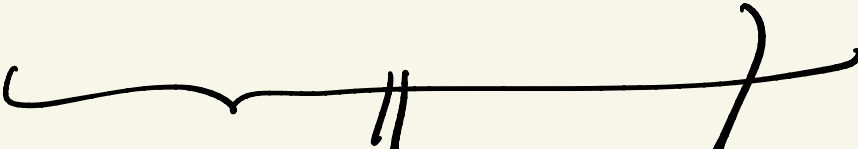
$N_R = \nu_R$
$N_L = C \bar{\nu}_R^T$

$$= C (\nu_R^\dagger \gamma^0)^T = C \gamma^0 \nu_R^*$$

↓

$$\mathcal{L}_Y(\nu) = y_0 \bar{l}_L i \sigma_2 \bar{L}^* \nu_R +$$

$$+ \frac{1}{2} M_R \nu_R^T C \nu_R + \text{h.c.}$$



$$\nu_R^T C \nu_R^*$$

$$\frac{1}{2} M_R \nu_R^T C \nu_R + \frac{1}{2} M_R^* \nu_R^T C^\dagger \nu_R^*$$

$$= \frac{1}{2} M_R^* N_L^T C N_L + \text{h.c.}$$

check: $N_L = C \bar{\nu}_R^T$

$$\underline{N_L^T C N_L} = \bar{\nu}_R C^T C C \bar{\nu}_R^T$$

$$= \nu_R^T \gamma^0 C (\nu_R^T \gamma^0)^T =$$

$$\begin{aligned}
&= V_R^\dagger \gamma^0 C \gamma^0 V_A^* \\
&= V_R^\dagger (-C) V_A^* = V_R^\dagger C^\dagger V_A^* \quad \downarrow
\end{aligned}$$

$$\Downarrow \quad \boxed{\begin{aligned} &\equiv u_D \\ &Y_D^* \psi \bar{V}_R V_L \end{aligned}}$$

$$+ \frac{1}{2} M_R V_R^T C V_R + \underline{\text{h.c.}}$$

$$\left\{ N_L = C \bar{V}_R^T \Rightarrow \bar{V}_R = N_L^T C \right\}$$

$$= u_D N_L^T C V_L + \frac{1}{2} N_L^T C N_L M_N$$

$$M_N = M_R^* \quad + \text{h.c.}$$

$\downarrow$ more gen.

$$= N_L^T C M_D \nu_L + \frac{1}{2} N_L^T C M_N N_L$$

$$= \cancel{\left(\frac{1}{2} \right)} \left(N_L^T C M_D \nu_L + \nu_L^T M_D^T C N_L \right)$$

$$+ \cancel{\left(\frac{1}{2} \right)} N_L^T C M_N N_L$$



$$M_{\nu N} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

assumption

$$M_N \gg M_D \neq \text{seesaw}$$

