Neutrino mass

Oscillations

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \boxed{\text{Pontecorvo}}$$

$\parallel$

$$\left.\begin{aligned} \nu_e &= \cos\theta\ \nu_1 + \sin\theta\ \nu_2 \\ \nu_\mu &= -\sin\theta\ \nu_1 + \cos\theta\ \nu_2 \end{aligned}\right\}$$ lepton mixing

$\Updownarrow$

$\nu_i \;:\; m_i \neq 0$ quark = Cabibbo

$\Downarrow$

$$P(\nu_e \to \nu_\mu) = \,?$$

- Maximal $\theta = \theta_{max} = 45^\circ$

- $\Delta m^2 \neq 0$

a) bigger $\Delta m^2 \Rightarrow$ bigger P

b) $E \to \infty \Rightarrow P \to 0$

c) bigger $L \Rightarrow$ bigger P

$\boxed{m/E \Rightarrow E \to \infty \Leftrightarrow m \to 0}$

$\Downarrow$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E}$$

$\Downarrow$ oscillations

$$\boxed{\Delta m^2 \neq 0 \quad \Rightarrow \quad m_\nu \neq 0}$$

- $\Delta m_A^2 \simeq 10^{-3}\, eV^2$ (atmospheric)

  $\theta_A \simeq 45^0$

- $\Delta m_\odot^2 \simeq 10^{-5}\, eV^2$

  $\theta_\odot \simeq 30^0$

$\Rightarrow$ $\boxed{\text{2 massive}}$

$\Downarrow$

$$\boxed{\text{SM} \neq \text{NOT complete}}$$

- if no fundamental theory

$\Downarrow$

$\boxed{\text{Effective theory}}$ $\boxed{\Delta L \neq 0}$

Weinberg '79

SM states

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R \qquad \cancel{\nu_R} \qquad \phi_0 = (v + h)$$

$\Downarrow$ Majorana

$$\underbrace{\nu_L^T \, C \, \nu_L}\; \frac{\phi_0 \, \phi_0}{\Lambda}$$

T_3 : $\frac{1}{2} + \frac{1}{2} \quad -\frac{1}{2}$

$\Uparrow$

$$\ell^T i\sigma_2 \, c \, \ell$$

Hint:

$$\bar{\nu}_e \quad \overline{e_L} \quad e_R \quad \phi_0$$

$$T_3 : \underbrace{\frac{1}{2} + 0 - \frac{1}{2} = 0}_{\text{good}}$$

$$\Downarrow$$

$$\mathcal{L}_{eff}^{(y)} = \nu_L^T c \, \nu_L \frac{\phi_0^2}{\Lambda}$$

$$= \nu_L^T c \, \nu_L \frac{(v+h)^2}{\Lambda}$$

$$\cong \nu_L^T C \nu_L \frac{v^2 + 2vh}{\Lambda}$$

$$\Downarrow$$

$$\boxed{m_\nu^M = \frac{v^2}{\Lambda}} \qquad \boxed{y_\nu = \frac{v}{\Lambda}}$$

$$\Updownarrow$$

$$m_\nu^M \, \nu_L^T C \nu_L + 2 \frac{v}{\Lambda} h \, \nu_L^T C \nu_L$$

(i) Neutrino = Majorana

$\Leftrightarrow \Delta L = 2$

$\Rightarrow$ neutrinoless double beta decay ($0\nu 2\beta$)

(ii) $y_\nu = \frac{v}{\Lambda} = \frac{m_\nu \cancel{v}}{v^{\cancel{2}}} =$

$M_W = \frac{g}{2} v \Rightarrow \qquad = \frac{g}{2} \frac{m_\nu}{M_W}$

$\Updownarrow$

$y_e = \frac{g}{2} \frac{m_e}{M_W}$

$$\boxed{m_\nu < 1\,\text{eV}} \quad (*)$$

KATRIN exp.

<u>0ν2β:</u> $\quad m_\nu \simeq (1/4 - 3/4)\,\text{eV}$?

$$\Rightarrow \quad g_\nu < \frac{10^{-9}}{100} = 10^{-11}$$

$$\Downarrow$$

$$h \to \nu\bar{\nu} \quad \Rightarrow \boxed{B(h \to \nu\bar{\nu}) \leq 10^{-20}}$$

• mass : $m_\nu = \frac{v^2}{\Lambda} \simeq \frac{M_W^2}{\Lambda}$

$$\Lambda = \frac{M_W^2}{m_\nu} \gtrsim \frac{10^4}{10^{-9}} = 10^{13}\,\text{GeV}$$

$\boxed{\text{hopeless}}$

$$\Uparrow$$

$\underline{\text{not}}$ $SU(2) \times U(1)$ inv.

$$\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}_L ; \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \xrightarrow{\text{unitary}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\Downarrow$$

$$\to \underbrace{(\ell_L^T C \, i\sigma_2 \, \ell_L)}_{0} \, \underbrace{(\Phi^T i\sigma_2 \, \Phi)}_{=\,0} \leftarrow$$

Λ

$$0 = \Phi_i \Phi_j (i\sigma_2)_{ij} = 0$$

$$\Downarrow$$

$$l_L^T i\sigma_2 \, C \, l_L = - l^T (i\sigma_2)^T C^T l$$

$$= - l^T (-i\sigma_2)(-C) l$$

$$= - l^T i\sigma_2 C l = 0$$

$$\boxed{\text{what to do?}}$$

$$\Downarrow$$

l = doublet, Φ = doublet

$$\Downarrow$$

$$\boxed{(1) \; (l_L^T i\sigma_2 \Phi) C (\Phi^T i\sigma_2 l_L) \frac{1}{\Lambda}}$$

+ 2 more ways $\Leftarrow$ $\boxed{\text{find them}}$

$\Rightarrow$ show all three equivalent

- expand $\Phi = \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$\Downarrow$

$$\ell^T i\sigma_2 \Phi = \nu_L^T (v+h)$$

$\Downarrow$

$$\boxed{(1) = (\nu_L^T C \nu_L)(v+h)^2 \frac{1}{\Lambda}}$$

neutrino Yukawa

- $S=0 \quad |\uparrow\downarrow - \downarrow\uparrow\rangle$

$$\Leftrightarrow \quad D_1^T \, i\sigma_2 \, D_2$$

- Lorentz

$$\boxed{\psi_1^T \, C \, \psi_2}$$

$$\boxed{\begin{gathered} C^T = -C \\ C\gamma_\mu C^{-1} = -\gamma_\mu^T \end{gathered}}$$

$$\psi^c = C(\bar{\psi})^T = C\gamma_0 \psi^*$$

$$C = i\sigma_2\gamma_0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

Seesaw mechanism

= (Poor man's) UV completion

Ultra Violet

$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \Big| \quad e_R, \quad \nu_R, \quad \Phi$

$\boxed{Y_{\nu_R} = 0}$

SM singlet

$$\mathcal{L}_y = y_e \, \bar{l}_L \, \Phi \, e_R \; + \; y_D \, \bar{l}_L \, i\sigma_2 \, \Phi^* \, \nu_R \; + h.c.$$

$\Downarrow$

$$Y_D\, \bar{\nu}_L \nu_R (v+h) +$$

$$+ Y_e\, \bar{e}_L e_R (v+h)$$

$$\Rightarrow \quad m_\nu = m_D = Y_D v$$

$$m_e = \qquad Y_e v$$

$$\Rightarrow \quad y_D = \frac{m_\nu}{v} = \frac{g}{2} \frac{m_\nu}{M_W}$$

$$B(h \to \nu\bar{\nu}) \leq 10^{-20} \quad : C$$

<u>Forgot</u>

$$\text{?} \quad \nu_R = \text{singlet}$$

(i) $\bar{\nu}_R \nu_R = 0$

$$\nu_R = \frac{1-\gamma_5}{2}\nu = R\nu$$

$$\bar{\nu}_R \nu_R = \nu_R^+ \gamma_0 \nu_R =$$

$$= (R\nu)^+ \gamma_0 R\nu =$$

$$= \nu^+ R \gamma_0 R \nu = \nu^+ \gamma_0 L R \nu$$

$$= 0$$

(ii) $\underbrace{\nu_R^T C \nu_R}_{\downarrow}$ $\qquad \underbrace{\cancel{\nu_L^T C \nu_L}}_{SU(2)\times U(1)}$

Majorana term

$$\Downarrow$$

$$\frac{1}{2} M_R \left(\nu_R^T C \nu_R + \nu_R^+ C^+ \nu_R^* \right)$$

$$\mathcal{L}(\nu_R) = i \bar{\nu}_R \gamma^\mu \partial_\mu \nu_R -$$

$$- \frac{1}{2} M_R \left(\nu_R^T C \nu_R + h.c. \right)$$

$$N_M = \nu_R + C \bar{\nu}_R^T = \nu_R + i \gamma_2 \nu_R^*$$

$$C = i \gamma_2 \gamma_0$$

$$N_M = \begin{pmatrix} 0 \\ u_R \end{pmatrix} + \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ u_R^* \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ u_R \end{pmatrix} + \begin{pmatrix} i\sigma_2 u_R^* \\ 0 \end{pmatrix}$$

4 comp.

$$\boxed{N_M = \begin{pmatrix} i\sigma_2 u_R^* \\ u_R \end{pmatrix}} \qquad \psi_D = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$$u_L \neq u_R$$

$$\Downarrow$$

• $\bar{N} N = \left(\bar{\nu}_R + \overline{C\bar{\nu}_R^T}\right)\left(\nu_R + C\bar{\nu}_R^T\right)$

$$= \underbrace{\bar{\nu}_R \nu_R}_{=0} + \overline{C\bar{\nu}_R^T}\,\nu_R + \cdots$$

$$= \nu_R^T C \nu_R + h.c.$$

• $\bar{\nu}_R \gamma^\mu \partial_\mu \nu_R = \frac{1}{2} \bar{N} \gamma_\mu \partial_\mu N$ (Maj)

$$\underbrace{\mathcal{L}(\nu_R) = \frac{1}{2}\left[\bar{N}\gamma^\mu \partial_\mu N - M_R \bar{N} N\right]}_{\boxed{\text{same as Dirac}}}$$

$$N = \nu_R + C\bar{\nu}_R^{\,T} = \nu_R + (\nu^c)_L$$

$$\boxed{\begin{array}{l} N_R = \nu_R \\ \hline N_L = C\bar{\nu}_R^{\,T} \end{array}} = C(\nu_R^\dagger \gamma^0)^T$$

$$= C\gamma_0 \nu_R^*$$

$$\mathcal{L}_y(\nu) = y_D \bar{l}_L i\sigma_2 \Phi^* \nu_R +$$

$$\underbrace{+\frac{1}{2} M_R \nu_R^T C \nu_R + h.c.}_{\parallel}$$

$$\nu_R^\dagger C \nu_R^*$$

$$\frac{1}{2} M_R \nu_R^T C \nu_R + \frac{1}{2} M_R^* \nu_R^\dagger C^\dagger \nu_R^*$$

$$= \frac{M_R^*}{2} N_L^T C N_L + h.c.$$

<u>check:</u> $N_L = C \bar{\nu}_R^T$

$$\underline{N_L^T C N_L} = \bar{\nu}_R C^T C C \bar{\nu}_R^T$$

$$= \nu_R^\dagger \gamma^0 C (\nu_R^\dagger \gamma_0)^T =$$

$$= \nu_R^{+} \gamma^0 C \gamma_0 \nu_R^{*}$$

$$= \nu_R^{+} (-C)\, \nu_R^{*} = \nu_R^{+} C^{+} \nu_R^{*}$$

$$\Downarrow$$

$$y_D v\, \overline{\nu_L} \nu_R + \boxed{\underbrace{y_D^{*} v\, \overline{\nu_R} \nu_L}_{\equiv m_D}}$$

$$+ \frac{1}{2} M_R\, \nu_R^{T} C \nu_R + \underline{h.c.}$$

$$\left\{ N_L = C \overline{\nu_R}^{T} \Rightarrow \overline{\nu_R} = N_L^{T} C \right\}$$

$$= m_D\, N_L^{T} C\, \nu_L + \frac{1}{2} N_L^{T} C N_L\, \underline{M_N} + h.c.$$

$$M_N = M_R^{*}$$

$\downarrow$ more gen.

$$= N_L^T C M_D \nu_L + \frac{1}{2} N_L^T C M_N N_L$$

$$= \cancel{\frac{1}{2}} \left(N_L^T C M_D \nu_L + \nu_L^T M_D^T C N_L \right)$$

$$+ \cancel{\frac{1}{2}} N_L^T C M_N N_L$$

$$\Downarrow$$

$$M_{\nu N} = \begin{array}{c} \nu \\ N \end{array} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$$

assumption $\boxed{M_N \gg M_D \Leftarrow \text{seesaw}}$