Neutrino Mass and Grand Unification

$\boxed{\text{Lecture XV}}$

7/12/2021

LMU
Winter 2021

Hierarchy "Problem"

vs

Doublet – Triplet Splitting

· <u>fermions</u>

$$\mathcal{L}_D = i\bar{f}\gamma^\mu\partial_\mu f - m_f^0 \bar{f} f \quad , \quad \bar{f} = f^\dagger\gamma^0$$

(a) $f \to e^{i\alpha} f$

$$\mathcal{L}_D = i\bar{f}_L\gamma^\mu\partial_\mu f_L + i\bar{f}_R\gamma^\mu\partial_\mu f_R$$

$$- m_f^0(\bar{f}_L f_R + \bar{f}_R f_L)$$

$$f_L \to e^{i\alpha} f_L \,, \quad f_R \to e^{i\alpha} f_R$$

(b) $m_f^g = 0 \Rightarrow$

$$f_L \to e^{i\beta} f_L, \quad f_R \to f_R$$

$$f_L \to f_L, \quad f_R \to e^{i\delta} f_R$$

or

(3) $f_L \to e^{i\beta} f_L, \quad f_R \to e^{-i\beta} f_R$

only 2 symmetries:

(a) $f \to e^{i\alpha} f$ $\qquad f_{L/R} = \frac{1 \pm \gamma_5}{2} f$

(b) $f \to e^{i\beta\gamma_5} f$

(a)·(b) $\quad f_L \to e^{i(\alpha+\beta)} f_L$

$$f_R \to e^{i(\alpha-\beta)} f_R$$

- $\alpha + \beta = 0 \qquad f_R \to e^{\cdots} f_R, \; f_L \to f_L$
- $\alpha - \beta = 0 \qquad f_R \to f_R, \; f_L \to e^{\cdots} f_L$

$$\Downarrow$$

$$\boxed{\begin{array}{l} m_f^0 = 0 \Rightarrow \text{chiral symm. (a) + (b)} \\ \Rightarrow \; m_f = 0 \text{ to all orders} \end{array}}$$

$$\Downarrow$$

$$\boxed{m_f = m_f^0 \Big[1 + \underbrace{\left(\frac{\alpha}{2\pi}\right)^n \ln \frac{\Lambda}{m_f^0}}_{\text{small}} \Big]}$$

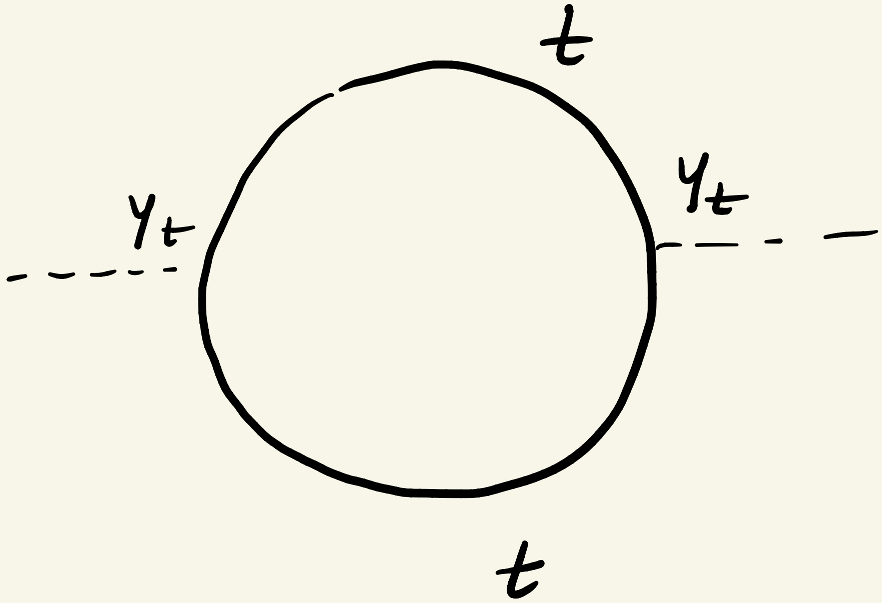
for $\Lambda \leq M_{pe}$

(chiral = protective)

- <u>scalars</u>

$m_0^2 \, \phi^+ \phi \leftarrow \boxed{\text{no}}$ protection

Higgs of SM



$$\Downarrow$$

$$\boxed{m^2 = m_0^2 + \frac{y_t^2}{16\pi^2}\left(\Lambda^2 + m_t^2\right)}$$

Higgs mass

large cut-off

$(\Lambda \gtrsim M_{GUT})$

<u>Fine Tuning</u> (FT) !

$$m_0 \sim \Lambda \frac{y_t}{4\pi} \simeq \Lambda/10$$

$$m_0^2 \simeq (5\text{ GeV})^2 \quad \therefore$$

$$m^2 = (100\text{ GeV})^2$$

$$m_\phi^2 = \lambda v_w^2\,, \quad M_W = g/2\, v_w$$

Hierarchy "Problem"!

Why (how) $M_W \ll \Lambda$?

(i) $\boxed{\text{no scalars}}$

(ii) $\boxed{\#\text{ of scalars} = \#\text{ of fermions}}$

(i) Why? $\Rightarrow$ Higgs = bound state

$\pi^0 \leftarrow$ bound by QCD (color)

Higgs $\leftarrow$ —"— Technicolor

Weinberg
Susskind '70s

(ii) <u>supersymmetry</u>

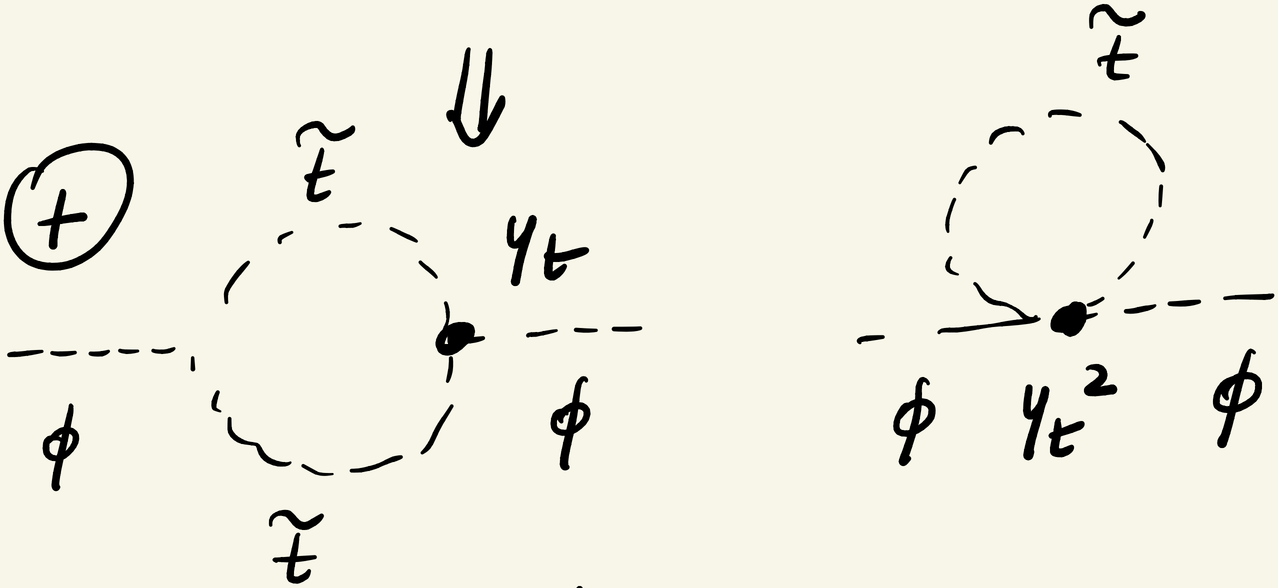
$$f \longleftrightarrow \tilde{f} \text{ (sfermion)}$$

$$(h) \text{ Higgs} \longleftrightarrow \tilde{h} \text{ (Higgsino)}$$

$$\Downarrow$$

$$t \Longrightarrow \tilde{t} \quad (\text{stop})$$

$\Downarrow$



$\Downarrow$

$$m^2 \equiv m_\phi^2 = m_0^2 + \frac{y_t^2}{16\pi^2}\left(\cancel{\Lambda^2} + m_t^2\right)$$

$$- \frac{y_t^2}{16\pi^2}\left(\cancel{\Lambda^2} + m_{\tilde{t}}^2\right)$$

$\Downarrow$

$$\boxed{m^2 = m_0^2 - \frac{y_t^2}{16\pi^2}\left(m_{\tilde t}^2 - \cancel{m_t^2}\right)}$$

- no cut-off dependence
- $\rightarrow$ Higgs mechanism

"<u>naturalness</u>" :

$$m_0 \simeq 100 \text{ GeV}$$

$$\frac{y_t}{4\pi} m_{\tilde t} \simeq 100 \text{ GeV}$$

$$(y_t/4\pi \simeq 1/10)$$

$\Downarrow$

$$\boxed{m_{\tilde{t}} \lesssim 1\ \text{TeV}}$$

How to quantify ?

- Higgs mechanism

  $y_t \simeq O(1)$ ⇐ 1982

- "naturalness"

- unif. was predicted ⇐ 1981

Doublet - Triplet Splitting

SU(5)

$$5_H = \begin{pmatrix} T \\ \hdashline \phi = D \end{pmatrix}$$

color triplet

weak doublet

24_H

$$V = \cdots \boxed{\beta \, 5_H^+ \langle 24_H^2 \rangle 5_H} + \mu_0^2 \, 5_H^+ 5_H$$

$$\langle 24_H \rangle = v_x \, \mathrm{diag}\,(1 \;\; 1 \;\; 1 \;\; -3/2 \;\; -3/2)$$

$$\Downarrow$$

$$m_T^2 = m_0^2 + \beta v_x^2 \simeq O(v_x^2)$$

$$m_D^2 = m_0^2 + 9/4\,\beta v_x^2 \simeq 0 \quad (v_W^2 \simeq 0)$$

$$\Rightarrow \boxed{\begin{aligned} m_T^2 &= \overbrace{m_0^2 + 9/4\,\beta v_x^2}^{0} - 9/4\,\beta v_x^2 + \beta v_x^2 \\ &= -5/4\,\beta v_x^2 \quad (\beta < 0) \end{aligned}}$$

$$\Uparrow$$

$$\boxed{\text{FT at tree-level}}$$

$m_T \simeq v_x$ (proton stability)

$$\Downarrow$$

$$m_0 \sim v_x \Rightarrow$$

$$\underbrace{m_0^2 \quad \text{vs} \quad \beta v_x^2}$$

FT of $10^{32}\,\text{GeV}^2 \to 0$

$$\boxed{\left.\begin{array}{l} T \Rightarrow \text{heavy} \\ D = \phi \Rightarrow \text{light} \end{array}\right\} \Rightarrow \text{FT}}$$

Proton stability

$GUT \supseteq SU(5)$

$\phi \subseteq 5_H$ (or bigger)

$\Downarrow$

$\boxed{D - T \text{ splitting}}$ Problem

SUSY $T \longleftrightarrow \tilde{T}$

as much as SM $\longrightarrow$ GUT

$\Downarrow$

SUSY SU(5) ! D + T +

partners

$\Downarrow$

T is heavy $(m_T \simeq v_X)$

$\Downarrow$

Fine Tune D-T mass difference

- SUSY GUT

$\Downarrow$

FT at tree level $\longrightarrow$ you can forget it

- GUT

$$\underbrace{m^2 = m_0^2 + \frac{y_t^2}{16\pi^2}\left(\Lambda^2 + \dots\right)}_{\text{FT at 1-loop level}}$$

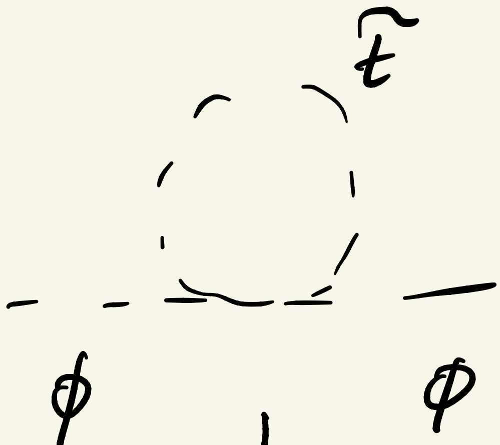
- eliminate FT by some group theory trick

$\Rightarrow$ susy is favored (tree)

<u>but</u>

- doubled # of states

- $m_{\tilde{p}} \leq \text{TeV}$ (naturalness)



$$\downarrow$$

$$-\frac{y_t^2}{16\pi^2} m_{\tilde{t}}$$

$$y_t = O(1) \Rightarrow m_{\tilde{t}} < \text{TeV}$$

- $\tilde{c}$?

$$-\frac{y_c^2}{16\pi^2} m_{\tilde{c}}^2 \Rightarrow m_{\tilde{c}} \leq 100\ \text{TeV}$$

and yet:

$$m_{\tilde{f}} = \Lambda_{susy}$$

assumption

- fermions = protected (chiral sym.)

$$m_f = m_f^0 \left[1 + \cdots \right]$$

- susy $\Rightarrow$ no difference between f and scalars

$$\Rightarrow m_\phi^2 = m_\phi^0 \left[1 + \cdots \right]$$

- chiral sym. protects fermions
- SUSY protects scalars

$$\boxed{\text{SUSY} \Rightarrow \text{Higgs is protected}}$$

<u>but</u>

$$\boxed{\begin{array}{l}\text{SUSY is broken} \\ \Rightarrow m_{\tilde{p}} = \text{small}\end{array}}$$

Minimal realistic extension of minimal SU(5)?

Failure of minimal SU(5):

- no unification **
- $m_e = m_d$ wrong ← ?
- $m_\nu = 0$ **

- $m_e = m_d$

$$\mathcal{L}_y = Y_d\, \bar{5}_F\, 10_F\, 5_H^* +$$

$$+ Y_u\, \underbrace{10_F\, 10_F\, 5_H}_{\text{5-index AS = \underline{invariant}}}$$

$$\Rightarrow \boxed{M_d = Y_d\, v_W = M_e^T}$$

$\boxed{\text{tree - level}}$

- $\Lambda \lessapprox M_{pe}$ (cut-off)

$\Downarrow$

$$\mathcal{L} = \mathcal{L}_{tree} + \underbrace{\mathcal{L}_{effective}}$$

compute
decays, scatterings

new physics

$$\mathcal{L}_{GUT} = \mathcal{L}_{tree}\left(1 + \left(\frac{M_*}{\Lambda}\right)^n\right)$$

$$\left.\begin{array}{l} M_X \simeq 10^{16}\ \text{GeV} \\ \Lambda \le 10^{19}\ \text{GeV} \end{array}\right\} \boxed{\le 10^{-3}}$$

$$y_e \simeq 10^{-5}, \quad y_d \simeq 10^{-4}, \quad y_s \simeq 10^{-3}$$

$$\Downarrow$$

$\boxed{\text{I cannot ignore } \mathcal{L}_{eff} \text{ operators}}$

$$\mathcal{L}_{SM} = \mathcal{L}_{tree}\left(1 + \underbrace{\left(\frac{M_W}{\Lambda}\right)^n}_{\text{small for large } \Lambda}\right)$$

$$\mathcal{L}_Y^{ren} = \{ d=4 \} = \text{tree level}$$

$d=5$

**

$\boxed{\text{Can you write } \mathcal{L}_Y(d=5) \text{ by adding } 24_H/\Lambda\text{?}}$

$$\text{correction} \sim \underbrace{\frac{\langle 2 \varphi_H \rangle}{\Lambda} \simeq \frac{v_x}{\Lambda}}_{\leq 10^{-3}}$$

$$\boxed{10\, M_X \leq \Lambda < M_P/10}$$

$$? + 10_F\; 10_F\; 10_H\; \frac{2\varphi_H}{\Lambda} \quad ?$$