Neutrino Mass and Grand Unification

Lecture XIV

3/12/2021

LMU
Fall 2021

SU(5) and Unification!

Can it be saved?

$$\frac{1}{\alpha(E_2)} = \frac{1}{\alpha_2(E_1)} + \frac{b}{2\pi} \ln E_2/E_1$$

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \underbrace{\frac{1}{3} T_S}_{\text{complex scalars}}$$

$+$ YM (AF)

$\boxed{1-2}$

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_2(M_W)} + \frac{b_2}{2\pi} \ln \frac{M_X}{M_W}$$

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_1(M_W)} + \frac{b_1}{2\pi} \ln \frac{M_X}{M_W}$$

$$\Downarrow$$

$$2\pi \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_2} \right) \Bigg|_{M_W} = (b_2 - b_1) \ln \frac{M_X}{M_W}$$

$$\boxed{T_{GB} = N} \quad \text{for } SU(N), \; N \geq 2$$

$$T_{GB} = 0, \quad \text{for } U(1)$$

$$\Rightarrow \quad b_2 = \frac{22}{3} - \frac{4}{3} n_g - \frac{1}{6}$$

Higgs

$$b_1 = 0 - \frac{4}{3} n_g - \frac{1}{10}$$

$$\left(b_1 = \frac{3}{5} b' = \frac{3}{5} b_Y \right)$$

$\Downarrow$

$$\boxed{M_X^{1-2} \cong 10^{13}\ \mathrm{GeV}}$$

$$\boxed{M_X^{2-3} \cong 10^{17}\ \mathrm{GeV}}$$

Who runs below M_X ?

- SM fermions
- W, Z, γ
- Higgs

but:

$$5_H = \begin{pmatrix} T \\ \hline D = \Phi \end{pmatrix} \quad T \to m_T > 10^{12}\,\text{GeV}, \quad D = \Phi \to \text{Higgs}$$

$$24_H = \begin{pmatrix} \text{octet of color} & \cancel{\Sigma_X, \Sigma_Y} \\ \cancel{\bar{\Sigma}_X, \bar{\Sigma}_Y} & \text{triplet of weak} \end{pmatrix} \quad \text{eaten}$$

eaten

$$+ \boxed{\text{singlet} \quad \text{SM} \quad \text{Higgs}}$$

$\Rightarrow$ no charge
no color
no flavor

$(T_i = 0, \quad i = 1, \dots, 12)$

$8_c, \quad 3_W = \text{physical}$

$$m_g = \frac{5}{4}\vartheta_x b \qquad \begin{cases} M_x = \cdots \vartheta_x \\ M''_y \end{cases}$$

$$\frac{b}{2}\,\mathrm{Tr}\,(2Y_H)^4$$

$$m_3 = 5 b \vartheta_x = 4 m_g$$

$$\boxed{M_x = \# g \vartheta_x \qquad g \simeq O(1)}$$

X <u>is</u> heavy!

$b = ?$ What if b small?

SM : electron $\quad m_e = y_e v_W$

$$\underset{\sim 100\ \text{GeV}}{\uparrow}$$

$$m_e \neq v_W$$

$$\doteq 10^{-5} v_W \;!!!$$

Q. What if $b \sim y_e \ll 1$?

· $\underline{m_3}$, $\underline{m_g}$ $=$?

3_W : $Y = 0 \Rightarrow b_1(3_W) = 0$

$$\delta b_1 \equiv b_1(3_W) = 0$$

$$b_2\,(3w) = -\frac{1}{3}\cdot\frac{1}{2}\cdot 2 = -1/3$$

real

$$\boxed{b_2\,(3w) = \delta b_2}$$

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_1(M_W)} + \frac{b_1}{2\pi}\ln\frac{M_X}{M_W}$$

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_2(M_W)} + \frac{b_2}{2\pi}\ln\frac{m_3}{M_W} + \frac{(b_2 + \delta b_2)}{2\pi}\ln\frac{M_X}{m_3}$$

$$= \frac{1}{\alpha_2(M_W)} + \frac{b_2}{2\pi}\ln\frac{M_X}{M_W} + \frac{\delta b_2}{2\pi}\ln\frac{M_X}{m_3}$$

$$\Downarrow$$

$$\left[\frac{1}{\alpha_1(M_W)} - \frac{1}{\alpha_2(M_W)}\right] 2\pi = (b_2 - b_1) \ln \frac{M_X}{M_W}$$

$$-\frac{1}{3} \ln \frac{M_X}{m_3}$$

negative

$$2\pi \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_2}\right)\Bigg|_{M_W} = (b_2 - b_1) \ln \frac{M_X^{\,0}}{M_W}$$

$$(m_3 = M_X)$$

$$= (b_2 - b_1) \ln \frac{M_X}{M_W} - \frac{1}{3} \ln \frac{M_X}{m_3}$$

$$\Downarrow$$

best: $m_3 \simeq M_W$

$$\Downarrow$$

$$M_X^{(1-2)} \leq 10^{14}\ \mathrm{GeV}$$

ruled out!

Q. What is the minimal realistic extension of minimal $SU(5)$?

A. $24_F \to (24_V, 24_H)$

$$\boxed{\begin{array}{l}\text{SUPER SYMMETRIC} \\ \quad SU(5)\end{array}}$$

Super Symmetry = Su Sy

= SUSY

$$\begin{array}{ccc} b & \longleftrightarrow & f \\ \uparrow & & \uparrow \\ \text{boson} & & \text{fermion} \end{array}$$

Gelfand, Likhtman '71

Wess, Zumino '74

$(f)\ e \longleftrightarrow \tilde{e}\ (b)$ (selectron)

$W \longleftrightarrow \tilde{W}$ (Wino)

$(b) \qquad (f)$

(b) $h \longleftrightarrow \tilde{h}$ (Higgsino) (f)

(f) $u \longleftrightarrow \tilde{u}$ (up squark) (b)

$$\boxed{\text{SUSY} = \text{broken}}$$

Theory (prejudice) :

$$\Lambda_{susy} = m_{\tilde{p}} - m_p \simeq \text{TeV}$$

(s-particle) particle

$\Downarrow$

sparticles mass above TeV

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S$$

$$\Downarrow$$

$$b^{susy} = ?$$

(g. boson) $\overset{(s=1)}{A}$ $\longleftrightarrow$ $\tilde{A}$ (gaugino)$^{(s=1/2)}$ (f)

(fermion) f $\longleftrightarrow$ $\tilde{f}$ (sfermion) (b)

$\uparrow$

$(s=0)$

$(s=0)$ h $\longleftrightarrow$ $\tilde{h}$ $(s=1/2)$

$p \longleftrightarrow \tilde{p}$

same internal symmetry

$$b = \frac{11}{3} T_{GB} - \frac{2}{3} T_F - \frac{1}{3} T_S \quad (*)$$

gauge boson A

gaugino $\tilde{A}$ $\longrightarrow$ how much ?

$$-\frac{2}{3} \times T_{GB}$$

fermions

gaugino

sfermions

$$b^{SUSY} = \left(\frac{11}{3} - \frac{2}{3}\right) T_{GB} - \left(\frac{2}{3} + \frac{1}{3}\right) T_F$$

$$- \left(\frac{1}{3} + \frac{2}{3}\right) T_S$$

Higgsino

$$\Downarrow$$

$$\boxed{b^{SUSY} = 3T_{GB} - T_F - T_S}$$

$$b_3 = 3\cdot 3 - 2n_g - 0 = 9 - 2n_g$$

$$b_2 = 3\cdot 2 - 2n_g - \frac{1}{2} = 6 - 2n_g - \frac{1}{2}$$

$$< 0 \quad (n_g = 3)$$

$$\boxed{\alpha_2 \neq \text{AF in SUSY}}$$

$$b_1 = -2n_g - \frac{3}{5}\cdot\frac{1}{2}$$

normalization

Marciano, G.S. '81

$$\overset{(SUSY)}{\Rightarrow} \sin^2\theta_W = 0.23$$

$$(\sin^2\theta_W)^{exp} = 0.20$$

$$\Rightarrow \boxed{m_t \simeq 200 \text{ GeV}}$$

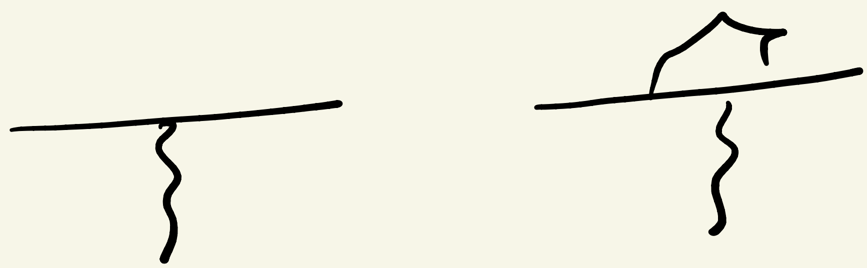
Scale of SUSY breaking!

$$\Lambda_{SUSY} = ?$$

why a prejudice $\Lambda_{SUSY} \simeq$ TeV?

$\Updownarrow$

$$\boxed{\text{Hierarchy Problem}}$$

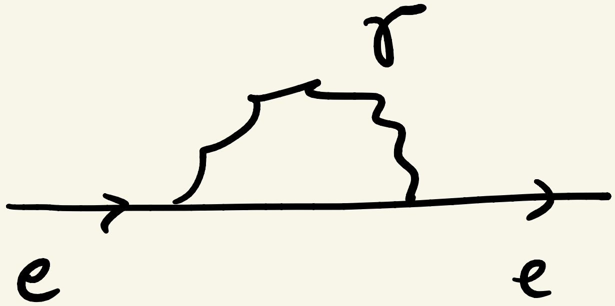


$+ \ldots$

$$e^{(E)} = e_0 \left(1 + \underbrace{\frac{e_0^2}{4\pi} \ln \Lambda/E}_{\text{small ``infinity''}}\right)$$

$$\Lambda \lesssim M_p \simeq 10^{19}\,\text{GeV}$$

$$\Rightarrow \quad \alpha \ln \Lambda/E < 1$$



Weisskopf 1930

$$m_e^{(E)} = m_e^0 + a\alpha\Lambda + b\alpha m_e^0 \ln \Lambda/E$$

Q1. Is $a \neq 0$ possible?

A. $a = 0$

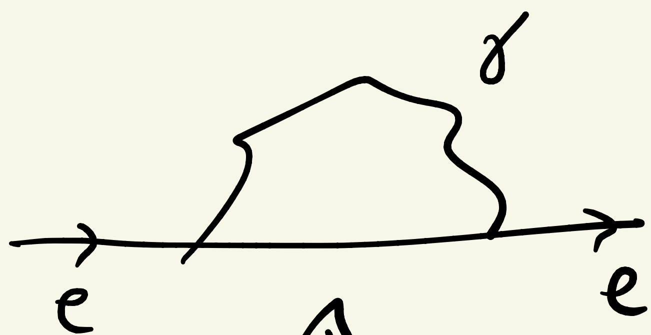
Q2. Why $a = 0$?

$$m_e^0 \, \bar{e} e = m_e^0 (\bar{e}_L e_R + \bar{e}_R e_L)$$

$\Rightarrow$ $\boxed{\text{if } m_e^0 = 0}$

chiral symmetry $\Rightarrow \begin{cases} e_L \to e^{i\alpha} e_L, & e_R \to e_R \\ e_L \to e_L, & e_R \to e^{i\beta} e_R \end{cases}$

· $\underline{m_e^0 = 0} \Rightarrow$ chiral symmetry



loops must preserve

$\Downarrow$

$m_e \propto m_e^0$, so that

$$m_e^0 \to 0 \Rightarrow m_e \to 0$$

(chiral symmetry)

$$\Downarrow$$

$$\boxed{m_e^{1\,loop}(E) = m_e^0\left(1 + \frac{\alpha}{\pi}\ln \Lambda/E\right)}$$

to all orders!

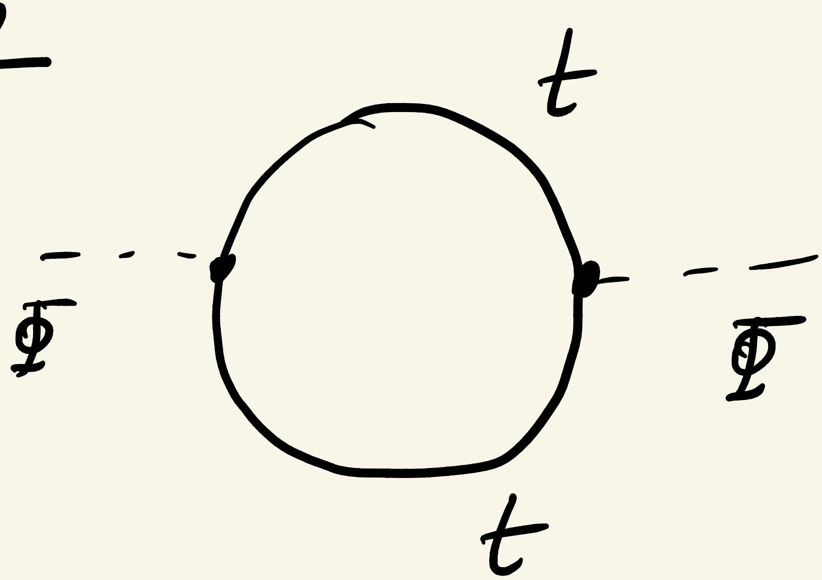
$$m_e^{n\,loop}(E) = m_e^0\left(1 + \#\underbrace{\left(\frac{\alpha}{\pi}\right)^n \ln \Lambda/E}_{\boxed{<1}}\right)$$

Higgs mass

$$m^2 \Phi^\dagger \Phi \nleftarrow \underline{\text{no}} \text{ "chiral" sym.}$$

$$\Downarrow$$

1 loop



$$m_\Phi^2 \,(1 \text{ loop}) \simeq \frac{y_t^2}{16\pi^2} \left(\equiv \frac{\alpha_t}{4\pi} \right)$$

$$\left(\Lambda^2 + m_t^2 \ln \frac{\Lambda}{E} \right)$$

"hierarchy problem"

GUT : $\Lambda > M_X$!

Fine-Tuning (FT)

$$m_\Phi^2 = m_0^2 + \frac{\alpha_t}{4\pi}\left(\Lambda^2 + m_t^2 \ln \frac{\Lambda}{m_\Phi}\right)$$

$$m_0^2 + \frac{\alpha_t}{4\pi}\Lambda^2 \simeq (100\ \text{GeV})^2$$

$$\wr$$

$(10^{16}\ \text{GeV})^2$ $\qquad$ $(10^{16}\ \text{GeV})^2$

$\boxed{\cdot \text{ chiral symmetry}}$

$$\mathcal{L}_D = i\bar{f}\gamma^\mu \partial_\mu f - m_f \bar{f} f$$

$\cdot\ m_f \neq 0 \Rightarrow \boxed{f \to e^{i\alpha} f}$

$\Leftrightarrow f_L \to e^{i\alpha} f_L,\ f_R \to e^{i\alpha} f_R$

<u>why?</u> $\bar{f} f = \bar{f}_L f_R + \bar{f}_R f_L$

$\cdot\ m_f = 0 \Rightarrow f_L \to e^{i\beta} f_L,\ f_R \to f_R$

$\left(f_L \to f_L,\ f_R \to e^{i\gamma} f_R \right)$

$\Leftrightarrow f_L \to e^{i\beta} f_L,\ f_R \to f_R$ +

$$\Downarrow$$

- $f \to e^{i\alpha} f$ (vector symmetry)

$$\underbrace{}_{\text{always}}$$

- $m_f = 0 \Rightarrow f \to e^{i\beta\gamma_5} f$ (chiral)

$$\Updownarrow$$

$$\underbrace{f_L \to e^{i\beta} f_L,\ f_R \to e^{-i\beta} f_R}_{L \neq R}$$