Neutrino Mass and Grand Unification

Lecture X

19/11/2021

LMU
Fall 2021

SU(5): (SSB) Yukawa sector,
p decay and all that (I)

1st stage: $\Sigma \to U \Sigma U^{+}$ (24_H)

(Adjoint)

$$\langle \Sigma_0 \rangle \cong M_{GUT} > 10^{15}\,\mathrm{GeV}$$

(p life-time)

$$SU(5) \xrightarrow[\langle \Sigma_0 \rangle]{} SU(3) \times SU(2) \times U(1)$$

$$\langle \Sigma_0 \rangle = v_X \,\mathrm{diag}\left(1 \;\; 1 \;\; 1 \;\; -3/2 \;\; -3/2\right)$$

$$v_X \equiv v_{GUT}$$

2nd stage

$$H \equiv 5_H = \begin{pmatrix} T \\ \hdashline \underbrace{\Phi \; (D)}_{\updownarrow \text{ doublet}} \end{pmatrix} \Big\} \text{ weak}$$

$$5_F = \begin{pmatrix} d \\ e^c \\ -\nu^c \end{pmatrix}_R$$

$$\Rightarrow \quad Q(T) = Q(d)$$

$$Y(T) = Y(d_R)$$

$$\boxed{\langle H \rangle = \langle 5_H \rangle = \begin{pmatrix} 0 \\ \langle \Phi \rangle \simeq M_W \end{pmatrix} \quad ??}$$

Decoupling: Ignore all the heavy fields in Σ

$(m_8, m_3, \ldots)$

propagate: $\sim \frac{E^2}{m_X^2} = \frac{E^2}{m_{GUT}^2}$

$E \simeq M_W$ (weak int. scale)

However: $\langle \Sigma_0 \rangle =$ fundamental

?

$$V = V(\Sigma) + V(H) + V(\Sigma, H)$$

last lecture

$$V(H) = \frac{\lambda}{4}\left(H^+H - v_W^2\right)^2$$

$$= -\frac{\mu_H^2}{2} H^+H + \frac{\lambda}{4}(H^+H)^2 + \text{const}$$

$$\boxed{V(\Sigma, H) = \alpha\, H^+H\, \mathrm{Tr}\,\Sigma^2 + \beta\, H^+ \Sigma^2 H}$$

crucial

$$\Sigma \longrightarrow \Sigma^0 = v_x \, \mathrm{diag} \, (1 \; 1 \; 1 \; -3/2 \; -3/2)$$

α: $\frac{15}{2} v_x^2 \alpha \; H^+ H$

mass term

$$\frac{-\mu_H^2(\text{eff})}{2} = \underbrace{\frac{-\mu_H^2}{2} + \frac{15}{2} \alpha v_x^2}_{\text{mass term (effective)}}$$

β: $\Sigma_0^2 = v_x^2 \left(1 \; 1 \; 1 \; 9/4 \; 9/4\right)$

$\beta \, H^+ \Sigma_0^2 H$ $\Leftarrow$ splits T from D (Φ)

$$m_T^2 = -\frac{\mu_H^2}{2} + \beta v_X^2$$

effective

$$m_D^2 \equiv m_\Phi^2 = -\frac{\mu_H^2}{2} + \frac{9}{4}\beta v_X^2 \simeq M_W^2$$

$$(m_h^2 = 2\mu_H^2)$$

$$M_W / M_X \simeq 0$$

$m_D^2 \simeq 0$ (compared to M_X)

$$\Rightarrow \boxed{\mu_H^2 \simeq 9/2\, \beta\, v_X^2} \quad (\beta = ?)$$

$$\Rightarrow m_T^2 = -\frac{\mu_H^2}{2} + \beta v_X^2 = \overbrace{-\frac{\mu_H^2}{2} + \frac{9}{9}\beta v_X^2}^{0}$$

$$- \frac{5}{9}\beta v_X^2$$

$$\Downarrow$$

$$\boxed{m_T^2 = -\frac{5}{4}\beta v_X^2}$$

($T =$ color triplet)

$$\Downarrow$$

$$\boxed{\beta < 0 \quad (m_T \gg m_D)}$$

$$\Downarrow$$

SM Higgs potential for Φ

$$V_H(\mathrm{eff}) = -\frac{\mu_H^2}{2}\Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2$$

find, effective (small on GUT scale)

$$\Downarrow$$

$$\boxed{\langle \Phi \rangle = \begin{pmatrix} 0 \\ v_W \end{pmatrix} \quad SU(2)}$$

$\Rightarrow$ "would be" goldstones

$$\Phi = \begin{pmatrix} G_W^+ \\ v + h + i\,G_Z \end{pmatrix}$$

G_W^+ = eaten by W^+

G_Z = —"— by Z

$\Downarrow$

$$\boxed{\begin{array}{c} T = \text{physical particle} \\ (\text{nobody can eat } T) \end{array}}$$

(X, Y) = doublet of $SU(2)$

T = singlet of $SU(2)$

$\Rightarrow$ $(\cancel{X}, \underline{Y})$ cannot eat T

$\Downarrow$ $\boxed{\text{Physics of } T}$ (D)

$$\boxed{\text{Yukawa sector}}$$

$$5_F = \begin{pmatrix} d \\ e^c \\ -\nu^c \end{pmatrix}_R \qquad \left(\bar{5}_F = \begin{pmatrix} d^c \\ \nu \\ e \end{pmatrix}_L \right)$$

$$10_F = \left(\begin{array}{c|cc} u^c & u & d \\ \hline & 0 & e^c \\ & -e^c & 0 \end{array}\right)_L$$

(AS)

<u>Yukawa</u>: $\boxed{y\, \bar{f}_R\, \phi\, f_L + h.c. \quad (\text{Lorentz})}$

$\Downarrow$

$$\mathcal{L}_y(SU(5)) = \bar{5}_F{}^i\, Y_1\, 10_{F\,ij}\, 5_H^{*\,j}$$

$i = 1, \dots, 5$

(H)

$$+\ \cancel{5^T_{F_i}\, C\, 5_{F_j}\, 10^{*\,ij}_H} \quad \underline{\text{no}}$$

$$+\ \underbrace{\underbrace{10^T_{F_{ij}}\, C\, Y_2\, 10_{F_{kl}}}_{\text{Lorentz (NOT SU(5)) inv.}}\ 5_{H_m}\, \epsilon^{ijklm}}_{\text{SU(5) inv. } \cancel{(?)}\ \boxed{\text{YES}}}$$

<u>SU(2)</u> $\qquad D_1^T\, i\sigma_2\, D_2 = D_1^T\, \epsilon\, D_2$

<u>SU(3)</u> $\qquad T_1^i\, T_2^j\, T_3^k\, \epsilon_{ijk}$

(a) <u>Masses</u>

$$5_H \equiv H \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_W \end{pmatrix}$$

$$5_{H_i} \to \langle 5_H \rangle_i = v_W \, \delta_{i5}$$

Q. Σ couples to fermions?

$$\bar{5}_F{}^i \, 10_{F\,jk} \, \Sigma^k_i$$

<u>sticky!</u>

$$\Sigma \to U \Sigma U^+$$

$$(5 \times \bar{5})$$

$$\bar{5}_F{}^i \, 5_{F\,j} \, \Sigma_i{}^j \overset{?}{=} \text{inv.}$$

$$\Downarrow$$

$$\underbrace{\bar{5}_F^{\,i}\, 5_{Fi}}_{\text{NOT Lorentz inv.}} \overset{?}{=} \text{invariant}$$

$$\cancel{\bar{5}_F\, 5_F = \bar{5}_{FR}\, 5_{FR}}$$

$$\boxed{\begin{array}{ll} & \underline{\text{Reminder}} \\ (\text{DIRAC}) & \bar{f}_L\, f_R \\ (\text{MAJORANA}) & f_L^T C f_L\,,\ f_R^T C f_R \end{array}}$$

$$\Downarrow$$

NO direct mass term

in SM

$$Q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R\,,\ d_R$$

(D) $\overline{Q_L}\, u_R$, $\overline{Q_L}\, d_R$ SU(2)

(M) $u_R^T \epsilon\, u_R$ (Y, color)

$\Rightarrow$ $\boxed{\text{SM} = \text{chiral theory}}$

$\Downarrow$

$\boxed{\text{MUST break } G_{SM} \text{ (Higgs)}}$

$$\boxed{\text{NO direct mass in } SU(5)}$$

$$\bar{5}_F = (\quad)_R, \qquad 10_F = (\quad)_L$$

(D) $\cancel{\bar{5}_F 5_F}$ Lorentz

(M) $5_F^T C\, 5_F$ $SU(5)$

(same for 10_F)

$$\Downarrow$$

$$\boxed{SU(5) = \text{chiral theory}}$$

(NO mass term)

$$Y_1 : \quad \bar{5}_F^{\,i}\, Y_1\, 10_{F\,ik}\, v_W\, \delta^{k5}$$

$$= v_W\, \bar{5}_F^{\,i}\, Y_1\, 10_{F\,i5}$$

$$(i = 1,2,3) \rightarrow v_W\, \bar{d}_R^{\,0}\, Y_1\, d_L^0 + h.c.$$

$$Y_1 \equiv Y_d \qquad d_L^0 = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}^0$$

$Y_1 \neq$ not diagonal

$$(M_D = Y_D v_W)$$

- - - - - - - -

$$(i = 4) \rightarrow v_W\, \bar{e}_R^c\, Y_1\, e_L^c + h.c.$$

$$e_L^c \equiv C \bar{e}_R^{\,T}, \quad e_R^c \equiv C \bar{e}_L^{\,T}$$

$$\overline{e^c_R}\, \gamma_1\, e^c_L = \overline{c\,\bar{e}_L^T}\ \gamma_1\, c\, \bar{e}_R^T$$

$$= \left[C\left(e_L^+ \gamma^0\right)^T\right]^+ \gamma^0 \gamma_1\, C\, \bar{e}_R^T$$

$$= \left(C\ \gamma_0\ e_L^*\right)^+ \gamma_0\ \gamma_1\, C\, \bar{e}_R^T$$

$$= e_L^T \gamma_0\, C^+ \gamma_0\, \gamma_1\, C\, \bar{e}_R^T$$

$$= e_L^T \underbrace{\left(-C^+ C\right)}_{-1} \gamma_1\, \bar{e}_R^T =$$

$$= \overline{e_R}\ \gamma_1^T\, e_L$$

$$\Downarrow$$

$$\vartheta_W\ \overline{e_R}\ \gamma_1^T\, e_L \quad \Rightarrow$$

$$\boxed{M_e = v_w Y_1^T = M_D^T}$$

masses "run"

$m_f(E)$

which E?

$$\boxed{M_e = M_D^T}$$ $(M_{GUT} = E)$

$$\boxed{m_e = m_d}$$

$m_b \simeq 3\, m_\tau$ ok?

$m_s \simeq 3\, m_\mu$ no

$m_d \simeq 3\, m_e$ no

$\boxed{E \simeq \text{GeV}}$

$\rightsquigarrow$ 5 MeV

$\boxed{\text{FAILURE ?!}}$

Y_2: $\quad 10^T_{F_{ij}}\, C Y_2\, 10_{F_{kl}} \langle 5_{Hm} \rangle\, \epsilon^{ijklm}$

$\to 10^T_{F\,ij}\, C Y_2\, 10_{F\,kl}\, v_w\, \epsilon^{ijkl5}$

$(12)(34) \left(u^c_L\right)^T C Y_2\, u_L\, v_w$

↑ color 3 ↑ color 3

$$\Rightarrow v_W \left(C \overline{u_R}^T\right)^T C Y_2 u_L =$$

$$= v_W \overline{u_R} C^T C Y_2 u_L = v_W \overline{u_R} Y_2 u_L$$

$$\Rightarrow \boxed{\begin{array}{l} M_u = Y_2 v_W \\ \Rightarrow Y_2 = Y_u \end{array}}$$

Yukawa: $\underbrace{10_F^T Y_2 10_F}$

$$\Rightarrow \boxed{Y_2 = Y_2^T}$$

$\boxed{\text{(Check!)}}$

$$\Downarrow$$

$$\boxed{M_u = M_u^T, \quad M_d = M_e^T}$$

$\Rightarrow$ Minimal $SU(5)$ = "completely" predictive

Stay tuned!

$$\mathcal{L}_Y = \bar{5}_F^{\,i}\, Y_1\, 10_{F_{ij}}\, 5_H^{*j} + h.c.$$

$\uparrow$

T: take out

$$T_1^* : \quad \bar{5}_F^{\,i}\, 10_{F\,i1}$$

$$-1 + 2/3 = -1/3$$

$$= \overline{d_R}\, y_d\, u_L^c \quad + \overline{e_R^c}\, y_d\, u_L \checkmark$$

$i = 1,2,3$ (color) $\qquad i = 4$

$$1/3 \quad -2/3$$

$$= -1/3 \checkmark \qquad + \overline{\nu_R^c}\, y_d\, d_L \checkmark$$

$i = 5$

$(-1/3)$

$$\Downarrow$$

$$\boxed{\left(\overline{d_R}\, y_d\, u_L^c + \overline{e_R^c}\, y_d\, u_L + \overline{\nu_R^c}\, y_d\, d_L\right) T^*}$$

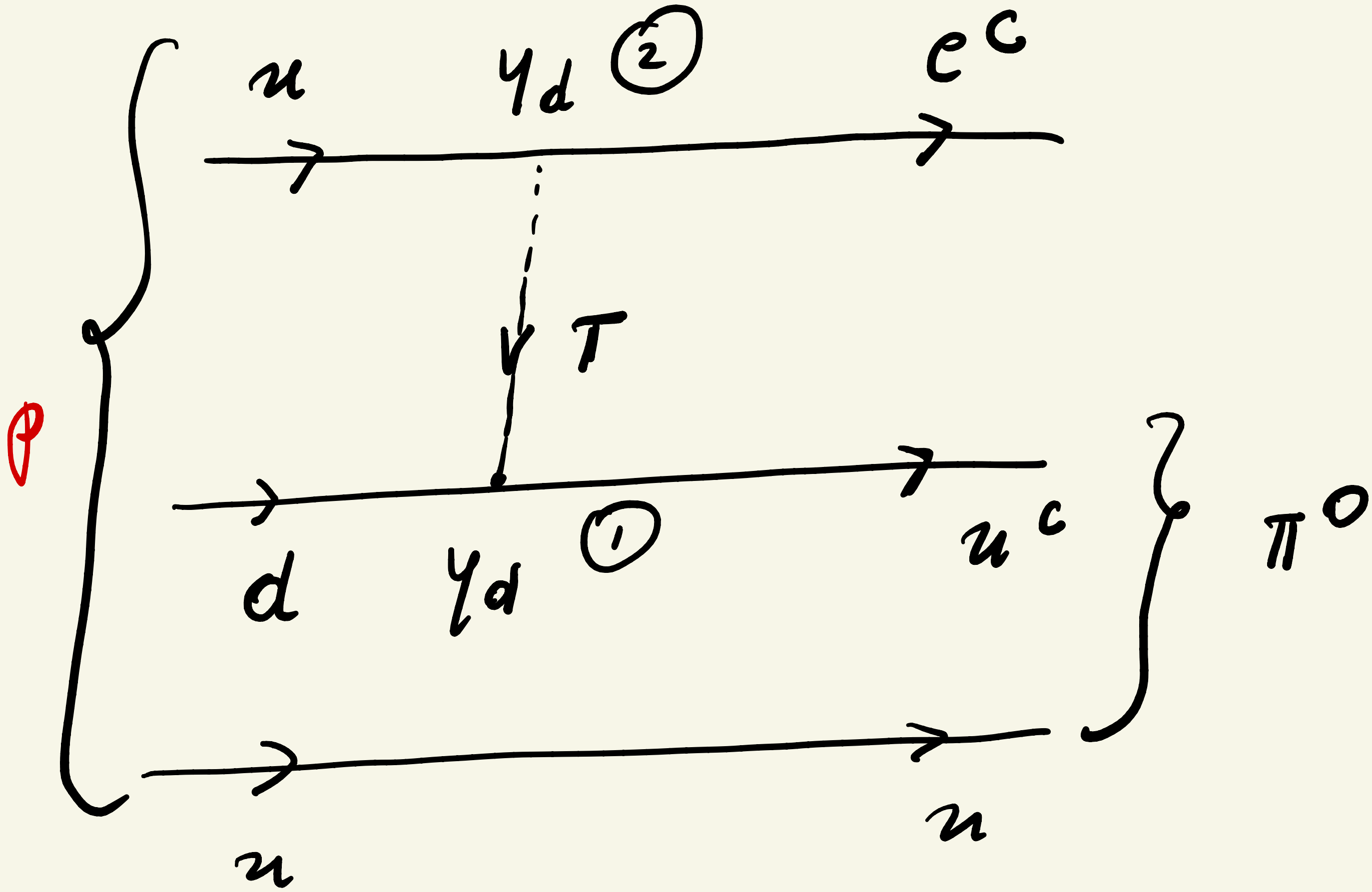
① ② ③ $+h.c.$

$$\Downarrow$$

$$B: \quad \underbrace{-1/3 - 1/3 = -2/3} \qquad \underbrace{0 - 1/3 = -1/3}$$

$$\Downarrow$$

B is broken $\overset{?}{\Rightarrow}$ p decay



$$\boxed{p \to \pi^0 + e^c}$$

$$\Rightarrow \frac{y_d^2}{m_T^2} \sim \frac{g^2}{M_X^2}$$

$$\Downarrow$$

$$m_T \simeq M_X \frac{y_d}{g} > 10^{15} \frac{y_d}{g}\ \text{GeV}$$

$$(\tau_p > 10^{34}\ \text{yr})$$

$$\Rightarrow \boxed{m_T \gtrsim 10^{12}\ \text{GeV}}$$

$$\Uparrow$$

p stability

- $m_T^2 = -\frac{\mu_H^2}{2} + \beta v_X^2 > (10^{12}\ \text{GeV})^2$

- $m_D^2 = \underbrace{-\frac{\mu_H^2}{2} + \frac{9}{4}\beta v_X^2 \simeq M_W^2}$

$$\Downarrow$$

$\boxed{\text{Fine Tuning (FT)}}$

between μ_H^2 and v_X^2

(each $> (10^{12}\,\text{GeV})^2$)

$\boxed{\text{D-T splitting}}$

<u>Called</u>: Problems ?

$\boxed{\text{NOT a problem, but to me it's ugly}}$

- String CP: problem or blessing?

G.S., Tello

inspire

- Natural Philosophy vs Philosophy of Naturalness

Senjanović

Generations: $y = \text{matrix}$

1 gen : $y = \text{number}$

Quantum:

$$\boxed{M_{\phi} = \text{matrices}}$$