Neutrino Mass

and

Grand Unification

Lecture VIII

12/11/2021

LMU

Fall 2021

SU(5) GUT : predictions

- SU(5)
- matter content : q, l

$$\bar{5}_F = \begin{pmatrix} d^c \\ --- \\ \nu \\ e \end{pmatrix}_L \qquad 10_F = \begin{pmatrix} u^c & u & d \\ & & e^c \end{pmatrix}_L$$

anti-symmetric

$\Downarrow$

besides gluons, W, Z, γ

+

$$X^\alpha, Y^\alpha; \quad \bar{X}^\alpha, \bar{Y}^\alpha$$

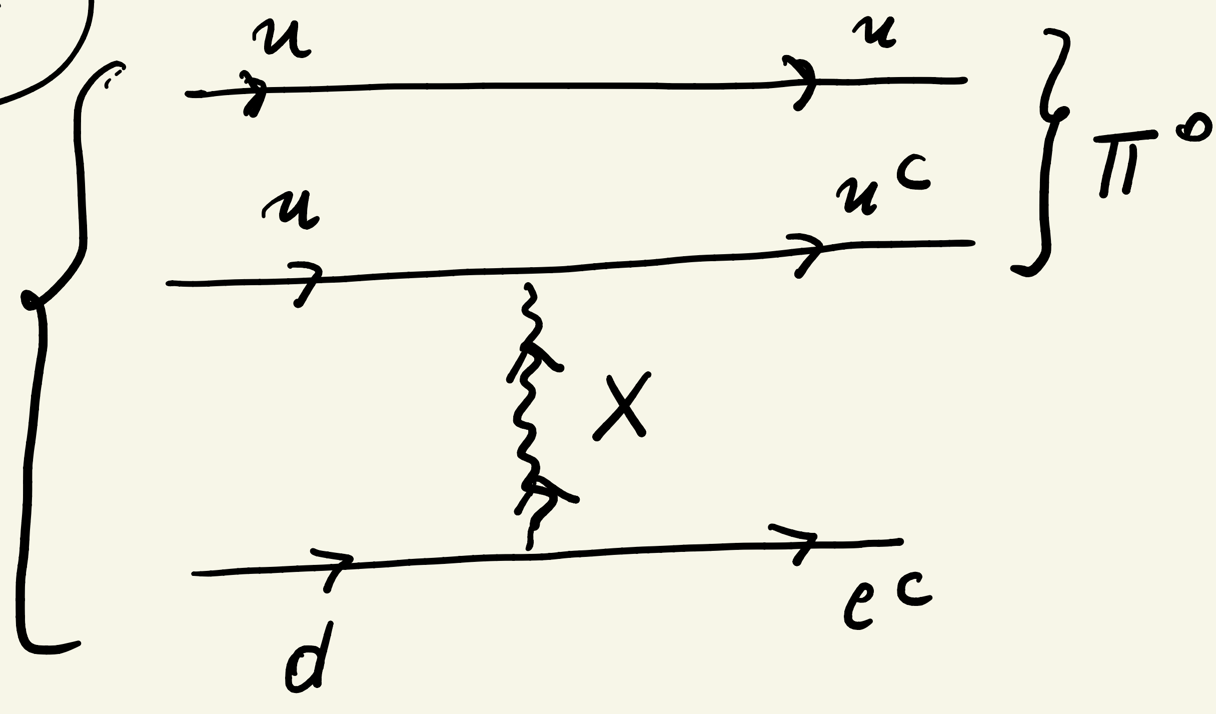
$\boxed{SU(2)}$

$$X_\mu \left[\underline{\overline{u^c} \gamma^\mu u} + \underline{\bar{d} \gamma^\mu e^c} + \bar{e} \gamma^\mu d^c \right]$$

$$Y_\mu \left[\underline{\overline{u^c} \gamma^\mu d} + \bar{u} \gamma^\mu e^c + \underline{\bar{\nu} \gamma^\mu d^c} \right]$$

$$+ \ h.c.$$

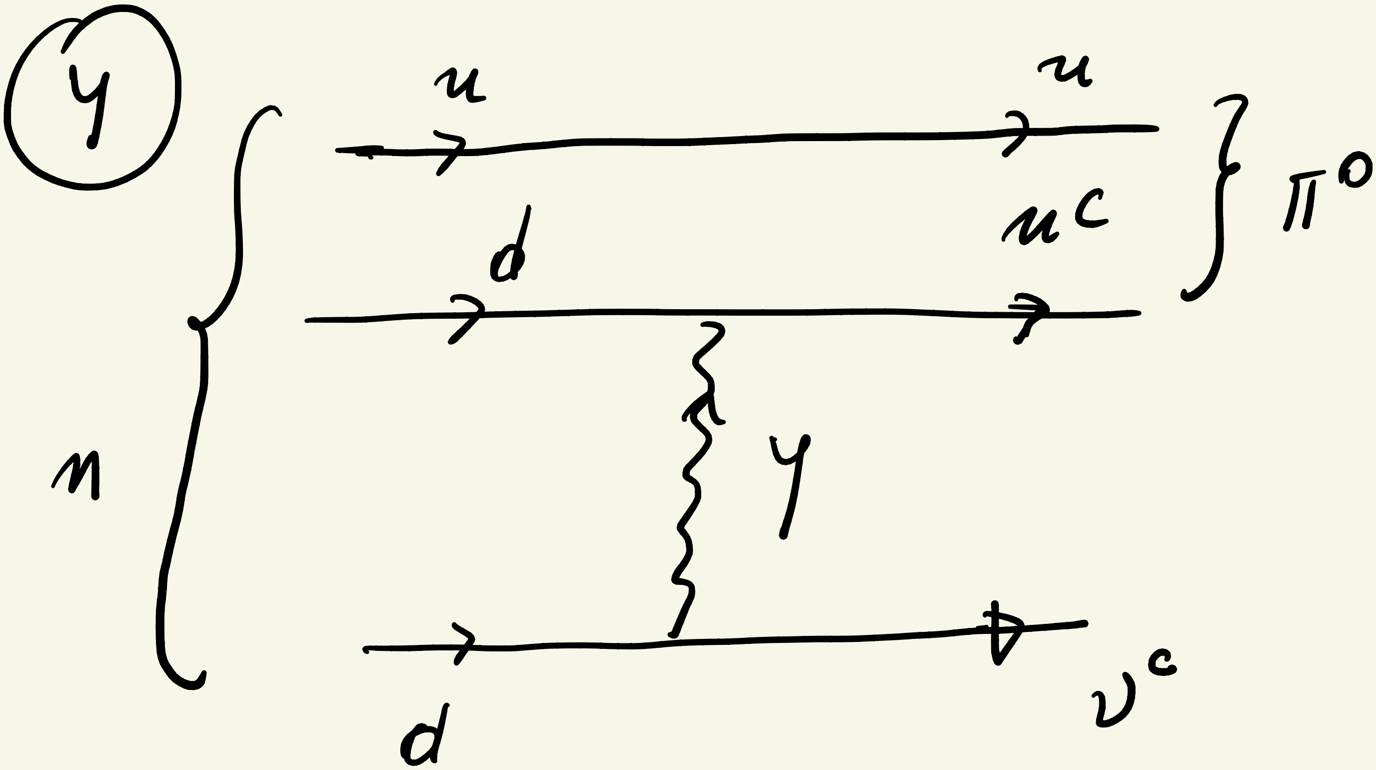
X



$$\boxed{p \longrightarrow e^c + \pi^0}$$

$$\Delta B \neq 0 , \quad \Delta L \neq 0$$

$$\{ \Delta (B-L) = 0 \}$$



$$\boxed{n \to \pi^0 + \nu^c}$$

$$\Delta B \neq 0, \quad \Delta L \neq 0$$

$$(\Delta (B-L) = 0)$$

$u \to e^c + \pi^-$ etc

- - - - -

$p \to \pi^0 + e^c$

Kamiokande
(Super)
50,000 tons of water

$$\boxed{\tau_p (p \to \pi^0 e^c) \gtrsim 10^{35}\, yr}$$

$$\boxed{\tau_p \equiv \tau_p (p \to \pi^0 e^c)}$$

- violation of charge

$$e \to \nu + \gamma$$

$$\tau(e \to \nu\gamma) \gtrsim 10^{28}\,\mathrm{yr}$$

- weak int. decay

$$\left.\begin{array}{l} \text{(i)}\ n \to p + e + \bar{\nu} \\ \tau_n \simeq 10\,\mathrm{min} \end{array}\right\} \begin{array}{l} \text{long!} \\ m_n \simeq m_p \end{array}$$

$$\boxed{\begin{array}{l} \text{(ii)}\ \mu \to e + \bar{\nu}_e + \nu_\mu \\ \tau_\mu \simeq 10^{-6}\,\mathrm{sec} \end{array}}$$

$$\Downarrow$$

$$\Gamma_\mu \propto \frac{1}{M_W^4} m_\mu^5$$

$$\frac{1}{M_W^2}$$

$m_\mu \simeq 200 m_e$

$\Updownarrow$

$m_e = 0$

$$\Downarrow$$

$$\Gamma_p \propto \frac{1}{M_X^4} m_p^5$$

$m_p \simeq$ GeV

$m_\pi \simeq 100$ MeV

$$\Downarrow \qquad \tau = \Gamma^{-1}$$

$$\Rightarrow \boxed{\tau_p / \tau_\mu = \left(\frac{M_X}{M_W}\right)^4 \left(\frac{m_\mu}{m_p}\right)^5}$$

$$\tau_p / \tau_\mu \gtrsim \frac{10^{35}\,\mathrm{yr}}{10^{-6}\,\mathrm{sec}} \simeq \frac{10^{42}}{10^{-6}} \simeq 10^{48}$$

$$\mathrm{yr} \simeq \pi \times 10^7\,\mathrm{sec} \simeq 3 \times 10^7\,\mathrm{sec}$$

$$\Downarrow$$

$$\left(M_X / M_W\right)^4 \gtrsim \left(\frac{m_p}{m_\mu}\right)^5 10^{48} \simeq 10^{53}$$

$$m_\mu \simeq 100\,\mathrm{GeV}$$

$$\boxed{M_X > 10^{13} M_W \simeq 10^{15}\,\text{GeV}}$$

- $M_X = ?$
  can I determine it?

M_Y : can it be low?

$\begin{pmatrix} X \\ Y \end{pmatrix}$: $SU(2)_L$ doublet

$$\boxed{\Delta M\,(SU(2)) \le M_W}$$

$\Rightarrow \boxed{M_X \simeq M_Y}$

$\Downarrow$

$\boxed{M_{GUT} = M_X}$ $> 10^{15}$ GeV

$\uparrow$

$\boxed{\text{unification}}$

$\Downarrow$

break $SU(5)$

$\longrightarrow U(1)_{em} \times SU(3)_c$

$$SU(5) \xrightarrow[(a)]{M_X} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\Big\downarrow \; M_W \;\; (b)$$

$$U(1)_{em} \times SU(3)_c$$

<u>(b) first</u>

Q. Which Higgs representation?

A. Hint: it must contain $SU(2)_L$ doublet

$\Downarrow$ minimal such repr.

$$\boxed{5_H}$$

$$5_F = \begin{pmatrix} d \\ e^c \\ -\nu^c \end{pmatrix}$$

$\left.\begin{matrix} e^c \\ -\nu^c \end{matrix}\right\}$ SU(2) doublet

$$5_H = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ \hline \phi^+ \\ \phi^0 \end{pmatrix}$$

$\left.\begin{matrix} T_1 \\ T_2 \\ T_3 \end{matrix}\right\}$ colour triplet Higgs

$$\boxed{Q_T = -1/3}$$

SM

$$\mathcal{L}_y = \bar{l}_L \, y_e \, \bar{\Phi} \, e_R$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \Rightarrow \boxed{\Phi = \text{doublet}}$$

$$\Downarrow$$

$$\boxed{\text{in } SU(5) \quad \Phi \simeq 5_H}$$

- Now part (a)

$$SU(5) \xrightarrow[\langle \Sigma \rangle]{} SU(3) \times SU(2) \times U(1)$$

$$(5 \to 321)$$

How to choose Σ ?

$$r(SM) = 4 \quad (= 2 + 1 + 1)$$

$$r(SU(5)) = 4$$

4 elem. of Cartan

$$C = \{[T_i, T_j] = 0\} =$$

$$\{\text{diagonal } T_i,\ T_r T_i = 0\}$$

$\Rightarrow$ 4 such T_i

$$\Downarrow$$

$$\boxed{\langle \Sigma \rangle \text{ must preserve rank}}$$

$SU(2)$: vector rep. breaks

$SU(2) \to SO(2) = U(1)$

$$\boxed{\underset{(\text{of } SO(3))}{\text{vector}} = \underset{(\text{of } SU(2))}{\text{adjoint}}}$$

$\Downarrow$

Σ = adjoint of $SU(5)$

$\Downarrow$

$\left| \Sigma \to U \Sigma U^{+}, \quad \Sigma = \Sigma^{+} \right|$

$$+ T_1 \Sigma = 0$$

- if $\langle \Sigma \rangle \neq 0$

$$\langle \Sigma \rangle \to U \langle \Sigma \rangle U^\dagger$$

$\to$ diagonal

$$\langle \Sigma \rangle = \langle \Sigma^\dagger \rangle$$

$$\Downarrow$$

$$\langle \Sigma \rangle = \mathrm{diagonal}\,(a_1, a_2, a_3, a_4, a_5)$$

$$\left(\sum a_i = 0\right)$$

$$\Rightarrow a_5 = -(a_1 + a_2 + a_3 + a_4)$$

$$\langle \Sigma \rangle \to U \langle \Sigma \rangle U^{+} =$$

$$= (1 + i\theta_a T_a) \langle \Sigma \rangle (1 - i\theta_a T_a)$$

$$= \langle \Sigma \rangle + i\theta_a \underbrace{[T_a, \langle \Sigma \rangle] + \cdots}$$

$$\hat{T}_a \langle \Sigma \rangle = [T_a, \langle \Sigma \rangle]$$

but $\langle \Sigma \rangle =$ diagonal

$T_i \in C =$ diagonal

$$\Rightarrow \boxed{\hat{T}_i \langle \Sigma \rangle = 0}$$

$$\Downarrow$$

$\boxed{\langle \Sigma \rangle \Rightarrow \text{preserves the reals}}$

$\boxed{\text{adjoint}}$

$\Sigma = \{5 \times \bar{5}\} \Rightarrow$ $\boxed{\text{24 real components}}$

$\mathrm{Tr}\, \Sigma = 0$

$\Sigma = \Sigma^{+}$

$\boxed{\text{SSB with } \Sigma}$

$U^{+}U = UU^{+} = 1$

$\Sigma \to U \Sigma U^{+} \Rightarrow \Sigma^{2} \to U \Sigma^{2} U^{+}$

$\Rightarrow \mathrm{Tr}\, \Sigma = \mathrm{inv.} = 0$

$\Sigma^{3} \to U \Sigma^{3} U^{+}$

$\Rightarrow \mathrm{Tr}\, \Sigma^{2} = \mathrm{inv.}$

$\Sigma^{4} \to U \Sigma^{4} U^{+}$

$$\Downarrow$$

$$V_\Sigma = -\frac{\mu^2}{2} \operatorname{Tr} \Sigma^2 + \frac{a}{4} \left(\operatorname{Tr} \Sigma^2 \right)^2 + \frac{b}{4} \operatorname{Tr} \Sigma^4 + \frac{m}{3} \operatorname{Tr} \Sigma^3$$

$$\Uparrow$$

- $m = 0 \quad (\Sigma \to -\Sigma)$
- $m \neq 0 \Leftarrow$ Guth, Weinberg '82

$$\boxed{\operatorname{Tr} \Sigma = 0 \quad \text{constraint}}$$

$\Downarrow$ Lagrange multiplier

$$\Swarrow$$

$$V = -\frac{\mu^2}{2} \operatorname{Tr} \Sigma^2 + a/4 \left(\operatorname{Tr} \Sigma^2 \right)^2 + b/4 \operatorname{Tr} \Sigma^4 + \alpha \operatorname{Tr} \Sigma$$

$$\partial V / \partial \alpha = 0$$

$$\langle \Sigma \rangle = \mathrm{diag}\,(a_1, a_2\, a_3\, a_4\, a_5)$$

$\Rightarrow$ minimize $V(\Sigma)$ and

find global minimum

$$\boxed{\text{HARD!}}$$

- <u>Instead</u>

$\langle \Sigma \rangle = ?$ $\therefore$ SM symmetry

$$\| $$

$$SU(3) \times SU(2) \times U(1)$$

$$a_i = ?$$

M_X

$\Downarrow$

$\mathrm{Tr}\langle\Sigma\rangle = 0$

$\Downarrow$

$$\boxed{\langle\Sigma\rangle = v\,(\underbrace{1,\ 1,\ 1}_{\text{unbroken } SU(3)_c};\ \underbrace{-3/2,\ -3/2}_{SU(2) \text{ unbroken}})}$$

$$\mathrm{Tr}(\Sigma)^2 = v^2\left(9/2 + 3\right) = \frac{15}{2}v^2$$

$$\mathrm{Tr}\langle\Sigma\rangle^4 = v^4\left(81/8 + 3\right) = \frac{105}{8}v^4$$

$\Downarrow$

$$\underline{\underline{V = -\frac{\mu^2}{2}\frac{15}{2}v^2}} + \frac{a}{4}\left(\frac{15}{2}v^2\right)^2 + $$

$$+ \frac{b}{2}\frac{105}{8}v^4$$

$\Downarrow$ $(15 \cdot 7 = 105)$

$$\frac{\partial V}{\partial \vartheta} = 0 = \Big[-\frac{15}{2}\mu^2 + \frac{a}{\cancel{4}}\frac{15}{2}\vartheta^2\frac{15}{2}\cdot\cancel{4}$$

$$+\frac{2b}{\cancel{4}}\frac{105}{8}\cancel{4}\vartheta^2\Big]\vartheta$$

$$= +\frac{15}{2}\vartheta\left[-\mu^2 + a\frac{15}{2}\vartheta^2 + \frac{7}{4}b\vartheta^2\right]$$

$$\vartheta = 0, \quad \boxed{(15a+7b)\vartheta^2 = 2\mu^2}$$

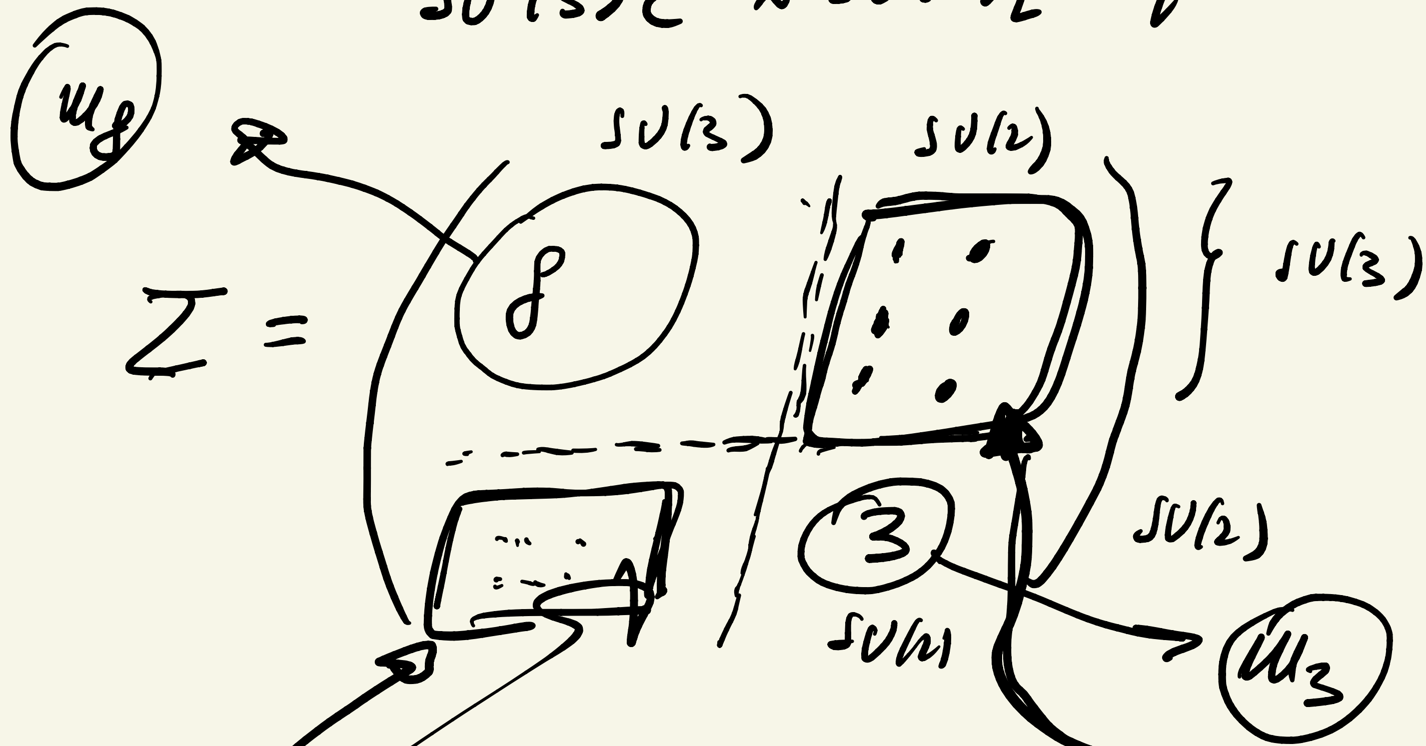
maximum!

local minimum?

all eigenvalues of 2nd derivatives > 0

Σ = 24 fields

$SU(3)_c \times SU(2)_L$ good



recall: some fields eaten

+ $\boxed{\text{singlet}} \leftarrow m_1$

$\Longrightarrow \boxed{m_1, m_3, m_8} \boxed{> 0}$

$$D = \begin{pmatrix} u \\ d \end{pmatrix}$$

dublet

$$\boxed{m_u - m_d \lesssim M_W}$$

$$m_e - m_\nu \simeq 10^{-5} M_W$$

$$m_c - m_s \simeq 10^{-2} M_W$$

$$m_t - m_b \simeq M_W$$

$$M_X - M_Y \lesssim M_W$$

$$\frac{m_e - m_\nu}{m_e + m_\nu} \simeq O(1)$$

$$\frac{M_X - M_Y}{M_X + M_Y} \lesssim \frac{M_W}{M_X} \lesssim 10^{-13}$$