Neutrino mass and Grand Unification

Lecture V

2/11/2021

LMU
Fall 2021

Standard EW Model:

Last lecture

Massive gauge bosons

① Proca

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$= \frac{1}{2} A_\mu \left[\left(\Box + m_A^2 \right) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu$$

$$\Rightarrow \left[\left(\Box + m_A^2 \right) g^{\mu\nu} - \partial^\mu \partial^\nu \right] A_\nu = 0$$

$$\Box \equiv \partial^\mu \partial_\mu$$

$$\Rightarrow \boxed{\partial^\mu A_\mu = 0}$$

$\Leftrightarrow$ 3 d.o.f.

$s = 1$ particle

$$\Delta_{\mu\nu}(A) = \frac{-i}{p^2 - m_A^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2} \right]$$

$\nearrow$

bad, when $p \gg m_A$

$\Rightarrow$ divergencies!

$\Updownarrow$ iff

A_μ = not coupled to conserved current

② Higgs U(1) gauge

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu \phi = \partial_\mu - i g Q A_\mu$$

$$Q \phi = \phi$$

$$\phi \to e^{i\alpha(x) Q} \phi \qquad U(1)$$

$$\phi = \frac{1}{\sqrt{2}}\left(h + \overset{\swarrow}{v} + iG\right)$$

$$V_0 = \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 \qquad (T=0)$$

$$\Downarrow \qquad m_A = g v$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu G)^2 + \frac{1}{2} m_A^2 A_\mu A^\mu$$

(bad)

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underline{m_A G \partial^\mu A_\mu}$$

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi m_A G\right)^2$$

$$\Downarrow$$

$$\mathcal{L} + \mathcal{L}_{gf} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{2}\xi m_A^2 G^2$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}m_A^2 A_\mu A^\mu$$

↑ new (beyond Proca)

⇓

$$D(G) = \frac{i}{p^2 - \xi m_A^2}$$

$$\Delta_{\mu\nu}(A) = \frac{-i}{p^2 - m_A^2}\left[g_{\mu\nu} + (\xi - 1)\frac{p_\mu p_\nu}{p^2 - \xi m_A^2}\right]$$

$G \Rightarrow$ <u>not</u> physical $\quad m_G^2 = \xi m_A^2$

<u>compute</u> $(\xi = 1)$ $\Big\}$ <u>all finite!</u>

<u>ξ independence!</u>

"unitary" gauge $\xi \to \infty$

$\Rightarrow m_G \to \infty \Leftrightarrow$ G decouples

$$\Delta_{\mu\nu}^{(A)} = \Delta_{\mu\nu} \ (\text{Proca})$$

SM (EW)

Goldstone 1961

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

(Goldstone) Higgs - Weinberg boson

$\Downarrow$

$$\mathcal{L}_y = \bar{q}_L\, y_d\, \Phi\, d_R + \bar{q}_L\, y_u\, i\sigma_2 \Phi^* u_R$$

$$+ \bar{l}_L\, y_e\, \Phi\, e_R \quad + \text{h.c.}$$

$$\Downarrow \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\bar{q}_L\, y_d\, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R \quad + \text{h.c.}$$

$$= \frac{1}{\sqrt{2}} (v+h)\, y_d\, \underbrace{(\bar{d}_L d_R + \bar{d}_R d_L)}_{\bar{d} d}$$

↑ Higgs int.

$$\Downarrow$$

$$m_d = \frac{1}{\sqrt{2}}\, y_d\, v$$

$$\mathcal{L}_h = \frac{1}{\sqrt{2}} y_f \, h \, \bar{f} f$$

$$f = d, \; u, \; e$$

↗ NO neutrino

$$\frac{1}{\sqrt{2}} y_d = \frac{m_d}{v}$$

$$= \frac{g}{2} \frac{m_d}{M_W}$$

$$M_W = \frac{g}{2} v = M_Z \cos\theta_W$$

$$\Downarrow$$

mass

$$\mathcal{L}_h = \left(\frac{g}{2} \frac{m_f}{M_W} \right) h \, \bar{f} f$$

$$e = g \sin\theta_W \Rightarrow g!$$

$v \to h + v$ | all the terms

$$\Downarrow$$

W, Z :

$$M_W^2 W_\mu^+ W^{\mu -} \left(1 + \frac{h}{v}\right)^2$$

$$\frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2$$

$$\Downarrow$$

$$M_W^2 W_\mu^+ W^{\mu -} + \underbrace{2 \frac{h}{v} M_W^2 W_\mu^+ W^{\mu -}}$$

$$\|$$

$$g M_W W_\mu^+ W^{\mu -}$$

Higgs ⟺ couples to mass
iff gives you mass

How to probe it?

(a) <u>indirect</u>

$h \Rightarrow$ decays

discovery } July 2012

$h \to \gamma\gamma$

$h \to b\bar{b}$, $\tau\bar{\tau}$, "$t\bar{t}$"

"W^+W^-", "ZZ"

μ ? getting there

compute: $h \to f\bar{f}$

$h \to WW^*$ ← off-shell

ZZ^* ↗

we accept Higgs as the source of mas

(b) direct

we live in the Higgs phase

$$\Phi_0 = v \neq 0$$

"Vacuum" carries Y, T_3

$$(Q_{em}\, \Phi_0 = 0)$$

Probe: heat it up

Kirzhnits 1972
Linde, -"- 1973
Weinberg 1974

$$V_T = V_0 + \boxed{\frac{1}{2} a T^2 \Phi^* \Phi}$$

$\Uparrow$

$T \gg v$

$k = \hbar = c = 1$

$\uparrow$

Boltzmann

dimensional

$(E_{kin} = kT)$

$$a = \lambda + g^2 \quad \text{(positive)}$$

$T \gg v$

$$V_T = \frac{1}{2}(a T^2 - \lambda v^2)\, \Phi^+ \Phi + \cdots$$

$\underbrace{\qquad}_{>0} \Rightarrow \boxed{\Phi_0 = 0}$

⇗

no symmetry breaking

⟺ high T symmetry restoration

$T_c \simeq v \simeq 100\ GeV$

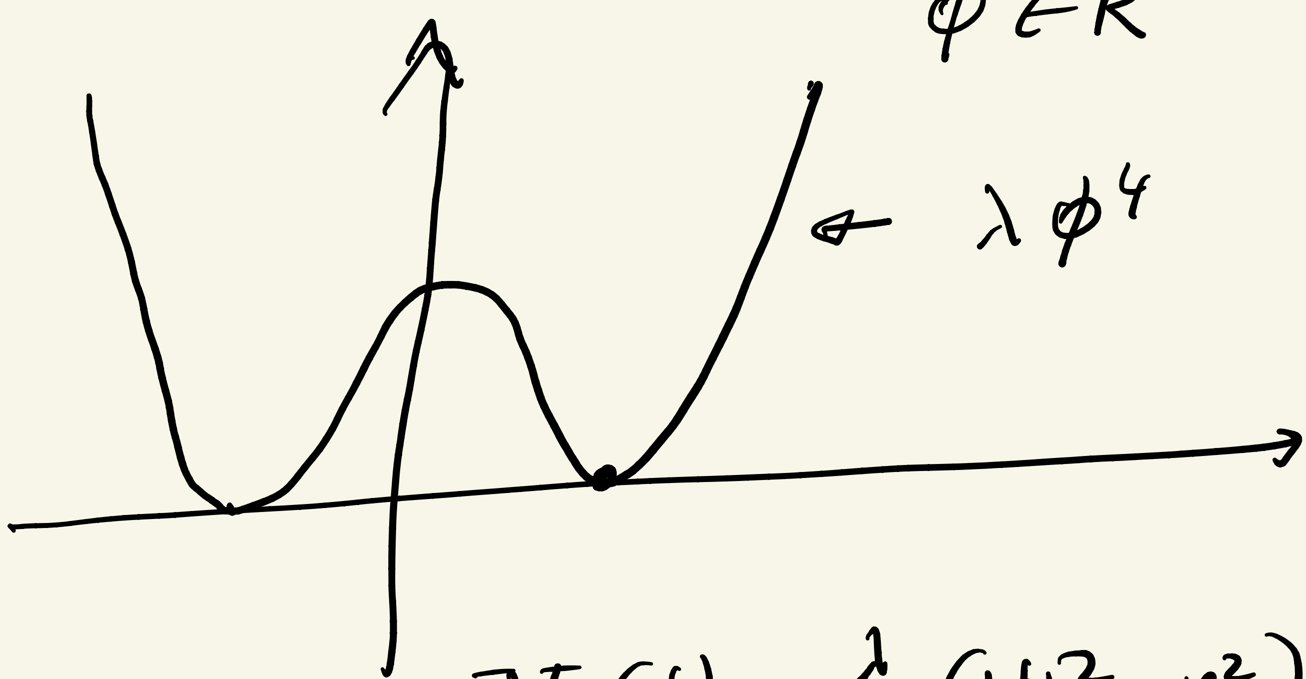
$eV \simeq 10^4\ ^\circ K$

$T_\odot \simeq 10\ MeV$

⇒ $T_c \Leftrightarrow$ early universe

DIRECT test of Higgs mechanism is impossible (for now)

$\phi \in R$



$$V(\phi) = \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2$$

$$m_h^2 = 2\lambda v^2 \simeq (125\ \text{GeV})^2$$

$$\lambda \simeq O(1/10 - 1)$$

Inflation: $\lambda_{inf.} \simeq 10^{-14}$

$\Downarrow$

We check Higgs indirectly

SM: we "predict" W mass

we do not predict

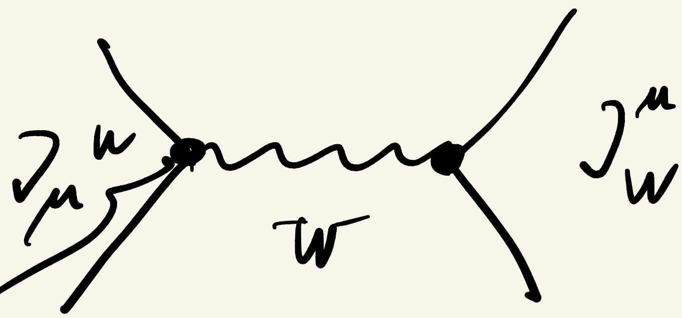
Higgs mass

$\nearrow$

People say

W mass

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



g

$\Downarrow$

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 M_W^2 \sin^2\theta_W}$$

↑ measure!

$\Rightarrow \quad M_W = 80 \text{ GeV}$

- Higgs $\quad m_h^2 = 2\lambda v^2$

$$V = \frac{\lambda}{4}\left[(h+v^2)^2 - v^2\right]^2$$

$$= \frac{\lambda}{4} h^4 + \frac{1}{2} m_h^2 h^2 + m_h h^3$$

$\uparrow$ int.

$$h + h \longrightarrow h + h$$

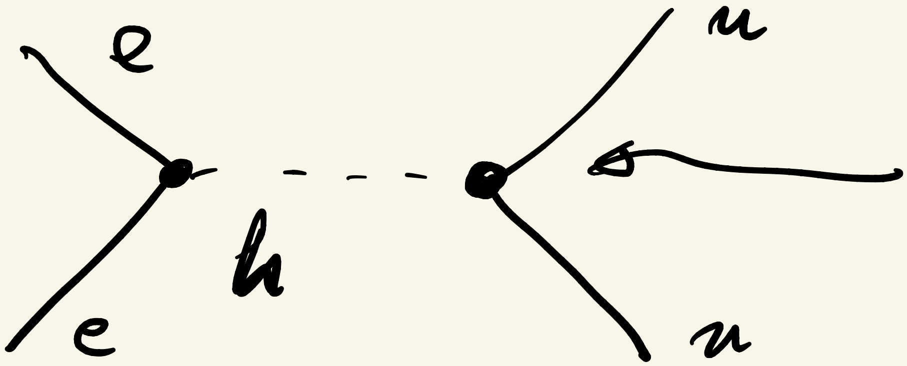
$\Rightarrow$ probes (measures) λ

Higgs mass = "predicted" as W mass

m_h = related to physical processes

$h \to e\bar{e} \Leftarrow$ probe m_e

all masses in SM = correlated to physical processes



$$y_u \simeq \frac{m_u}{m_W} \ll 1$$

$$m_h \sim m_W$$

$$\frac{G_{Higgs}}{\sqrt{2}} = \frac{y_e\, y_u}{m_h^2} =$$

$$= \frac{g^2}{4}\,\frac{m_e\, m_u}{m_h^2}$$

measure g $\Longrightarrow$ m_h from G_{Higgs}

Probe of mass $\Longleftrightarrow$

mass related 1–1 to

a physical process

and NOT predicting

m_e / m_μ ----

$\Downarrow$

first, you must know

the origin of mass itself

Natural philosophy

vs

philosophy of naturalness

G.S.

---

phase transition

$$T > T_c \cong 100\ \text{GeV}$$

$$\phi_0 = v$$

what phase transition?

1st, 2nd, ... ?

1st phase transition

$\Rightarrow$ gravitational waves

Riotto, ?

$m_h < 50$ GeV | prediction for baryogenesis

SM: $\rightarrow$ Shaposhnikov et al. $\Leftarrow$

Laine $\Rightarrow$ "smooth crossover"

Michal: Mukhonov
book