LMU GUT Course

Lecture IX

1/12/2020

LMU

Fall 2020

$\underline{SU(3)}$ (ew)

L

$$b_L = \begin{pmatrix} p \\ n \\ \cdots \\ p^c \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} e^c \\ \nu^c \\ \cdots \\ e \end{pmatrix}_R = l_R = 3_F$$

leptons

$$(p^c)_L \equiv C\,\bar{p}_R^{\,T}$$

$$3_F = F \text{ of } SU(3)$$

$$3_F \to U\,3_F \qquad (e^c)_R = C\,\bar{e}_L^{\,T}$$

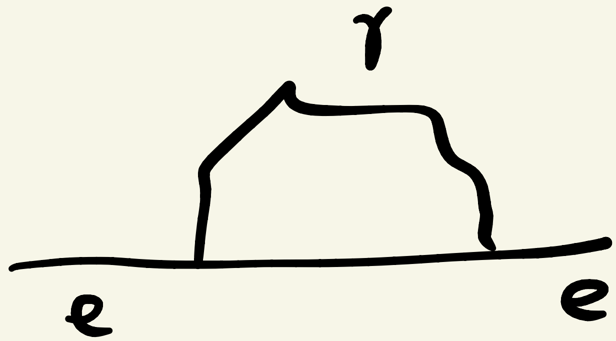
$$(\nu^c)_R = C\,\bar{\nu}_L^{\,T}$$

$$\cancel{\boxed{\bar{b}_L\, M\, l_R = \text{invariant}}}$$

$\boxed{M = 0}$ $\longrightarrow$ $\boxed{\text{more symmetry}}$

$\boxed{\Delta B = 2}$

$\boxed{Q \in \Delta}$



Weisskopf
1930

$$m_e = m_e^0 \left[1 + \frac{\alpha}{4\pi} \ln \frac{\Lambda}{m_e} \right]$$

$$m_e^0 = 0 \Rightarrow m_e = 0$$

$\boxed{\text{to any loop}}$

$$\boxed{m_e = m_e^0 \left[1 + \left(\frac{\alpha}{4\pi}\right)^{47} \ln \frac{\Lambda}{m_e} \right]}$$

- $m_e^0 = 0 \Rightarrow \mathcal{L}_{kin} = \mathcal{L}_{kin}(e_L)$
$+ \mathcal{L}_{kin}(e_R)$

$\Rightarrow e_L \to e^{i\alpha} e_L, \; e_R \to e_R$

chiral $U(1)_L \times U(1)_R$

$e_L \to e_L, \; e_R \to e^{i\beta} e_R$

- any order in pert. theory
$\to$ chiral symmetry

$\boxed{\text{'t Hooft "naturalness" (technical)}}$

physical parameter $\leftrightarrow$

$$\boxed{\begin{gathered}\Theta^0 = 0 \Rightarrow \text{more symmetry} \\ \Rightarrow \Theta = 0 \ \text{to all orders}\end{gathered}}$$

$$\Theta = \Theta^0 \Big[1 + \underbrace{\left(\frac{\alpha}{4\pi}\right)^n \ln \frac{\Lambda}{\Theta}}_{\text{small "infinity"}} \Big]$$

n - loop

$$\boxed{\Theta^0 \neq \text{special point}}$$

small "infinity"

SM:

$m_t \simeq M_W \quad (y_t = 1)$

$m_e \simeq 10^{-5} M_W \quad (y_e \simeq 10^{-5})$

$$\boxed{\tau_p \gtrsim 10^{34}\,\mathrm{yr}}$$

$$\boxed{\tau_\mu \simeq 10^{-6}\,\mathrm{sec}}$$

- $\theta_0 = 0 \longrightarrow \theta_0 = \varepsilon$ ~ also protected

[exact symmetry]

$m_e^0 = 0$ chiral

$\theta_0 = 2\epsilon$

$\downarrow$

$\theta_0 = 10\epsilon$

$\downarrow$

$\theta_0 = 10^{10}\epsilon$

$\boxed{\text{QED}}$

Charge = conserved

$U(1)$ = exact

• technical naturalness

(i) fermion mass = "naturally" small

$$\boxed{m_f = m_f^0\left[1+\frac{\alpha}{4\pi}\ln\frac{\Lambda}{m_e}\right]}$$

• no naturalness

Higgs mass

$$m_H^2 = m_0^2 + \frac{\alpha}{4\pi}\Lambda^2$$

$m_0 = 0 \not\Rightarrow$ more symmetry

"fermions are special"

their mass is protected

$$\boxed{m_\nu \ll m_e}$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix} W$$

"technical" naturalness

– "natural" naturalness

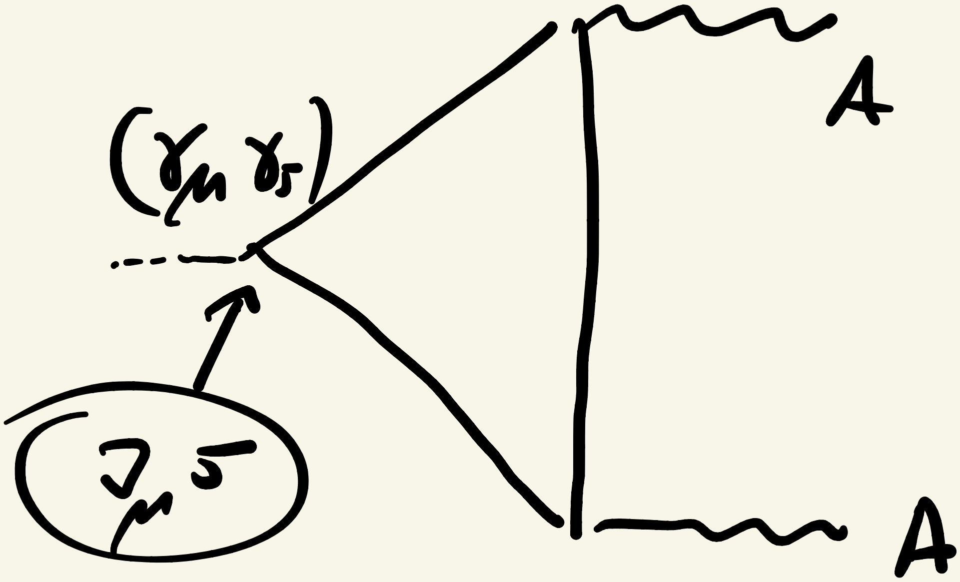
ANOMALY

$$\psi \to e^{i\alpha\gamma_5}\psi \quad \text{chiral}$$

$$\left(\psi_L \to e^{i\alpha}\psi_L,\ \psi_R \to e^{-i\alpha}\psi_R\right)$$

$$\Rightarrow \partial_\mu J^\mu_5 \propto \epsilon_{\mu\nu\alpha\rho} F^{\mu\nu}_a F^{\alpha\rho}_a$$

$a = 1 \ldots\ \ldots$



$\downarrow$ gauge anomalies

QED

$$\partial^\mu j_\mu = 0$$

$$\Downarrow$$

Ward identities

$$\Downarrow$$

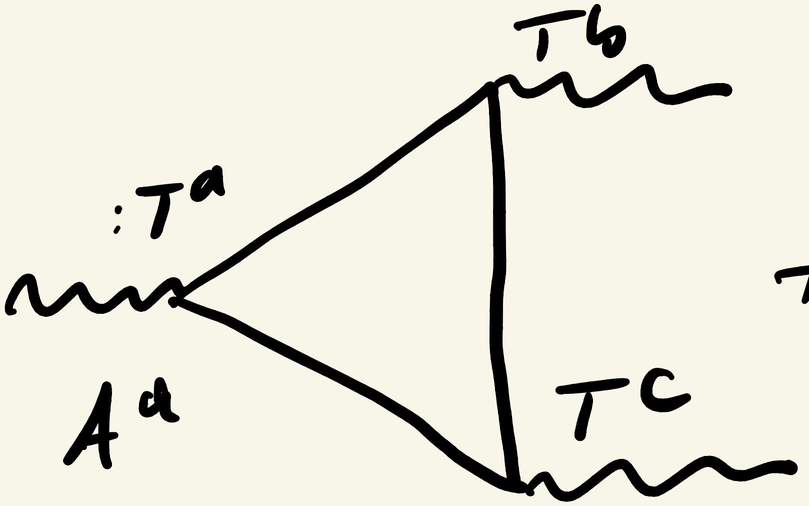
gauge theory

$$\partial^\mu j^5_\mu = 0$$

anomaly cancellation!!

a MUST

$$\Downarrow$$



+ perm.

<u>massless fermions</u>

$$D_\mu = \partial_\mu - ig\, T_a A_\mu^a$$

$$\Downarrow$$

$$\boxed{\begin{array}{l} A_{abc} \propto \mathrm{Tr}\{T_a, T_b\}\, T_c = d_{abc} \\ \text{(anomaly coeff)} \\ \quad \downarrow \\ \quad = 0 \end{array}}$$

$$\boxed{\text{chiral rep.}}$$

(i) $\psi_L \quad , \quad \psi_R'$

(ii) $\psi_L \quad , \quad (\psi'^c)_L = C\,\overline{\psi_R'}^T$

$$D_\mu \psi_L = \partial_\mu - ig\, T_a{}^L A_\mu{}^a$$

$$D_\mu \psi_R = \partial_\mu - ig\, T_a{}^R A_\mu{}^a$$

$$\boxed{\begin{array}{c} T_a{}^L \leftrightarrow T_a{}^R \\ \text{not related} \end{array}}$$

<u>Example:</u> SM

<u>SU(2)</u> $\quad T_a{}^L = \frac{\sigma_a}{2} \qquad T_a{}^R = 0$

- $T_a{}^L = T_a{}^R$

$$\Downarrow$$

<u>no</u> anomaly

$$\psi = \psi_L + \psi_R$$

$$\Rightarrow \quad D_\mu \psi = \partial_\mu - ig\, T_a \psi$$

$$A_\mu \left(\overline{\psi_L}\, \gamma_\mu T_a{}^L \psi_L + \overline{\psi_R}\, \gamma^\mu T_a{}^R \psi_R \right)$$

$$= A_\mu\, \overline{\psi}\, \gamma^\mu T_a \psi$$

$$\boxed{\text{example : QED}}$$

$$\Downarrow$$

no γ_5

$$\Rightarrow \boxed{\text{no anomaly}}$$

• $T_a^R = T_a^L \Rightarrow$ <u>no</u> anomaly

$$T_a^R = S\,T_a^L\,S^{-1} \quad \text{unitary}$$

$$\boxed{\begin{aligned} \Rightarrow\ & \mathrm{Tr}\,\{T_a^R, T_b^R\}\,T_c^R = \\ &= \mathrm{Tr}\,\{T_a^L, T_b^L\}\,T_c^L \end{aligned}}$$

$\begin{pmatrix} u \\ d \end{pmatrix}_L \Leftarrow$ same chirality

$\Updownarrow$

$\begin{pmatrix} \vdots \\ | \end{pmatrix}_L \Leftarrow$ any Repr. at gauge

$$\boxed{\psi_R \to (\psi^c)_L = C\bar{\psi}_R^T}$$

$$\boxed{\text{anomalies must add up to zero}}$$

- $\psi \to U\psi$ $\qquad T_a^\dagger = T_a$

$$U = e^{i\theta_a T_a} \qquad \mathrm{Tr}\, T_a = 0$$

$$\psi^c = C\bar{\psi}^T \propto \psi^*$$

$$\bar{\psi} = \psi^\dagger \gamma^0 \Rightarrow \psi^T = \gamma_0 \psi^*$$

$$\psi^c \to e^{-i\theta_a T_a^*}\psi^c$$

$$= e^{i\theta_a T_a^c}\psi^c$$

- if $\exists$ unitary S s.t.

$$T_a^* = -S T_a S^{-1}$$

$$\Downarrow$$

anomaly vanishes

<u>Proof:</u>

$$T_a^c = -T_a^* = -T_a^T$$

$$T_a = T_a^+$$

$$\mathrm{Tr}\{T_a^c, T_b^c\} T_c^c$$

$$= -\mathrm{Tr}\{T_a^T, T_b^T\} T_c^T$$

$$= -\mathrm{Tr}\left(T_c \{T_b, T_a\}\right)^T$$

$$= -\mathrm{Tr}\, T_c \{T_b, T_a\}$$

$$= -\mathrm{Tr}\{T_b, T_a\} T_c$$

$$= -\mathrm{Tr}\,\{T_a, T_b\}T_c = -d_{abc}$$

$$A_{abc} \propto d_{abc} = -d_{abc} \propto -A_{abc}$$

$$\Rightarrow \boxed{A_{abc} = 0}$$

example $\boxed{SU(2)}$

$$T_a = \sigma_a/2$$

$$\mathrm{Tr}\,\{\sigma_a, \sigma_b\}\sigma_c = 0$$

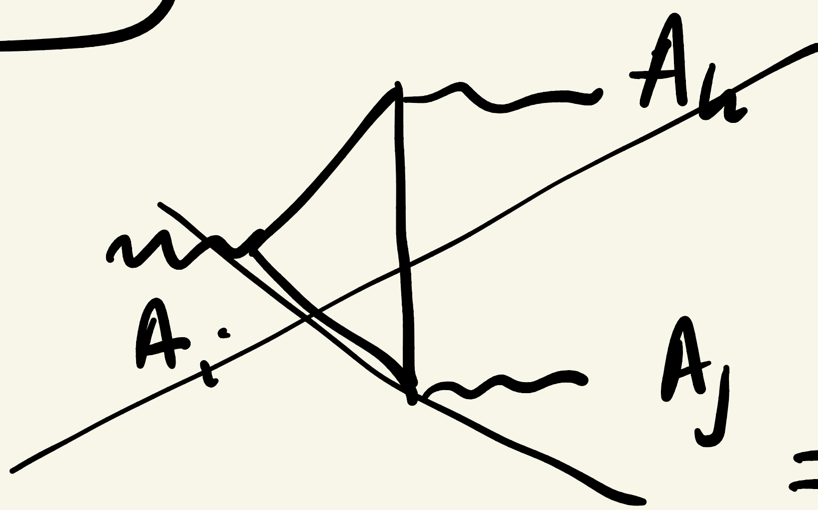
since $\{\sigma_a, \sigma_b\} = 2\delta_{ab}$

$$\mathrm{Tr}\,\sigma_a = 0$$

$$\boxed{\begin{array}{c} \sigma_a^* = -\sigma_2 \sigma_a \sigma_2 \\ S = \sigma_2 \end{array}}$$

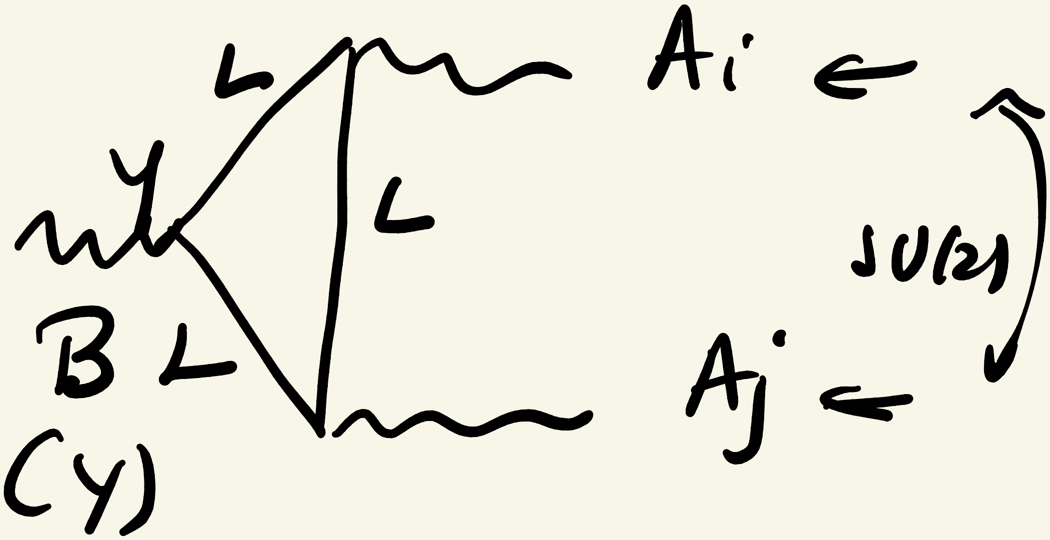
$$\boxed{\begin{array}{c}\text{NOT true in } SU(3) \\ \{\lambda_a, \lambda_b\} \neq 2\delta_{ab}\end{array}}$$

$\boxed{\text{SM}}$



$i, j, k = 1, 2, 3$

$= 0$



$i = j = 3 \Rightarrow \boxed{\mathrm{Tr}\, Y_L = 0}$

$$Q = T_3 + Y/2 \quad (*)$$

$$\mathrm{Tr}\, T_3 = 0$$

$\Delta T_3 = 1$
in doublet

$$\boxed{\mathrm{Tr}\, Q_L = 0}$$

$$\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L + \begin{pmatrix} \nu \\ e \end{pmatrix}_L}_{\mathrm{Tr}\,(\text{over this})}$$

$$\boxed{\begin{aligned} q_u &= q_d + 1 \\ q_\nu &= q_e + 1 \end{aligned}} \quad (*)$$

$$\Downarrow$$

$$3(q_u + q_d) + q_\nu + q_e = 0$$

$$\boxed{3(2q_d + 1) + q_e + 1 = 0}$$

$\boxed{q \leftarrow l \text{ charge relation}}$

- μ, ν_μ, s, c must be there in order to cancel anomaly

add

$$\underbrace{Q_L \;,\; Q_R}_{\text{same } SU(2)} \qquad (T_a^L = T_a^R)$$

$\Rightarrow$ no anomaly

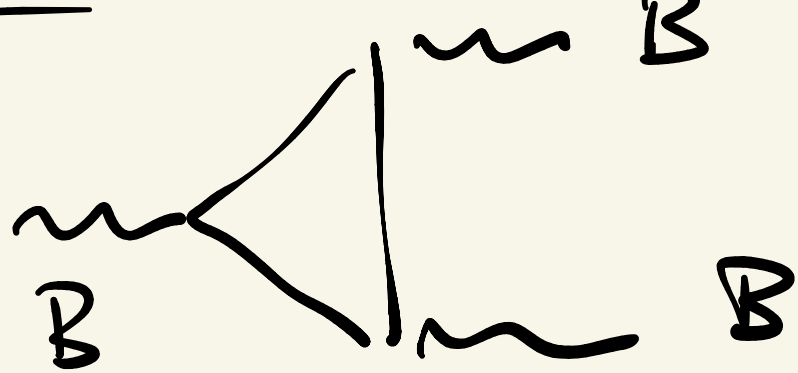
$\boxed{\text{charge } (Q_L, Q_R) = \text{arbitrary}}$

$= 1.3745\ldots$

$SU(2) \times U(1) \longrightarrow U(1)$

<u>no</u> monopoles

<u>SM</u>



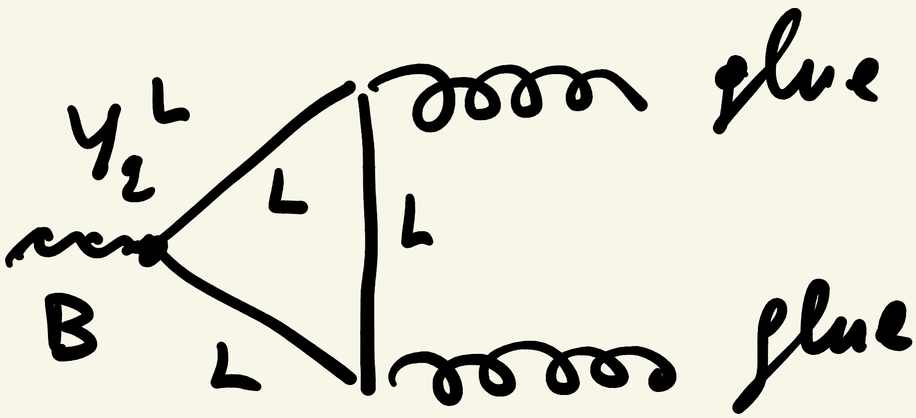
$\mathrm{Tr}\, Y^3 = 0$

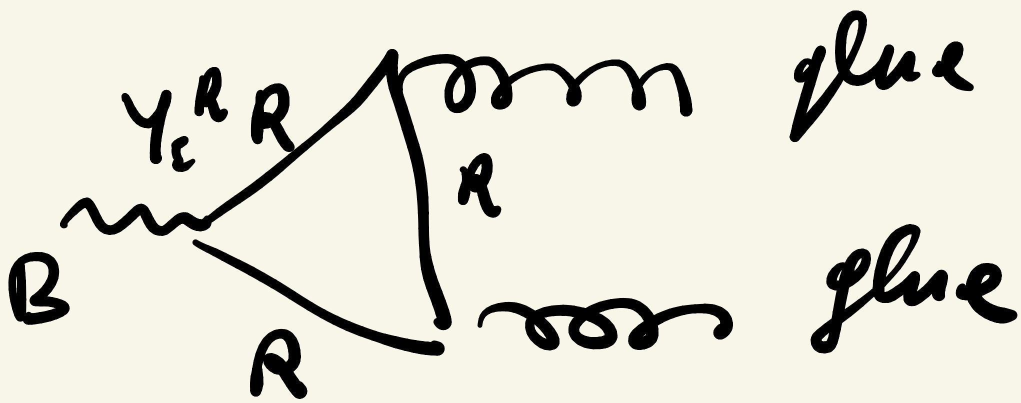
Most direct anomaly cancellation

$T_a^L = T_a^R$ (vector-like)

$$\boxed{\text{QCD} = \text{vector-like}}$$

$$J^\mu = \bar{\psi}\gamma^\mu\psi$$





$$\mathrm{Tr}\, Y_q^L = \mathrm{Tr}\, Y_q^R \quad \text{(quarks)}$$

$$Y = Q - T_3$$

$$\Rightarrow \boxed{\mathrm{Tr}\, Q_q^L = \mathrm{Tr}\, Q_q^R}$$

vector-like

$$F_L \longleftrightarrow F_R$$

$$T_a^L = T_a^R$$

$\underline{M}\ \bar{F}_L F_R$ is allowed

$\underline{M} \to 0$ is OK

but $M \to \infty$ must decouple

$$\begin{pmatrix} e^c \\ \nu^c \\ \cdots \\ e \end{pmatrix}_R \Longleftrightarrow \begin{pmatrix} p \\ n \\ \cdots \\ p^c \end{pmatrix}_L$$

$$\Uparrow \qquad T_a^L = T_a^R$$

anomalies

$$(e^c)_R = C\,\bar{e}_L^{\,T}$$

$$(\nu^c)_R = C\,\bar{\nu}_L^{\,T}$$

LH weak