

LMU GUT Course

Lecture VIII

27/11/2020

Fall 2020  
LMU



$$SU(3) \supseteq SU(2) \times U(1)$$

• quarks

$$\rho = uud \quad u = ccc$$

$$q_u = 2/3 \quad q_d = -1/3$$

$w \leftrightarrow LH \text{ fermions}$

$$F = \begin{pmatrix} u \\ d \\ \bar{D} \end{pmatrix}_L$$

$$F = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$F \rightarrow U_3 F$$

$SU(3) \times L_{\text{curv}} U(1)$

L      R

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \rightarrow U_2 \begin{pmatrix} f'_1 \\ f'_2 \end{pmatrix}$$

$U_3$  acts on LH fermions

$\rightarrow \text{RH} - 11 -$

separate

$$Q_{\text{em}} = \sum_{\text{Cartan}} c_\alpha T_\alpha$$

Cartan

$$(T_3, T_8)$$

$$\boxed{T_\nu Q_{\text{em}} = 0}$$

$$3_L = F_L = \begin{pmatrix} u \\ d \\ \bar{d} \end{pmatrix}_L^W \quad \begin{pmatrix} u \\ d \\ \bar{d} \end{pmatrix}_R$$

$$\boxed{\frac{d_R}{D_R}, \frac{u_R}{D_R}} \quad ? \rightarrow$$

$SU(2)$  Rights

$$\psi_R \rightarrow (\psi^c)_L = C \bar{\ell}_R^T$$

$$C^T = -C$$

Dirac  $C$

$$C \gamma_\mu C^T = -\gamma_\mu^T$$

$$C^T C = 1$$

$$\begin{array}{ccc} u & \leftarrow & 2/3 \\ d & \leftarrow & -1/3 \\ D & & -1/3 \end{array}$$

charges

$$\Rightarrow \begin{array}{ccc} \psi_1 & \rightarrow & 2/3 \\ \psi_2 & \rightarrow & -1/3 \\ \hline \psi_3 & = & -1/3 \end{array}$$

$\psi_R$  must be  $\psi_3$  because

$\pi_R \rightarrow SU(2)$  singlet

$(\begin{matrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{matrix}) \rightarrow$  only  $\psi_3 =$  singlet

$SU(2) \times U(1)$

Glashow '61

A

easy

(P, n)

(v, e)

NO  $\gamma$  couls

V - A

Marsden, Sudarshan  
'5+

$\Leftrightarrow p_L, u_L$  have weak int

$$\begin{pmatrix} p \\ u \\ \bar{p}^c \end{pmatrix}_L \Leftrightarrow \begin{pmatrix} u^c \\ p^c \end{pmatrix}_R$$



$$(u^c)_R = C \bar{u}_L^T$$

$$(p^c)_R = C \bar{p}_L^T$$

$$Q_F = Q_P = 1$$

$M_R \sim (u^c)_L = \text{singlet}$

$$P^c = ? = p^c$$

$$Q = \sum_a c_a T_a$$

Coulomb

$$Q_{\text{singlet}} = 0$$

$$T_a \text{ singlet} = 0$$

Given :  $(P^c)_L = (p^c)_L$

$$(\gamma^c)_L = C \bar{\psi}_R^\dagger \gamma^c$$

$\star$

SM weak int. singlet

$$(\gamma^c)_L = C \delta_0 \bar{\psi}_R^\dagger \gamma^c \quad (c = i, \delta_1, \delta_2)$$

$$= i \gamma_2 \psi_R^*$$

$$= \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \psi_R^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 \psi_R^* \\ 0 \end{pmatrix} \cancel{+ LH}$$

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$L(R) = \frac{1 \pm \gamma_5}{2}$$

$$L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$(Y^c) = c \gamma_0 \psi_R^*$$

$$= C \gamma_0 \frac{1-\gamma_5}{2} \gamma^*$$

$$R(\gamma^c) = \frac{1-\gamma_5}{2} C \gamma_0 \frac{1-\gamma_5}{2} \gamma^*$$

$$= C \frac{1-\gamma_5}{2} \gamma_0 \frac{1-\gamma_5}{2} \gamma^*$$

$$= C \gamma_0 \frac{1+\gamma_5}{2} \frac{1-\gamma_5}{2} \gamma^*$$

$\overline{\quad} = 0$

$$L(\gamma^c) = C \gamma_0 \frac{1-\gamma_5}{2} \frac{1-\gamma_5}{2} \gamma^*$$

$$\boxed{\gamma^c(\gamma_L) = LH}$$

$(4^c)_L \leftrightarrow$  no weak int.

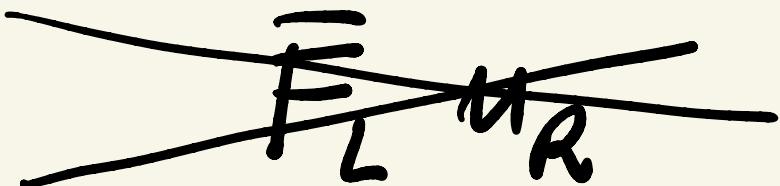
$$(4^c)_R \equiv c \bar{4}_L^\top$$

weak int.

$$F = \begin{pmatrix} p \\ u \\ d \\ \bar{p}^c \\ \bar{u}^c \\ \bar{d}^c \end{pmatrix}_L \quad \leftarrow \quad (u^c)_L$$

$\bar{u}_R^c = \text{singlet}$

$$\bar{f} f = \bar{f}_L f_R + \bar{f}_R f_L$$



by  $SU(3)$

SM:  $H = D \equiv$  doublet

$$\overline{\Phi}_{(3)} = \begin{pmatrix} 0 \\ - \\ ? \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \\ \phi^- \end{pmatrix}$$

$\langle \phi^0 \rangle = v \epsilon u \Leftrightarrow$  SM situation

$$L_y = \bar{F}_L Y_u \bar{\Phi} u_R + h.c.$$

$$\rightarrow \bar{F}_L U_3^+ \circled{Y_u} \bar{U}_3 \bar{\Phi} u_R \dots$$

$$= \bar{F}_L Y_u \underbrace{U_3^+ \bar{U}_3}_{1} \bar{\Phi} u_R \dots$$

SU(2)  $\mathcal{D} \rightarrow \mathcal{V} D$

$$D^T \cdot \sigma_2 \cdot D = D^T \in D = \text{right}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\sum_{i,j} D_i \cdot D_j = \text{Invert}$$

SU(3)  $T (\tau_i, i=1, 2, 3)$

$$T \rightarrow U T : \tau_i \rightarrow U_{ij} \cdot \bar{\tau}_j$$

$$\sum_{ijk} \tau_i \cdot \bar{\tau}_j \cdot \bar{\tau}_k \rightarrow$$

$$\sum_{ijk} U_{ia'} U_{jb'} U_{kc'} T_i T_j T_k$$

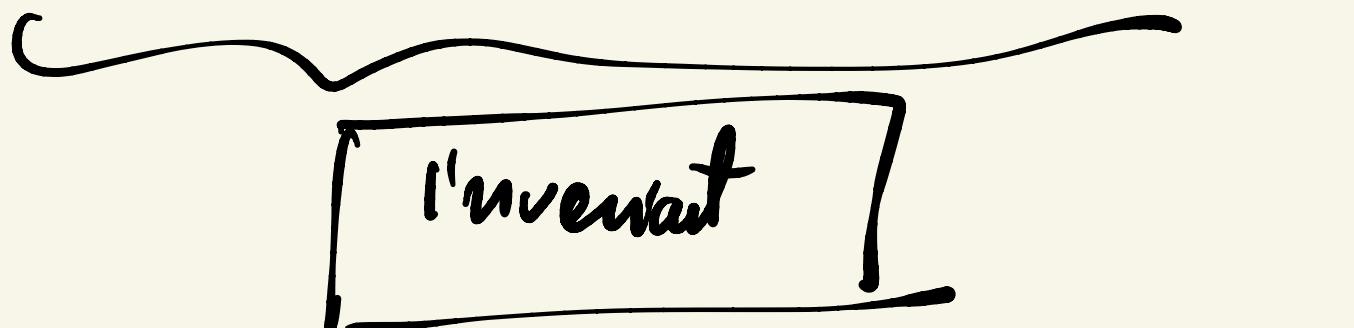
$$= \epsilon_{ijk} U_{i1} U_{jj'} U_{uu'} T_i T_{j'} T_{u'},$$

$$+ \epsilon_{iju}$$

$$= \epsilon_{123} U_{i1} (U_{2j} U_{3u'} - U_{3j} U_{2u'})$$

$$T_i T_j T_{u'}$$

$$= \epsilon_{i'j'u'} \underbrace{(\det U)}_1 T_i T_j T_{u'}$$



$$\left| \sum_{ijk} F_{ki}^T C F_{lj} \Phi_k Y_p \right|$$

$$F_L = \begin{pmatrix} 1 \\ n \\ pc \end{pmatrix}_L$$

Proton Yukawa

$$\bar{\Phi}_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

$$F_L^T C F_{L3} \bar{\Phi}_{02} \gamma_p$$

$$= P_L^T C (P_C)_L v \gamma_p$$

$$= P_L^T C C \bar{P}_R^T v \gamma_p \quad \boxed{C^2 = -1}$$

$$= - P_L^T \bar{P}_R v \gamma_p$$

$$C C^T = 1$$

$$C = -C^T$$

$$= \bar{p}_R p_L v y_P \quad ! \quad C = i \cdot \bar{v} \gamma_5$$

Emeda



$$\begin{pmatrix} k^+ \\ k^0 \end{pmatrix} \quad k^+ = u \bar{s} \quad \bar{u} d = \bar{r}^+ \\ k^0 = d \bar{s} \quad \frac{1}{2} \times \frac{1}{2}$$

$$D_\mu S' = (\partial_\mu - g \bar{T}_\alpha t_\mu^\alpha) S'$$

$$F = \left( \begin{array}{c} p \\ u \\ -p_c \end{array} \right)_L \} \text{SU}(2)$$

$$\left[ (u^c)_L \right]^{u_R \text{ circ}}$$

$$\Phi = \begin{pmatrix} \phi^+ \\ -\phi^0 \\ s^- \end{pmatrix}$$

- $\bar{F}_L \gamma_\mu u_R \bar{\Phi} + h.c.$  }  $\mathcal{L}_Y$
- $\bar{F}_{L_i}^\tau c \gamma_\mu F_{L_j} \bar{\Phi}_u \epsilon_{ijk} + h.c.$

$$m_p = m_u \Rightarrow \underbrace{y_p = g_u}_{u, \text{weak}} !$$

SM  $(\begin{smallmatrix} u \\ d \end{smallmatrix})_L$   $u_R, d_R$

$$\mathcal{L}_q = (\bar{u} \bar{d})_L \underbrace{\bar{\Phi}}_{\text{doublet}} y_\sigma d_R + \bar{\Phi} = \begin{pmatrix} \psi^+ \\ \psi^0 \end{pmatrix}$$

$$+ (\bar{u} \bar{d})_L^{\star} \Sigma \bar{\Phi}^* \gamma_\mu u_R + h.c.$$

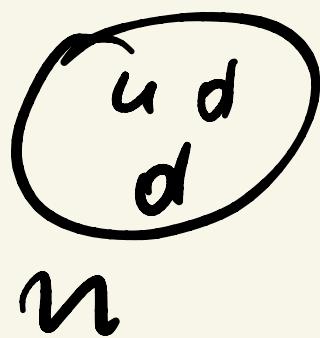
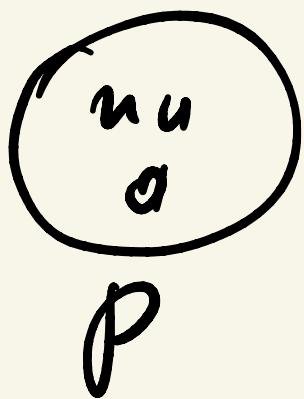
$$m_u = \gamma_u v \quad \langle \bar{\Phi} \rangle = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$m_d = \gamma_d v$$


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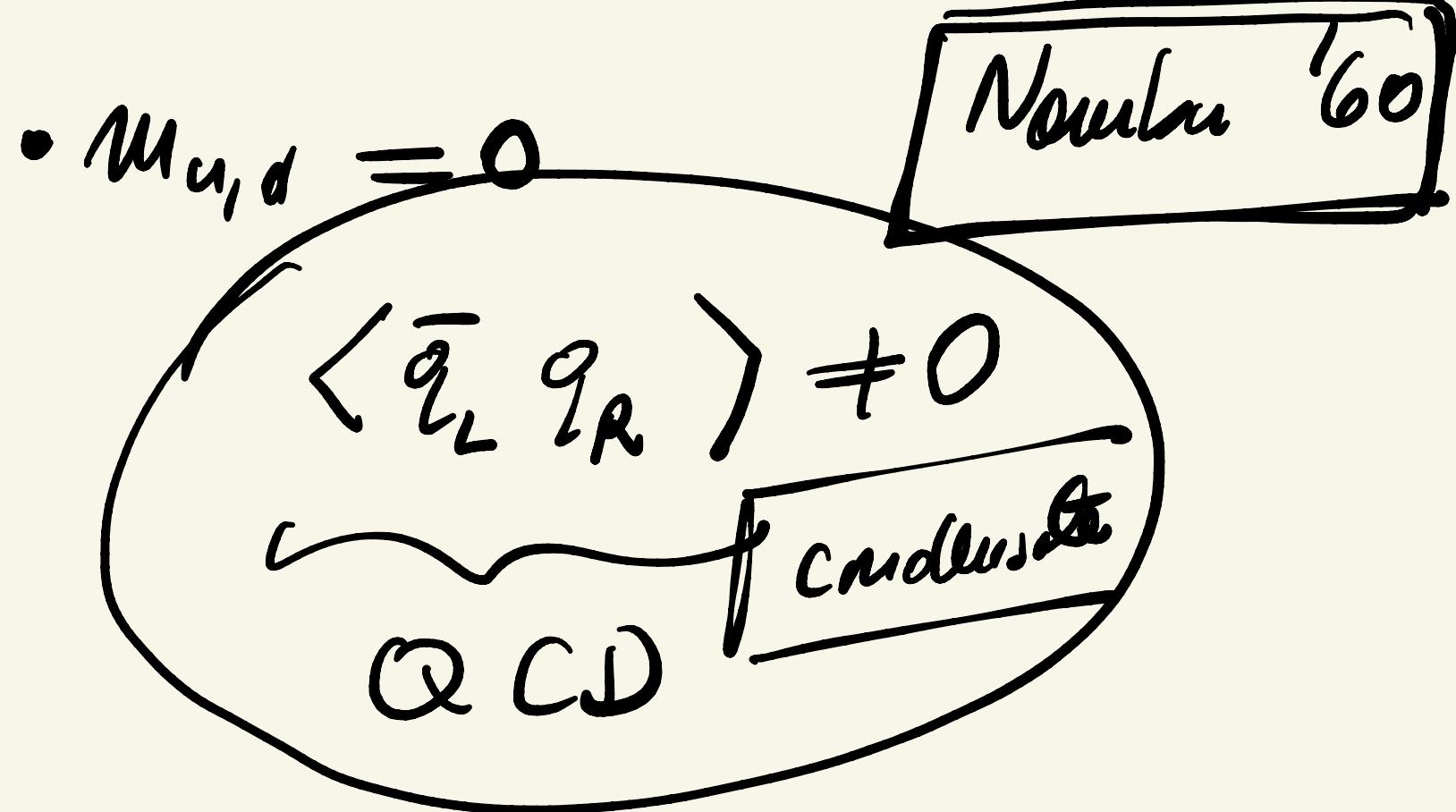
$$\gamma_{u,d} \ll 1$$

$$m_{u,d} \approx 0$$



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$$\gamma_t = 1, \quad \gamma_b = 1/20$$



$q_L \rightarrow e^{i\alpha} q_L, \quad q_R \rightarrow q_R$

local symmetry

$$L_{\text{kin}} = i \bar{\psi}_L \partial^\mu \gamma_\mu \psi + i \bar{\psi}_R \gamma^\mu \partial_\mu \psi$$

$\downarrow \qquad \qquad \qquad \downarrow$   
 $D_\mu \qquad \qquad D_\mu$

Dirac symmetry  $\leftrightarrow \bar{S} S \& B$

$\Rightarrow$  Nambu-Goldstone  $\downarrow$   
'1961'

•  $\langle \phi \rangle \neq 0$

$$\phi = a + h$$



$$m_h = \sqrt{\lambda} v$$

$\pi$  = Nambu-Goldstone bosons

$$m_{u,d} \neq 0$$

$$\Rightarrow \boxed{m_\pi^4 \simeq m_g \Lambda_{\text{QCD}}^3}$$

## Leptons

$$\left( \begin{array}{c} e^c \\ \nu^c \\ \cdots \\ e \end{array} \right)_R$$

$$(e^c)_R = c \bar{e}_L^T$$

has same charge as proton

SU(3) theory

of  $\phi, u + \nu, e$

$\nu=2$  SU(3) breaking  
↓

$$r=2 \quad SU(2) \times U(1)$$

adjoint  $\Sigma \rightarrow V\Sigma V^+$

$\Sigma_0 \rightarrow V\Sigma_0 V^+ = \text{diagonal}$

→  
vacuum

$$\Sigma^+ = \Sigma$$



values = preserved

$$[\bar{\Sigma}_0, \text{Cartan}] = [\Sigma_0, T_\alpha]_{\alpha \in \text{Cartan}}$$

diagonal

$$\Sigma_0 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} v_{\text{new}}$$

$$F = \left( -\frac{n}{pc} \right)_L^P \gamma^w \times \left( \frac{e^c}{v^c} \right)_R^e$$

$$m_x \approx f v_{mcw} \gg m_w$$

$$b_L = \begin{pmatrix} p \\ n \\ pc \end{pmatrix}_L^P \xrightarrow{\text{mirror}} b_R = \begin{pmatrix} e^c \\ v^c \\ e \end{pmatrix}_R^e$$

$$\bar{b} l = \text{invariant}$$

$$b_L \perp \! \! \! \perp b_R = \text{invariant}$$

$$\bar{p}_L e_R^c \rightarrow \boxed{\bar{e}^c = \phi}$$

- $\underline{M} \rightarrow 0 \Rightarrow p = \text{proton}$   
 $e^c = \text{positron}$
- $\bar{b}_L y_{uv} \sum e_R$  ellmed
- $\rightarrow \bar{b}_L y_{uv} U^\dagger U \Sigma U^\dagger U e_R$
- $= \bar{b}_L y_{uv} \sum e_R$  invariant

$$\Rightarrow \boxed{y_{uv} \rightarrow 0}$$

$y_{\text{new}} \ll g$

$M \ll g_{\text{re}}$

particles = SM particles

( $\gamma, l, W, Z, h$ )

Mass from Higgs

Model at leptons



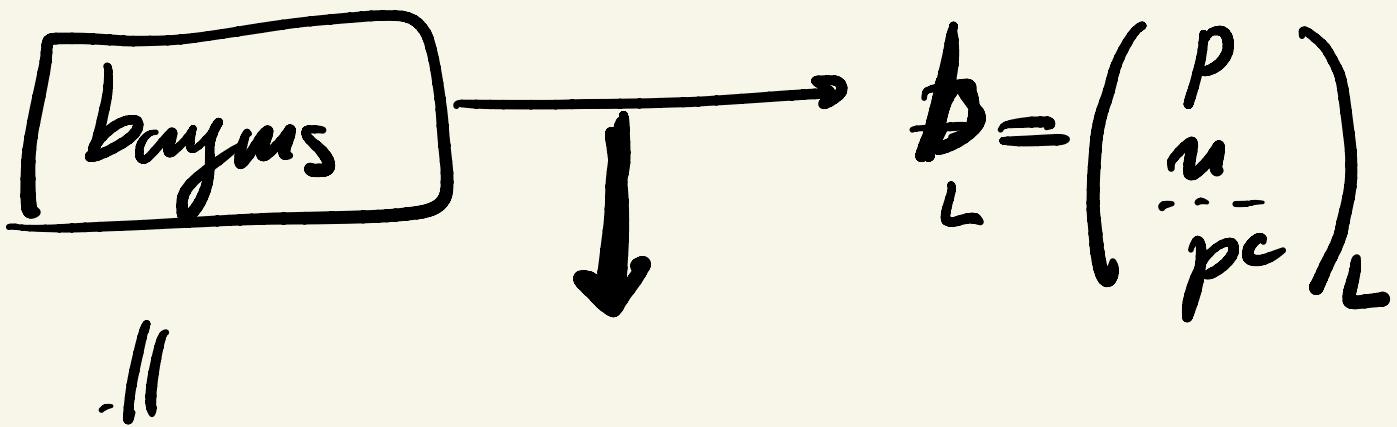
$$\begin{pmatrix} e^c \\ v^c \\ e \end{pmatrix}_R \Rightarrow \begin{pmatrix} v \\ -e^c \\ -e^c \end{pmatrix}_L$$

$\overbrace{\quad\quad\quad}$   $\bar{\Phi} \Rightarrow \text{it works! ?}$

Anomalies

kill anomalies .

geug theory = anomaly free



$\downarrow$

$L_H$  - weak int.

leptons

$$Q = \sum c_a \bar{t}_a$$

$$\begin{aligned} l &= \begin{pmatrix} e^c \\ \nu^c \\ \cdots \\ e \end{pmatrix}_R \\ \rightarrow & \end{aligned}$$

same as b

$$e_R^c \equiv c \bar{e}_L^T$$

$$SM \quad Q_{dm} = T_3 + \gamma_L$$

$g_u$   
 $g_d$  = orbiting, up to

$$g_u = g_d + 1$$

$$g_u - g_d = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$$

$\bar{b}_L \quad M \quad \ell_R$

$M \ll y^u$

$\ell_R \rightarrow -\ell_R$

$b_L \rightarrow b_L$

$M \rightarrow 0 \Rightarrow$  were symmetric

$w_p = w_u \Leftrightarrow y_p = y_u$

$y_p = y_u \Rightarrow SO(2)$

on

char :  $SO(2) \Rightarrow y_p = y_u$

$G \rightarrow H$  (global)

$u_G$                    $u_H$

# at quanta

$\Rightarrow$  # of Neumann-Goldstone

$$= u_G - u_H$$

= # of broken quanta

$$u_e = 0$$

$$\langle \bar{q}_L q_R \rangle \neq 0$$

$$\frac{3}{2} + \frac{3}{2} = 1_Q^3$$

$$= 3$$

global  $SU(2)_L \times SU(2)_R$

↓      3 broken fls.

$SU(2)_{L+R}$

$\Rightarrow 3 \text{ NG bosons} = \text{pians}$

$$m_T^2 f_\pi^2 = u_e \Lambda_{QCD}^3$$

$$u_e \rightarrow 0 \quad \Rightarrow \quad m_T \rightarrow 0$$

$\text{pians} = \text{pseudoscalar-NG bosons}$

$$u_e \ll \Lambda_{QCD}$$

$$\begin{pmatrix} v^c \\ e^c \end{pmatrix}_R = SM$$

$(\bar{e})_L \leftarrow$  def. at left

$$\gamma_R \leftrightarrow (\gamma^c)_L \equiv C \bar{\Psi}_R^\dagger$$

$$e_L + e_R$$

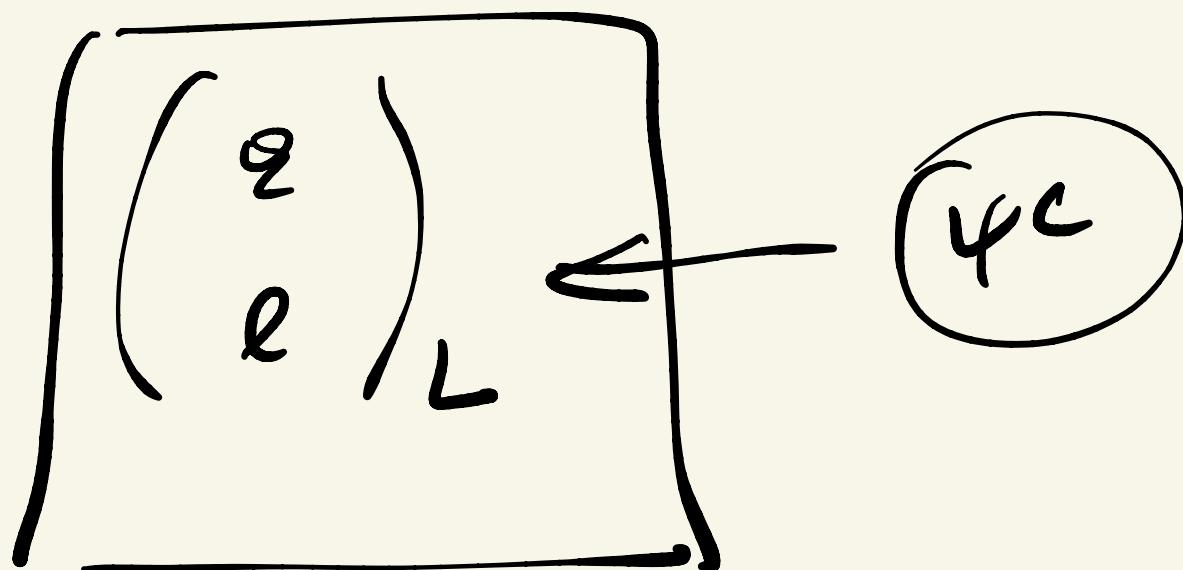
$$2 + 2 = 4$$

or

$$(e)_c + (e^c)_L = 4$$

$$2 + 2$$

GUT - work with L



$SU(3) \times U(1)$

Lee, Weinberg

'77

$$x \left[ \begin{pmatrix} p \\ u \\ -\bar{d} \\ pc \end{pmatrix}_L \right] (PR_L)$$

$$\boxed{x_\mu \bar{p}_L \delta^\mu(p^c)_L}$$

$$\leftarrow \overbrace{B(x_\mu)}^{+2} = +2$$

$\Rightarrow B$  is conserved

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c$$

$$[T_a, T_b] = i f_{abc} T_c$$

$$F_{\mu\nu}^a F^{\mu\nu a} \rightarrow (2m) f_{abc} A_\mu^b A_\nu^c$$



$$B(W) = 0$$

$$\bar{p} \sigma^\mu \gamma_5 W_\mu$$

break R

$B =$  longer number