LMU GUT Course

Lecture XXVI

12/2/2021

$$\boxed{SO(10)}$$

- $OO^T = 1 \qquad O = O_{10}$

  $\det O = 1$

  $$\phi \longrightarrow O\,\Phi \qquad \underline{\phi = 10 \text{ dim.}}$$

Motivation(s)

- real repr. = anomaly free

- $SO(10) \supseteq SO(6) \times SO(4)$

  $$10 = (6, 1) + (1, 4)$$

  $$\phi_1, \phi_2, \ldots \phi_{10}$$

$\phi_1 \cdots \phi_6 \; = \;$ 6 dim of $SO(6)$

$\phi_7 \cdots \phi_{10} \; = \;$ 4 dim of $SO(4)$

$$SO(6) = SU(4)$$

$$SO(4) = SU(2)_L \times SU(2)_R$$

$$\boxed{G_{PS} \subseteq SO(10)}$$

$\underline{SU(5)}$

$\bar{5}_F, \quad 10_F$

$\;\; \| \qquad\quad \|$

$\begin{pmatrix} d^c \\ \nu \\ e \end{pmatrix}_L \qquad ?$

anomaly:

$\bar{5}_F \longleftrightarrow 10_F$ cancel

$SU(2) \times SU(2) \times SU(4)$

$\begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_L$

$\begin{pmatrix} u_R & \nu_R \\ d_R & e_R \end{pmatrix}$

anomaly $= 0$:

$L \longleftrightarrow R$ cancel

Gauge sector

$$A = A_\mu^a T_a =$$

$$= A_\mu^{ij} L_{ij}$$

$$L_{ij} = -i(\delta_{in}\delta_{je} - \delta_{ie}\delta_{jn})$$

$$\# \text{ of gen} = \frac{10 \cdot 9}{2} = 45$$

- SU(5) decomposition

$$45 = 24 + 21$$

$$24 \to SU(5) \to SM + (X, Y)^\alpha$$

$$21 = 10 + \overline{10} + 1$$

$$5 \times 5 = \underset{(A)}{10} + \underset{(S)}{15}$$

$$5 \times \bar{5} = \underset{(\text{Adj})}{24} + \underset{(\text{singlet})}{1}$$

$SO(6)$ $\quad \phi_1 \, \text{--} \, \phi_6$

$$\left.\begin{matrix} \phi_1 \pm i\,\phi_2 \\ \phi_3 \pm i\,\phi_4 \\ \phi_5 \pm i\,\phi_6 \end{matrix}\right\} \boxed{3 + 3^*}$$

$$T_{3c} = \frac{1}{2}\left(L_{12} - L_{34}\right)$$

$$T_{8c} = N_8\left(L_{12} + L_{34} - L_{56}\right)$$

$$T_{15c} = N_{15}\left(L_{12} + L_{34} + L_{56}\right)$$

$\underline{SO(10)}$

$$\left.\begin{array}{c} \phi_1 \pm i\,\phi_2 \\ \phi_3 \pm i\,\phi_4 \\ \phi_5 \pm i\,\phi_6 \\ \phi_7 \pm i\,\phi_8 \\ \phi_9 \pm i\,\phi_{10} \end{array}\right\} (5+5^*)$$

<u>gauge bosons</u>

$$(10 \times 10)_A = (5+\bar{5}) \times (5+\bar{5})_A$$

$$= 10 + \overline{10} + 24 + 1$$

$$10_F = (u^c;\ (u\ d);\ e^c)$$

see below

X_{new} ?

$(2/3)$?

$$\boxed{(x';\ y')^\alpha + (\bar{x}', \bar{y}')^\alpha}$$

(p decay?)

$\rightarrow$ $\boxed{W'^{+},\ W'^{-},\ Z'\ ?}$

- <u>PS decomposition</u>

$$10 = (6,1) + (1,4)$$

$$(10 \times 10)_{AS} = (6 \times 6_A, 1) + (1, 4 \times 4_A) + (6,4)$$

$$= \underset{①}{(15_{PS}, 1)} + \underset{②}{(1_{PS}, 6)} + \underset{③}{(6,4)}$$

① = PS gauge boson :

$$(15_{PS}, 1) = \left(8\ \text{gluons} + \underline{\underline{X_{PS}}} + \bar{X}_{PS} + B_{BL}\right)$$

$$SU(4) \times \underbrace{SU(2)_L \times SU(2)_R}_{6 \,=\, (3_L,\, 1) + (1,\, 3_R)}$$

② 4 of $SO(4) = (2_L, 2_R)$

$$(4 \times 4)_A \longrightarrow 6 = (3_L, 1) + (1, 3_R)$$

$$(3_L, 1) = \vec{A}_L \qquad (1, 3_R) = \vec{A}_R$$

$$\boxed{(1_{PS}, 6) = \vec{A}_L + \vec{A}_R}$$

$SU(2)_L \times SU(2)_R$

$\left(\frac{Y}{2} = T_{3R} + \frac{B-L}{2} \right.$ quantized $\left.\right)$

$$\boxed{Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}}$$

3 $\quad (6, 4) = (3_c + \bar{3}_c, 4)$

$$4 = (2_L, 2_R)$$

$$\Downarrow$$

$$\boxed{\begin{pmatrix} x & x' \\ y & y' \end{pmatrix}} \quad \boxed{\text{bi-doublet}}$$

$45 = 21$ of Pati-Salam

$(\vec{A}_L, \vec{A}_R, \text{glue}, X_{PS}, \overline{X_{PS}}, B_{BL})$

$+ \; 24 = L \Leftrightarrow R$ sym of (X, Y)

$\Delta B \neq 0 \neq 0$ int.

$$X_\mu^{(L)}\left[\bar{u}_L^c \gamma^\mu u_L + \bar{d}_L \gamma^\mu e_L^c + \cdots\right]$$

$\underline{SU(2)_L}$

$$Y_\mu^{(L)}\left[\bar{u}_L^c \gamma^\mu d_L + \bar{u}_L \gamma^\mu e_L^c + \cdots\right]$$

$$\boxed{X', Y' \text{ int. ?}}$$

$$X_\mu'^{(R)}\left[\bar{u}_R^c \gamma^\mu u_R + \bar{d}_R \gamma^\mu e_R^c + \cdots\right]$$

Y'_μ [complete it!]

$$\boxed{\text{Complete gauge boson sector}}$$

- $\underline{\text{matter}}$ = fermion = (q, l)

$n_f = \# f \geq 15$ (together)

$$\boxed{n_f(\min) = 16}$$

$$\boxed{SO(10) \longrightarrow Spin(10)}$$

$$d(\psi) = 2^5 = 32 = 16_+ + 16_-$$

$\{\Gamma_1, \ldots, \Gamma_{10}\}$ Clifford

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$$

$$\Gamma_{FIVE} = (-i)^5 \Gamma_1 \cdots \Gamma_{10} = \begin{pmatrix} \mathbb{1}_5 & \\ & -\mathbb{1}_5 \end{pmatrix}$$

$$\Sigma_{ij} = \frac{1}{4i}[\Gamma_i, \Gamma_j]$$

$(2\Sigma_{2n-1,2n})$: ± 1 eigenvalues (ϵ_i)

$$\psi_+ : \boxed{\Gamma_{FIVE}\, \psi_+ = \psi_+}$$

$$\boxed{\begin{array}{c} \psi_+ = |\epsilon_1 \epsilon_2 \epsilon_3 \epsilon_4 \epsilon_5\rangle_L \\ \uparrow \quad \uparrow \qquad\qquad \uparrow \\ L_{12} \;\; L_{34} \;\; \cdots \;\; L_{910} \\ \prod_{i=1}^{5} \epsilon_i = 1 \end{array}}$$

$|+++++\rangle_L$ (1)

$|+++--\rangle_L$ (10) $\binom{5}{3}$

$|+----\rangle_L$ (5) $\binom{5}{4}$

$$\psi_+ \equiv 16_F = N_L + (10_F + \bar{5}_F)$$

singlet of $SU(5)$

family of observed fermions

SU(5) decomposition

- $\displaystyle\prod_{\epsilon_i = 1}^{5} = +1$ $\qquad \epsilon_1\epsilon_2\epsilon_3 = \pm 1$

$$\epsilon_4\epsilon_5 = \pm 1$$

$$16_F = (\underbrace{4_+ + 4_-}_{PS}, \underbrace{2_L + 2_R}_{SU(2)_L \times SU(2)_R})$$

$$= \begin{pmatrix} u & \nu \\ d & e \end{pmatrix}_L + \begin{pmatrix} d^c & e^c \\ u^c & \nu^c \end{pmatrix}_L$$

G

LR symmetry in $SO(10)$ =

$= C$ (charge conjugation)

<u>LR</u>: <u>P</u>: $f_L \to f_R$

$$C: \; f_L \to (f^c)_L \equiv C \bar{f}_R^*$$

$$\Downarrow$$

$$\boxed{C = SO(10) \text{ gauge rotation (finite)}}$$

<u>SU(2)</u> $\begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix}$ $\quad UU^\dagger = 1$

$\det U = 1$

$$U = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \subseteq SU(2)$$

$$U = e^{i\pi\sigma_3} = e^{i2\pi\sigma_3/2}$$

Slansky '1980

posted! complete review of GUT group theory

"Group theory for ——]

~~in $SO(10)$! discrete new parity (D)~~

$\underline{P}$: $\vec{W_L} \xrightarrow{P} \vec{W_R}$

(gen.) <u>parity</u>

<u>QED</u>

$$\left.\begin{array}{l} A_i \xrightarrow{P} -A_i \\ A_0 \xrightarrow{P} A_0 \end{array}\right\} \text{parity}$$

also: $f \xrightarrow{P} \gamma^0 f \Leftrightarrow f_L \xrightarrow{P} f_R$

C : $\vec{W_L} \xrightarrow{C} \vec{W_R}^*$

$$f_L \xrightarrow{C} (f^c)_L$$

$$\boxed{\begin{pmatrix} u \\ d \end{pmatrix}_L \leftrightarrow \begin{pmatrix} d^c \\ u^c \end{pmatrix}_R}$$

Γ_{FIVE} : $\psi = \psi_+ + \cancel{\psi_-}$

$\boxed{\Gamma_{FIVE} = 1}$

$\boxed{\text{all the fermions}}$

C: acts inside ψ_+

→ <u>no</u> connection with Γ_{FIVE}

SM + $\boxed{\text{singlet lepton}}$

often called RH neutrino

→ makes <u>no</u> sense

$$\left(\nu_R \; ; \; N_L \equiv C \bar{\nu}_R^T \right)$$

$\boxed{\text{nonmes}}$

- ν_L is just a nome $\Leftrightarrow$

$$\boxed{\bar{\nu}_L \gamma^\mu e_L W_\mu^+}$$

$$= \underbrace{\overline{(e^c)_R}} \gamma^\mu (\nu^c)_R W_\mu^+$$ not useful

- LR: $\nu_L \overset{P}{\longleftrightarrow} \nu_R$

$$\boxed{\nu_L \overset{C}{\longleftrightarrow} (\nu^c)_L \equiv C \bar{\nu}_R^T}$$

• SM + singlet fermion

mass term $\bar{\nu}_L \nu_R$

$$\boxed{\nu_L \longleftrightarrow (\nu^c)_R \equiv C\bar{\nu}_L^T}$$

$$\boxed{\nu_L \longleftrightarrow e_L \quad \text{with } W}$$

charge $-1 = 3\,\ell\,d$

$$\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$$

Dirac mass term

$$\| \quad \boxed{(\nu^c)_L\, C\, \nu_L}$$

(Majorana)

$$\boxed{\text{every mass term} = \text{Dirac or Majorana}}$$

$$\cancel{\nu_L \neq (\nu^c)_L}$$

$\boxed{\text{\underline{no} connection!}}$

$$\overline{\nu_R}\,\nu_L = \underbrace{(\nu^c)_L^T}_{N_L \text{ (Green)}}\, C\, \nu_L$$

Max.

||

lepton # on

$$\boxed{N_L \equiv C\bar{\nu}_R^{\,T}}$$

Max: $\boxed{\nu_R = \text{lepton}}$

Goran: $N_c = \text{anti-lepton}$

$$LR \Rightarrow \nu_L + \boxed{\nu_R\ (N_c)}$$

LR symmetry

$$\begin{pmatrix}\nu\\ e\end{pmatrix}_L \qquad \begin{pmatrix}\nu\\ e\end{pmatrix}_R$$

$$\begin{pmatrix}\nu\\ e\end{pmatrix}_L \qquad \begin{pmatrix}e^c\\ \nu^c\end{pmatrix}_R$$

$$\boxed{C \subseteq SO(10)} \qquad \boxed{\psi_+ = \begin{pmatrix}f\\ f^c\end{pmatrix}_L}$$

$\underline{SU(2)}$ $\quad \ell_L^T\, C\, \underbrace{(i\sigma_2)}_{= B}\, \ell_L$

$$\boxed{C = \text{Lorentz } B}$$

- C = gauged

$$f_L \longleftrightarrow (f^c)_L$$

- ~~P = gauged?~~

~~$f_L \leftrightarrow f_R$~~

- LR = theory of neutrino

$$\text{LR rotation} = \overset{\text{mass}}{P} \text{ or } C$$

neutral — "neutral" - gauged

- PS : P or C

$$\boxed{SO(10) \Rightarrow LR = C}$$

- ~~$4^+ 4^+$~~ forbidden

$$\psi^T B \psi \qquad B \propto \Gamma_1 \cdots \Gamma_9$$

$$\Downarrow$$

$$\Downarrow \quad B = \text{off-diagonal}$$

$$\psi^T B \psi \to \cancel{\psi_+^T \psi_-}$$

there is
<u>no</u> ψ_- !

- $\boxed{\text{anomaly}}$

PS = anomaly free

$$L + R$$

$$(1+\gamma_5) + (1-\gamma_5)$$

$$\Downarrow$$

$SO(10)$ = anomaly free – trivial

<u>$Spin(10)$?</u>

$\mathrm{Tr}\,\{\Sigma_{ij}, \Sigma_{kl}\}\,\Sigma_{mn} \;\propto$ <u>anomaly</u>

$= \cancel{\#\,\delta_{ij}\,\delta_{kl}\,\delta_{mn}}$

$= \#\,\epsilon_{ijklmn}$ does <u>not</u> exist in $Spin(10)$

$$\boxed{SO(2N) = \text{anomaly free}}$$

$$\boxed{\underline{\text{exception}}:\; SO(6) = SU(4)}$$

$SU(n) \Rightarrow$ anomalous

exception: $SU(2) = SO(3)$

$SU(4) \Leftrightarrow$ anomaly

$$f_L = (2_L, 1_R, 4_c)$$
$$f_R = (1_L, 2_R, 4_c)$$

$\boxed{\text{cancel}}$

$\boxed{SO(10)}$ $45 = 21$ (PS)

$+ \boxed{24 \; (X \, Y \; X' \, Y')}$

$\boxed{\text{anomaly free}}$

Georgi '74 Fritzsch, Minkowski '74

$\boxed{SO(10) = \text{free} \to SU(5) = \text{free}}$
$\boxed{\text{anomaly}}$

$\boxed{\text{Higgs sector}}$

break $SO(10)$ — new Higgs

Yukawa int. "old" Higgs

• $\boxed{\text{Yukawa sector}}$

$$\psi^T B \Gamma_i \psi \sim \phi_i$$

$$\downarrow$$

$$\psi^T S^T B \Gamma_i S \psi = \psi^T B \underbrace{S^+ \Gamma_i S}_{O_{ij} \Gamma_j} \psi$$

inv. $\psi^T B \psi \to \psi^T S^T B S \psi$

$= \psi^T B S^\dagger S \psi = \psi^T B \psi$

(def.) $\boxed{S^T B = B S^\dagger}$

$\Downarrow$

$B =$ off-diagonal

$\Gamma_i =$ –"–

- $\psi_+^T \underbrace{B \Gamma_i}_{\text{diag}} \psi_+ \quad \phi_i \; (10_H)$

$16 \times 16 = 10 +$

$10 = 5 + \bar{5}$ in $SU(5)$

$= (6,1) + (1,4) = (2_L, 2_R)$

- $\cancel{\psi_+^T B \Gamma_i \Gamma_j \psi_+}$ (?)

off - diagonal

- $\psi_+^T B \Gamma_i \Gamma_j \Gamma_k \psi_+ \quad \Phi_{[ijk]} \quad (120_H)$

$$\frac{5 \cdot \cancel{10} \cdot \overset{3}{\cancel{9}} \cdot 8}{\cancel{3} \cdot \cancel{2} \cdot 1} = 120$$

- $\cancel{\psi_+^T B \Gamma_i \Gamma_j \Gamma_k \Gamma_l \psi_+} \quad (\text{off})$

- $\psi_+^T B \underbrace{\Gamma_i \Gamma_j \Gamma_k \Gamma_l \Gamma_m} \psi_+ \quad \Phi_{[ijklm]}$

$$\frac{\cancel{10} \cdot 9 \cdot \overset{2}{\cancel{8}} \cdot 7 \cdot \cancel{6}^{2}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 252$$

$$\left(\Phi_{[ijklm]} \pm \frac{i^5}{5!} \epsilon_{ijklmnpqrs} \Phi_{[npqrs]} \right)$$

self + anti self-dual

$$\boxed{252_H = 126_H + \overline{126}_H}$$

$$\boxed{\Gamma_1\Gamma_2\cdots\Gamma_{10}(-i)^5 = 1}$$

$$\Rightarrow \boxed{\Gamma_i\Gamma_j\Gamma_k\Gamma_l\Gamma_m = \pm\frac{i^5}{5!}\epsilon_{ijklmnpqrs}\Gamma_n\Gamma_p\cdots\Gamma_s}$$

$$16\times 16 = 10_H + 120_H + 126_H$$

$$256 = \underbrace{\qquad\qquad}_{256}$$

$$\boxed{\text{Choice of Yukawa?}}$$

$$10_H = \text{minimal}$$

$$\langle 10_H\rangle = (6,1) + \langle(1,4)\rangle \rightsquigarrow (2_L, 2_R)$$

$$\Rightarrow \boxed{\begin{aligned} m_d &= m_e \\ m_u &= m_D\ (?) \end{aligned}}$$

wrong

$\underline{SO(2)}$ $\quad \psi^T B \Gamma_i \psi \; \phi_i \; (2_H)$

$$= \psi_+^+ \psi_+ \underbrace{(\phi_1 + i\phi_2)} +$$

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} \qquad \cancel{+ \psi_- \psi_- (\phi_1 - i\phi_2)}$$

$$u \to e^{i\theta/2} u$$
$$v \to e^{-i\theta/2} v$$

$$= \boxed{uu \, (\Phi_+)}$$
$$\cancel{+ vv \, (\Phi_-)}$$

- $126_H = \; ?$ in PS

$$126_H = (10_H \times 10_H \times 10_H \times 10_H \times 10_H)_{AS}$$

$\boxed{\text{do it!}}$

- In $\underline{SU(5)}$?

$$(\underbrace{10 \times \cdots 10}_{5})_{AS} = [\underbrace{(5 + \bar{5}) \times \cdots}_{5}]_{AS}$$

$$= \underbrace{\bar{5} \times \bar{5} \times \bar{5} \times \bar{5} \times \bar{5}}_{\text{singlet}} + \cdots$$

$$126_H = \underbrace{1_{SU(5)}} + \cdots$$

$$\langle 126_H \rangle = \langle \underbrace{1_{SU(5)}} \rangle$$

$$SO(10) \longrightarrow SU(5)$$

$$\Rightarrow m_{\nu_R} = \boxed{m_N = \langle 126_H \rangle}$$

$$\Downarrow$$

$\boxed{\text{seesaw mechanism}}$

$$\oplus \langle 126_H \rangle = \cdots \boxed{\text{has SM doublets}}$$

$$\Downarrow$$

$\boxed{\langle 126_H \rangle}$ under PS

$$10 = (1,4) + (6,1)$$

$$\underbrace{(10 \times \cdots 10)}_{5\ \text{times}} \doteq \underbrace{(15,4)}_{\boxed{\text{show}}} + \cdots$$

$SU(5)$ 1 → vacs N

$\langle 16_H \rangle$ → $3 m_d = - m_e$

$$\boxed{\langle 15 \rangle_{PS} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}}$$

$\Downarrow$

$\boxed{10_H + 126_H}$ $\boxed{\text{complete theory}}$

$\boxed{\text{Bajc, et. al '05}}$

- $\boxed{SO(10) \longrightarrow SM}$

$\boxed{\text{which Higgs?}}$

$\boxed{126_H}$

- $m_N \neq 0 \Rightarrow$ seesaw
- $m_e = -3 m_d$ $(M_e = -3 M_d)$

$m_D = -3 m_u$ $(M_D = -3 M_u)$

$$\Downarrow$$

$$M_\nu = -M_D^T \frac{1}{M_N} M_D =$$

$$= -9 M_u^T \frac{1}{M_N} M_u$$

<u>not</u> good for $\binom{1^{st}}{3^{rd}}$ <u>gen.</u>

$\Rightarrow M_{\text{minimal}} = 10_H + 126_H$

Too big !?

$\boxed{10_H + 126_H = \text{complete for Yukawa and GUT breaking?}}$

$\langle 126_H \rangle = (1_s \text{ of } SU(5))$

$+ \langle 24 \ldots \rangle$

$SU(5)$, $\boxed{\text{but who breaks } SU(5)\,?}$

$\Rightarrow$ $\boxed{\text{new GUT Higgs}}$

$\Downarrow$

ADJOINT $= 45_H$

$$\cdot\ 45_H = \left(10_H \times 10_H\right)_{AS} =$$

$$= \underbrace{(4,6)}_{(x,y,x',y')} + \underbrace{(1,15)}_{\downarrow\ PS} + \underbrace{(6,1)}_{\| \atop 3_L + 3_R}$$

$$SO(10) \longrightarrow \langle 45_H \rangle \longrightarrow PS\ ??$$

$$\langle 3_L \rangle = 0$$

$$\langle 3_R \rangle \neq 0 \quad \text{possible}$$

$$\boxed{SU(2)_L \times SU(4)_c \times U(1)_R}$$

A. NO! $\longrightarrow$ $\langle 45 \rangle$ cannot break $SO(10)$ to PS

$\cdot\ 45 = 24 + 10 + \overline{10} + 1$ in $SU(5)$

$SO(10) \longrightarrow SU(5)$ YES

$\langle 45 \rangle_H = \langle 1 \rangle$ / $\boxed{\text{global minimum}}$

$$\boxed{\begin{aligned} SO(10) \supset\ & SU(5) \times U(1) \\ & PS \\ & SO(9) \\ & SO(8) \times SO(2) \end{aligned}}$$

Ling-Fong Li '74

$\boxed{\text{global minima}} \Longleftarrow SO(2n), SO(n)$

with A, S, A_S --

$\Downarrow$ Michel's conjecture

$$\boxed{\begin{aligned} &G \longrightarrow \text{maximal subgroup} \\ &\langle \text{adjoint} \rangle \end{aligned}}$$

- $45_n^T = -45_n \longrightarrow \boxed{O\, 45_n\, O^T}$

$$\phi_i \phi_j \longrightarrow O_{in} O_{je} \phi_n \phi_e$$

$$= O_{in} \phi_n \phi_e O^T_{ej}$$

- $H = U d U^\dagger$ (adjoint of $SU(n)$)

  diagonal

  hermitian

- $$\langle 45_n \rangle \longrightarrow O \begin{pmatrix} a_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & & & \\ & a_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & & \\ & & \ddots & \\ & & & a_5 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{pmatrix} O^T$$

- $\langle 45 \rangle_H : \ SO(10) \to SU(5)$

$$\boxed{45_H + 126_H + 10_H = \underline{\text{not}} \text{ enough}}$$

- $54_H =$ symmetric

$$\boxed{54_H \to O\, 54_H\, O^T}$$

$$\langle 54 \rangle_H \Rightarrow O\, d_H\, O^T$$

$\hookrightarrow$ diagonal

$$54 = \frac{10\cdot 11}{2} = 55$$

- $\boxed{\mathrm{Tr}\, 54 = \text{inv.}}$

PS: $(10_H \times 10_H)_S = [(6,1)+(1,4)] \times [(6,1)+(1,4)]_S$

$$= \underbrace{(21,1)}_{21} + \underbrace{(6,4)}_{24} + \underbrace{(1,10)}_{10}$$

$21 = 20 + 1 = $ two

$(20+1, 1) + \ldots$

$$\boxed{10 + \overline{10} \text{ in } SO(4)}$$

- $54_H = (20, 1) + (6,4) + \boxed{(1,10)}$

$6 = 3 + \bar{3}$

$20 = 10 + \overline{10}$

$1 \to 4$

vev? NO!

breaks color

$$10 = \underbrace{6 + 3}_{2/3 \quad -2/3} + 1_{-2} \leftarrow \text{maybe this guy}$$

$\langle 10 \rangle \neq 0$? Is this ok???

$\langle 1_{-2} \rangle$! $\quad Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$

$$= 0 + 0 + (-1) = -1$$

$\Rightarrow$ $\boxed{\underline{\text{no}} \text{ vev !}}$

__last:__ $(1, 10)$ $\quad (4 \times 4)_s =$

$$= (2_L, 2_R) \times (2_L, 2_R)$$

$$= \underbrace{(3_L, 3_R)}_{10} + (1_L, 1_R)$$

no vev $\Rightarrow$

$$10 = 9 + 1$$

$$4 \times 4 = (3_L, 3_R) + (1_L, 1_R) = 10$$

$$+ (3_L, 1_R) + (1_L, 3_R) = 0$$

$$\Downarrow$$

$$\boxed{\langle 54_H \rangle = \langle 1_L, 1_R \rangle \neq 0 \text{ — OK}}$$

$$54_H \Longleftrightarrow 24_H$$

in $SO(10)$ in $SU(5)$

$\downarrow$ $\hookrightarrow U\, 24_H\, U^\dagger$

$O\, 54_H\, O^T \Rightarrow$ $\Rightarrow (24)_H = \text{diag}$

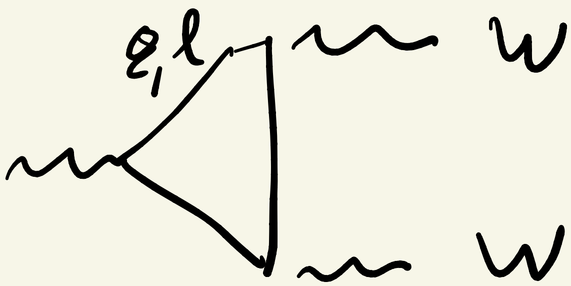
$(54_H) = \text{diagonal}$

$$\boxed{54_H + 126_H + 10_H} \;\; ??$$

• $\underbrace{126_H = \text{too big}}$

$\boxed{B-L}$ ← great role

(i) $B-L$ = anomaly free in SM
($q \leftrightarrow l$)



= gauge symm. in SO(10), PS,
LR

(ii) $Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$

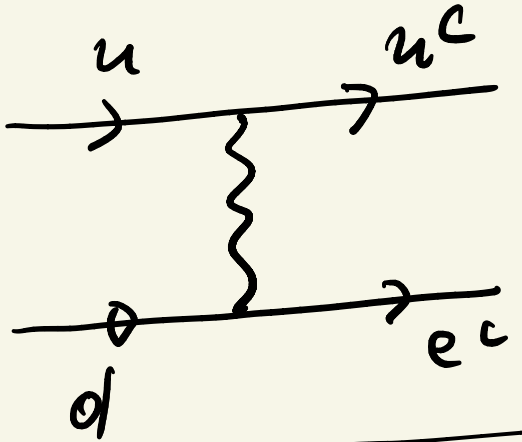
(iii) <u>baryon decay</u>

↓

$$X_\mu \left[\overline{u^c_L} \gamma^\mu u_L + \overline{d_L} \gamma^\mu e^c_L \right]$$

$$\Updownarrow \; L \leftrightarrow R$$

$$X'_\mu \left[\; \cdots \; \right]$$



$(q\,q\,q\,l)$

$\boxed{\text{B - L symmetry}}$

- $\boxed{\Delta B \neq 0}$

$\underbrace{q\,q\,q}_{\text{color}}\, l$ or $\underbrace{q\,q\,q}_{\text{color}}\, l^c$

SM symmetry

Weinberg 79

$\boxed{\text{G.S., 2009}}$

$$p \to \pi^0 + e^+$$

$$n \to \pi^- + e^+$$

$$\cancel{\pi^+ + e}$$

$$\cancel{p \to \pi^0 + \pi^+ + e}$$

$\boxed{\text{selection rules}}$

$$\boxed{\begin{array}{c} \langle 126_H \rangle \to \cancel{B-L} \\ \updownarrow \\ \langle 16_H \rangle \to \cancel{B-L} \end{array}}$$

$$16_F = (\underbrace{u_L \; u^c_L \; d_L \; d^c_L \; \nu_L}_{SM} \;\; N_L)$$

SM singlet

GUT Higgs

45_H, 54_H, 210_H

2-antisym 2-sym. 4-anti

$\cancel{(B-L)}$ 16_H, 126_H $\cancel{(B-L)}$

Yukawa Higgs

10_H, 120_H, 126_H

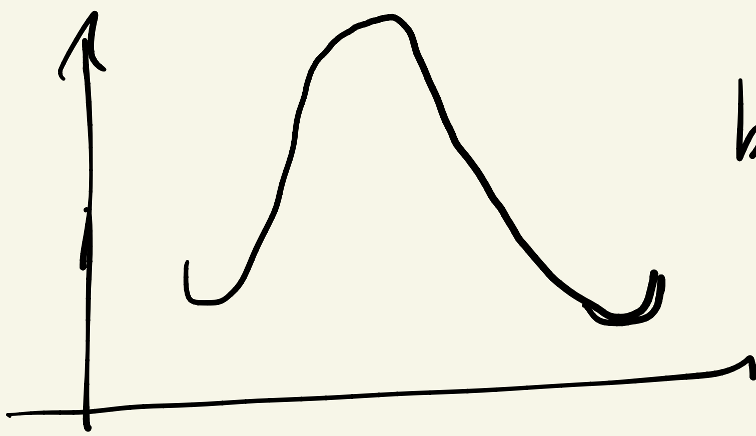
- local minimum

unstable world

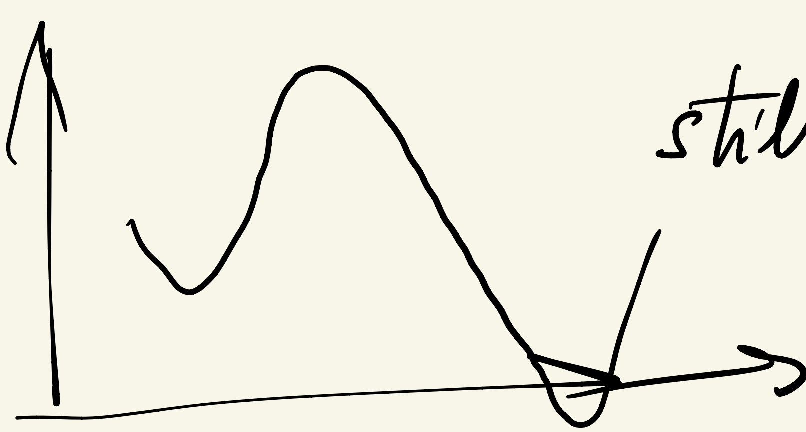
tunneling —

not so good

better



best



still ok

$$\boxed{\tau_{\text{tunneling}} > \tau_{\text{universe}}}$$ 04

Coleman '74

Coleman, de Lucia '76

$SO(10) \xrightarrow[\langle 54 \rangle_H]{} PS \xrightarrow[\langle 45 \rangle_H]{} LR$

$\langle 54 \rangle_H$ = local minimum

$$SO(10) \xrightarrow[\langle 54_H \rangle]{} \begin{cases} SO(5) \times SO(5) \\ SO(9) \end{cases}$$