LMU GUT Course

Lecture XX

22/1/202

SU(5): Neutrino mass

- $M_d = M_e^T \quad (\bar{5}_F 10_F 5_H^*)$ (1)
- $M_u^T = M_u \quad (10_F 10_F 5_H)$ (2)

$\Downarrow$

(a) p decays determined as $f(V_{CKM})$

(b) $m_d^{(i)} = m_e^{(i)}$ $\quad i = 1, 2, 3$

at M_{GUT}

$\Downarrow$

wrong $\quad \frac{m_b}{m_s} = \frac{m_\tau}{m_\mu}$

- unification fails

- $m_\nu = 0$ $\qquad F = 10_F + \bar{5}_F$

$\Uparrow$ $\qquad = \text{SM fermions}$

$\Rightarrow$ the same in SM

- q - l mass relations

Q. Why $m_d = m_e$?

A. Well, Yukawas have SU(5)

$\Downarrow$

e, d live in $\bar{5}_F$

$\Downarrow$

$$y_e = y_d$$

$\langle 5_H \rangle \neq 0$ breaks $\underline{\underline{SU(5)}}$

$\downarrow$ down to what?

$$\langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix} \quad \underline{\underline{SU(4)}}$$

$$SU(5) \xrightarrow[\langle 5_H \rangle]{} SU(4)$$

$$\begin{pmatrix} d^r \\ d^y \\ d^b \\ e^c \\ \nu^c \end{pmatrix} \quad \underbrace{SU(4)_c, \; SU(4)_{PS}}_{\underbrace{d \leftrightarrow e}_{\text{same repr.}}}$$

$\cancel{SU(4)_{PS}}$ by $\langle 24 \rangle_H$

$$\langle 24\rangle_H = \mathrm{diag}\,(1,1,1,-3/2,-3/2)\, v_X$$

but (24_H) is not coupled to fermions

$$\boxed{SM = \text{renormalizable theory}}$$

$$\mathcal{L}(d\le 4) + O(d>4)$$

(induced)

$$\mathcal{L}_Y = \bar{q}_L Y_d d_R \Phi + \cdots$$

do not write $d=5,6,\ldots$

you do <u>not</u> write

$$+ \frac{q\,q\,q\,l}{\Lambda_{\cancel{B}}^2} \quad (d = 4 \times 3/2 = 6)$$

GUT

$$\boxed{\Lambda_{\cancel{B}} = M_X}$$

<u>SM</u>: higher dim. operator = = small

$$\Longleftrightarrow \boxed{\Lambda_{new} \; (\Lambda_{\cancel{B}}) \gg M_W}$$

LHC: $\boxed{\Lambda_{new} > \text{TeV}}$

$M_{W'} > 5\ \text{TeV}$

GUT

Renormalisable theory

$$\mathcal{L}_Y = 10_F\, 10_F\, 5_H + \bar{5}_F\, 10_F\, 5_H^*$$

$(d=4)$

$\Leftarrow\!\!\!\exists$ $\Lambda_{new} \gg M_X$

$$\underbrace{+ \ \bar{5}_F \frac{24_H}{\Lambda_{new}} 10_F\, 5_H^*}_{\boxed{\text{example}}} \quad (?)$$

$$\bar{5}_F^{\,i} \frac{(24_H)_i^{\,k}}{\Lambda_{new}} 10_{kj}\, 5_H^{*\,j}$$

$\langle 24 \rangle$ breaks $SU(4)_{PS}$

$\Downarrow$

$$\bullet\quad \underset{\substack{\| \\ \bar{d}_R}}{\bar{5}_F{}^{1}}\ \frac{\overset{\equiv v_X}{\langle 24_H\rangle^1_1}}{\Lambda_{new}}\ \underset{\substack{\| \\ d_L}}{10^F_{15}}\ \underset{\substack{\| \\ v}}{\langle 5_H^*\rangle^5}$$

$$10^F_L = \begin{pmatrix} u^c & u\ d \\ & e^c \end{pmatrix}_L$$

$$\Rightarrow \boxed{\bar{d}_R d_L \frac{v_X}{\Lambda_{new}} v} \qquad \underbrace{}_{\stackrel{!}{=} -3/2\, v_X}$$

$$\bullet\quad \underset{\substack{\| \\ \bar{e}^c_R}}{\bar{5}_F{}^{4}}\ \frac{\langle 24_H\rangle^4_4}{\Lambda_{new}}\ \underset{\substack{\| \\ e^c_L}}{10^F_{45}}\ v$$

$$\Downarrow$$

$$\bar{e}_R e_L \left(-\frac{3}{2}\frac{v_X}{\Lambda_{new}}\right) v$$

$$\Downarrow$$

$$\bar{d}d: \left(y_d + \frac{v_x}{\Lambda_{new}} \right) v$$

coeff.

$$\bar{e}e: \left(y_d + \frac{-\frac{3}{2} v_x}{\Lambda_{new}} \right) v$$

SM: $\Lambda_{new} = \underline{\underline{M_{pe}}} \; (M_{GUT})$

SU(5) $\Lambda_{new} = \underline{\underline{M_{pe}}}$

$$M_x = 10^{16}\,\mathrm{GeV}$$

$$M_{pe} = 10^{18}\,\mathrm{GeV}$$

$$\Downarrow$$

$$\boxed{\frac{v_x}{\Lambda_{new}} \simeq \frac{M_x}{M_{pe}} \leq 10^{-2}}$$

$$\Downarrow$$

even the gravity (M_{pe}) effects are <u>not</u> negligible!

- $y_\mu \simeq y_s \simeq 10^{-3}$
- $y_d, y_u \sim 10^{-4}$, $y_e \simeq 10^{-5}$

$$\boxed{\begin{aligned} m_e &= \text{MeV} \\ m_t &= 100\ \text{GeV} \end{aligned}}$$

$\Downarrow$

$m_e = m_d$ NOT good

$\Downarrow$ Options

$$\boxed{\bullet\ d=5\ \left(M_X/M_{pe}\right)\ \text{natural}}$$

• add new Higgs

$$\bar{5}_F\, 10_F \big(\underbrace{5_H^*}_{\Phi_1} + \underbrace{45_H}_{\Phi_2}\big)$$

$$10_F\, 10_F\, (5_H + 45_H + 50_H)$$

$$(45_H)^{k}_{[ij]} \longrightarrow \text{messy}$$

AS

⇓

lot of new states

⇓

populate desert

⇓

lose predictions

Oto: minimal SM

$$\Rightarrow \quad y_f = \frac{m_f}{M_W}\, g/2$$

$$y\, h \bar{f} f$$

$$\Rightarrow \Gamma(h \to f\bar{f}) = \left(\frac{m_f}{M_W}\right)^2 \frac{g^2}{4} \times \frac{m_h}{8\pi} (?)$$

$$\Downarrow$$

with more Higgs?

$$\mathcal{L}_Y = \bar{q}_L \left(Y_1^d \Phi_1 + Y_2^d \Phi_2 \right) d_R + h.c.$$

$$\Rightarrow M_d = \left(Y_1^d v_1 + Y_2^d v_2 \right)$$

$$\Downarrow \qquad \langle \Phi_i \rangle = v_i \qquad i = 1,2$$

$$U_L^d M_d U_R^{d\dagger} = D_d \text{ (diagonal mass)}$$

$$\Downarrow$$

Y_1^d, Y_2^d are <u>not</u> diagonal

$\Rightarrow$ lose the $\boxed{\text{mass – decay rate prediction}}$

<u>LHC</u>: $\Rightarrow$

$$\left.\begin{aligned} Y_t &= \frac{g}{2}\frac{m_t}{M_W} \\ Y_b &= \frac{g}{2}\frac{m_b}{M_W} \\ Y_\tau &= \frac{g}{2}\frac{m_\tau}{M_W} \end{aligned}\right\} \underline{\underline{3^{rd}}}$$

<u>no</u> information on 1^{st}, 2^{nd}

$$\Downarrow$$

Consistent with SU(5)

correction to the SM $y_f \leftrightarrow m_f$

is $\sim \frac{M_X}{M_{pl}} \simeq 10^{-2}$

it can only affect 2nd, 1st gen.

$\Downarrow$

d = 5 operator is consistent

with $y_3 = \frac{m_3}{M_W} g/2$

3 = 3rd gen.

- Could I use 5_H^1, 5_H^2?

~~(45_H could do the job)~~

Why not?

KM work: 2 gen no $\cancel{CP}$

$\exists$ 3rd gen $\Rightarrow$ $\cancel{CP}$

$$\langle 5_H \rangle^i = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}^i \qquad i = 1, \ldots, 1001$$

$$5_F = \begin{pmatrix} d \\ \hline e^c \\ \nu^c \end{pmatrix}$$ carries charge

$\Rightarrow$ $SU(4)_{PS}$

Why? $\cancel{SU(4)_{PS}} \Leftarrow \langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \\ \vdots \end{pmatrix}$

breaks Q_{em}

$\boxed{\text{$5_H$ can never work, independent of their \#}}$

$\Downarrow$

(i) 24_H (ii) 45_H

$\frac{}{\Lambda_{new}}$

· $d=5$ int. (24_H)

$\Downarrow$

no predictions for p decay

- no unif.

$\boxed{\bullet \quad m_\nu = 0}$ $\Big\}$ $\nexists$ new states

$$\Downarrow$$

$$\boxed{24_F}$$

- $m_\nu \neq 0 \iff$ seesaw

SM: e_L, e_R $\checkmark$

$$\Downarrow$$

$y_e \, \bar{e}_L e_R \Phi \Rightarrow$

$m_e = y_e v$

$m_\nu \neq 0 \Rightarrow \exists \, \nu_R$

$\Rightarrow y_\nu \, \bar{\ell}_L \nu_R \Phi^* \neq 0$

$$\cancel{m_\nu = y_\nu v}$$

$$\mathcal{L} = i \bar{\nu}_R \gamma^\mu \partial_\mu \nu_R + \underbrace{\frac{1}{2} \nu_R^T C \nu_R}\, M_R + h.c.$$

$$\boxed{C^T = -C}$$

$$\Rightarrow \quad m_{\nu_R} = M_R$$

$$\nu_M = \nu_R + C \bar{\nu}_R^T \equiv \underline{\text{Majorana}}$$

particle

anti-particle

$$\nu_M = \nu_R + C \gamma_0 \nu_R^*$$

$$\Rightarrow \boxed{\bar{\nu}_M \gamma^\mu \partial_\mu \nu_M = 2 \bar{\nu}_R \gamma^\mu \partial_\mu \nu_R \quad (*)}$$

Check!!

$$\bar{\nu}_M \nu_M = \nu_R^T C \nu_R + \nu_R^\dagger C^\dagger \nu_R^*$$

$$\Downarrow$$

$$\mathcal{L}(\nu_R) = i \bar{\nu}_R \gamma^\mu \partial_\mu \nu_R - \frac{1}{2} M_R \nu_R^T C \nu_R + h.c.$$

$$\Downarrow$$

$$= \cancel{\frac{1}{2}} \underbrace{\left(i \bar{\nu}_M \gamma^\mu \partial_\mu \nu_M - M_R \bar{\nu}_M \nu_M \right)}_{\sim \text{Dirac}}$$

$\Longrightarrow$ $\underline{M_R}$ = mass term

$$\psi_D = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

$$\psi_M = \begin{pmatrix} u \\ -i\sigma_2 u^* \end{pmatrix} = \begin{pmatrix} i\sigma_2 v^* \\ v \end{pmatrix}$$

$$\psi_M = \psi_L + C\bar{\psi}_L^T$$

$$= \psi_R + C\bar{\psi}_R^T$$

$C\bar{\psi}_L^T =$ right-handed

$$C\bar{\psi}_L^T \equiv C\gamma_0\psi_L^* \qquad C \equiv i\gamma_2\gamma_0$$

$$\Rightarrow R\, C\bar{\psi}_L^T = R\,C\gamma_0\psi_L^* =$$

$$R \equiv \frac{1-\gamma_5}{2} \qquad = CR\gamma_0\psi_L^* =$$

$$= C\gamma_0 L\psi_L^* = C\gamma_0\psi_L^*$$

$$\boxed{C\bar{\psi}_L^T \equiv (\psi^c)_R}$$

$$\boxed{C\bar{\psi}_R^T = (\psi^c)_L}$$

$$\Downarrow \ SM$$

$$v\, y_D\left(\overline{\nu_L}\,\nu_R + \overline{\nu_R}\,\nu_L\right) + \frac{1}{2} M_R\, \nu_R^T C \nu_R$$

$$\Downarrow$$

$$\begin{array}{c} \\ \nu_L \\ \nu_R \end{array} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \checkmark \qquad m_D \equiv y_D v$$

<u>seesaw!</u>

$$M_R \gg m_D$$

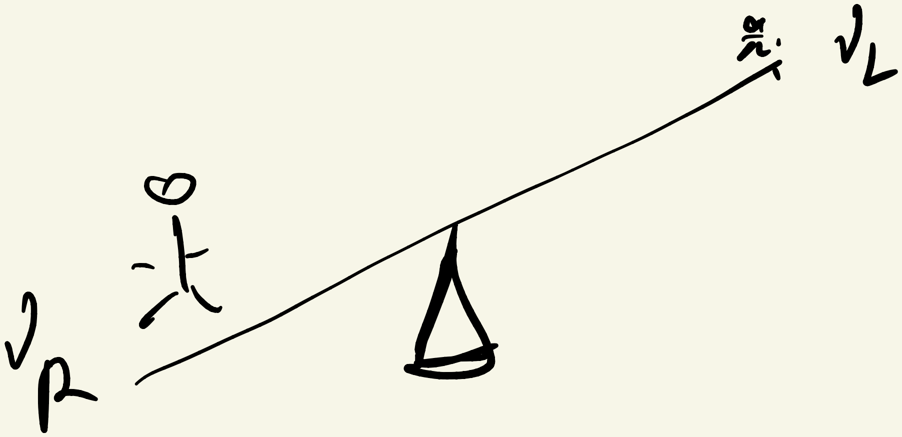
$$\Downarrow$$

$$m_{\nu_L} = -\frac{m_D{}^2}{M_R} \qquad (\nu \simeq \nu_L)$$

$$M_{\nu_R} \simeq M_R$$

$$\det = M_R\left(-\frac{m_D^2}{M_R}\right) = -m_D^2$$

✓



$$24_F = (8_c, 1_w, Y=0)_F \equiv O_F$$

$$+ \boxed{(1_c, 3_w, Y=0)_F \equiv T_F}$$

$$+ \boxed{(1_c, 1_w, Y=0)_F \equiv S_F}$$

$$+ \left(3_c, 2_w, Y=\frac{5}{6}\right)_F +$$

$$(\bar{3}_c, 2_w, Y=-5/6)_F$$

$\sim \nu_R$

$\boxed{\text{SM singlet}}$

$\Downarrow$

$\boxed{\text{seesaw mechanism}}$

$$\bar{5}_F{}^i\, 24_F{}_i^j\, 5_{H j}$$

$$24 \to U\, 24\, U^+$$

$$24_i{}^j$$

$$\underbrace{\overset{\uparrow}{\underset{D}{\bar{l}}}\ \overset{\uparrow}{\underset{D}{\bar{\Phi}}}}_{\text{doublets}}\ \nu_R^*$$

$$D \times D = \overset{(1)}{S} + \overset{(3)}{T}$$