Prof. Dr. Viatcheslav Mukhanov Winter term 2019/2020

Exercises on Quantum Mechanics II (TM1/TV)

Problem set 11, discussed January 13 - January 17, 2019

Exercise 64 (Central Tutorial)

Let's consider the hydrogen atom ignoring the spins of the electron and of the proton. In order to have non-zero transition probability from an energy state $|\,\psi_a\rangle$ to another one $|\,\psi_b\rangle$, the matrix element of the dipole moment operator $\boldsymbol{D}_{ba} \equiv e\,\langle\psi_b\,|\,\hat{\boldsymbol{r}}\,|\,\psi_a\rangle$ must be non-zero too ($\hat{\boldsymbol{r}} = (\hat{x}, \hat{y}, \hat{z})$). The conditions to have non-zero transition probabilities are called *selection rules.*

(i) Which are good quantum numbers that characterize the state $|\,\psi\rangle$ of the hydrogen atom? To what do they physically correspond?

(ii) Which values can those quantum numbers have? What is the degeneracy of the state for a given energy level?

(iii) Recalling the definition of the angular momentum operator $\hat{\boldsymbol{L}} = (\hat{L}_x, \hat{L}_y, \hat{L}_z)$, calculate

$$\left[\hat{L}_z, \hat{x}\right] \quad , \quad \left[\hat{L}_z, \hat{y}\right] \quad , \quad \left[\hat{L}_z, \hat{z}\right] \tag{1}$$

(iv) Using the results of the previous point and taking the expectation values of those between two different states $|\,\psi_a\rangle$ and $|\,\psi_b\rangle$, derive the selection rules to have $\boldsymbol{D}_{ab} \neq 0$.

(v) These are not the only selection rules. Recall the definition of the Casimir operator $\hat{L}^2$; what is the action of this operator on an eigenstate $|\,\psi\rangle$? Prove that

$$\left[\hat{L}^2, \left[\hat{L}^2, \hat{\boldsymbol{r}}\right]\right] = 2\hbar^2(\hat{\boldsymbol{r}}\hat{L}^2 + \hat{L}^2\hat{\boldsymbol{r}}) \tag{2}$$

(vi) Use the previous results to find other selection rules for the transition between $|\,\psi_a\rangle$ and $|\,\psi_b\rangle$.

(vii) What do these selection rules correspond physically to?

Solution

(i) From the course of QMI we know that a state of the hydrogen atom (ignoring spins) is characterized by three quantum numbers: n for the energy level, l for the total angular momentum and m for the projection of the angular momentum on the $\hat{z}$-axis. Therefore, not considering super-positions of states, we can write the state as

$$|\,\psi\rangle = |\,n, l, m\rangle \tag{3}$$

(ii) The quantum number for the energy n can take all positive integer values, i.e. $n \in \mathbb{N}$; l can take integer values in $[0, n-1]$; lastly, m can take integer values in $[-l, +l]$.
For a fixed l, m has $2l+1$ possibilities. In the same way, for a fixed n, l takes values from 0 to $n-1$. Therefore the degeneracy for a given energy level specified by n is

$$\sum_{l=0}^{n-1}(2l+1) = 2\sum_{l=0}^{n-1}(l) + n = 2\frac{(n-1)n}{2} + n = n^2 \tag{4}$$

(iii) The angular momentum operators for each component are

$$\hat{L}_x = \hat{y}\,\hat{p}_z - \hat{z}\,\hat{p}_y \qquad \hat{L}_y = \hat{z}\,\hat{p}_x - \hat{x}\,\hat{p}_z \qquad \hat{L}_z = \hat{x}\,\hat{p}_y - \hat{y}\,\hat{p}_x \qquad \text{with} \qquad \hat{\mathbf{L}} = \begin{pmatrix} \hat{L}_x \\ \hat{L}_y \\ \hat{L}_z \end{pmatrix}$$

Using Heisenberg commutation relations

$$[\hat{r}_j, \hat{p}_k] = i\hbar\delta_{jk}\mathbb{1} \quad , \quad \hat{\boldsymbol{r}} = (\hat{x}, \hat{y}, \hat{z}) \quad , \quad \hat{\boldsymbol{p}} = (\hat{p}_x, \hat{p}_y, \hat{p}_z) \tag{5}$$

it follows that:

$$\left[\hat{L}_z, \hat{z}\right] = 0 \tag{6}$$

since $\hat{L}_z$ does not depend on $\hat{z}$ or $\hat{p}_z$.

$$\begin{aligned} \left[\hat{L}_z, \hat{x}\right] &= [\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{x}] = 0 - [\hat{y}\hat{p}_x, \hat{x}] = i\hbar\hat{y} \\ \left[\hat{L}_z, \hat{y}\right] &= [\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{y}] = [\hat{x}\hat{p}_y, \hat{y}] - 0 = -i\hbar\hat{x} \end{aligned} \tag{7}$$

(iv) Let's calculate the matrix element of $\hat{\boldsymbol{r}}$ with these information. As we know the action of $\hat{L}_z$ on $|\,n, l, m\rangle$ is given by

$$\hat{L}_z\,|\,n, l, m\rangle = \hbar m\,|\,n, l, m\rangle \tag{8}$$

$$\begin{aligned} 0 &= \Big\langle \underbrace{n', l', m'}_{\psi_b} \Big| \left[\hat{L}_z, \hat{z}\right] \Big| \underbrace{n, l, m}_{\psi_a} \Big\rangle = \Big\langle n', l', m' \Big| \left[\hat{L}_z, \hat{z}\right] \Big| n, l, m \Big\rangle = \Big\langle n', l', m' \Big| \hat{L}_z\hat{z} - \hat{z}\hat{L}_z \Big| n, l, m \Big\rangle = \\ &= \hbar(m' - m)\,\langle n', l', m'\,|\,\hat{z}\,|\,n, l, m\rangle \end{aligned} \tag{9}$$

Therefore $\langle n', l', m'\,|\,\hat{z}\,|\,n, l, m\rangle$ can be different from 0 only if $m = m'$.
Doing the same with the other two commutators we find that:

$$\begin{cases} i\hbar\,\langle n', l', m'\,|\,\hat{y}\,|\,n, l, m\rangle = \hbar(m' - m)\,\langle n', l', m'\,|\,\hat{x}\,|\,n, l, m\rangle \\ -i\hbar\,\langle n', l', m'\,|\,\hat{x}\,|\,n, l, m\rangle = \hbar(m' - m)\,\langle n', l', m'\,|\,\hat{y}\,|\,n, l, m\rangle \end{cases} \tag{10}$$

The solution found before $m' = m$ would lead to $\langle n', l', m'\,|\,\hat{x}\,|\,n, l, m\rangle = \langle n', l', m'\,|\,\hat{y}\,|\,n, l, m\rangle = 0$. Solving this system with $m' \neq m$ we find that:

$$\hbar^2\,\langle n', l', m'\,|\,\hat{y}\,|\,n, l, m\rangle = \hbar^2(m' - m)^2\,\langle n', l', m'\,|\,\hat{y}\,|\,n, l, m\rangle \tag{11}$$

It is true only if:

$$\hbar^2 = \hbar^2(m' - m)^2 \qquad \Longleftrightarrow \qquad m' - m = \pm 1 \tag{12}$$

Therefore the conditions to have a non-zero dipole moment expectation value are

$$m' - m \equiv \Delta m = 0, \pm 1 \tag{13}$$

(v) The casimir operator is defined as:

$$\hat{\boldsymbol{L}}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 \tag{14}$$

Its action on a state $\psi \equiv |\,n, l, m\rangle$ is given by

$$\hat{\boldsymbol{L}}^2\,|\,n, l, m\rangle = \hbar^2 l(l + 1)\,|\,n, l, m\rangle \tag{15}$$

To prove the relation we first prove other useful relations. In the following we are going to use Einstein's convention with Euclidean metric.

$$\left[\hat{\boldsymbol{L}}^2, \hat{L}_j\right] = \left[\hat{L}_i\hat{L}_i, \hat{L}_j\right] = \hat{L}_i\left[\hat{L}_i, \hat{L}_j\right] + \left[\hat{L}_i, \hat{L}_j\right]\hat{L}_i = i\hbar\epsilon_{ijk}(\hat{L}_i\hat{L}_k + \hat{L}_k\hat{L}_i) \overset{\text{symm.}}{=} 0 \tag{16}$$

where the commutation relation between angular momentum operators has been used:

$$\left[\hat{L}_i, \hat{L}_j\right] = i\hbar\epsilon_{ijk}\hat{L}_k \tag{17}$$

Lastly:

$$\left[\hat{L}_i, \hat{x}_j\right] = \epsilon_{inm}\left[\hat{x}_n\hat{p}_m, \hat{x}_j\right] = \epsilon_{inm}\hat{x}_n\left[\hat{p}_m, \hat{x}_j\right] = -i\hbar\epsilon_{inm}\hat{x}_n\delta_{mj} = i\hbar\epsilon_{ijk}\hat{x}_k \tag{18}$$

We can now start the computation.

$$\begin{aligned}\left[\hat{\boldsymbol{L}}^2, \hat{x}_n\right] &= \hat{L}_i\left[\hat{L}_i, \hat{x}_n\right] + \left[\hat{L}_i, \hat{x}_n\right]\hat{L}_i = i\hbar\epsilon_{inj}(\hat{L}_i\hat{x}_j + \hat{x}_j\hat{L}_i) \overset{Eq.18}{=} i\hbar\epsilon_{inj}(\epsilon_{ijk}\hat{x}_k + 2\hat{x}_j\hat{L}_i) = \\ &= 2i\hbar(\epsilon_{ijn}\hat{x}_i\hat{L}_j - i\hbar\hat{x}_n)\end{aligned} \tag{19}$$

Therefore:

$$\begin{aligned}\left[\hat{\boldsymbol{L}}^2, \hat{x}\right] &= 2i\hbar(\hat{y}\hat{L}_z - \hat{z}\hat{L}_y - i\hbar\hat{x}) \\ \left[\hat{\boldsymbol{L}}^2, \hat{y}\right] &= 2i\hbar(\hat{z}\hat{L}_x - \hat{x}\hat{L}_z - i\hbar\hat{y}) \\ \left[\hat{\boldsymbol{L}}^2, \hat{z}\right] &= 2i\hbar(\hat{x}\hat{L}_y - \hat{y}\hat{L}_x - i\hbar\hat{z})\end{aligned} \tag{20}$$

We now calculate the commutator $\left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{z}\right]\right]$:

$$\left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{z}\right]\right] = \left[\hat{\boldsymbol{L}}^2, 2i\hbar(\hat{x}\hat{L}_y - \hat{y}\hat{L}_x - i\hbar\hat{z})\right] \overset{Eq.16}{=} 2i\hbar\left(\left[\hat{\boldsymbol{L}}^2, \hat{x}\right]\hat{L}_y - \left[\hat{\boldsymbol{L}}^2, \hat{y}\right]\hat{L}_x - i\hbar\left[\hat{\boldsymbol{L}}^2, \hat{z}\right]\right) \tag{21}$$

We analyze the first two terms separately:

$$\begin{aligned}\left[\hat{\boldsymbol{L}}^2, \hat{x}\right]\hat{L}_y &= 2i\hbar(\hat{y}\hat{L}_z - \hat{z}\hat{L}_y - i\hbar\hat{x})\hat{L}_y = 2i\hbar(\hat{y}\hat{L}_z - i\hbar\hat{x})\hat{L}_y - 2i\hbar\hat{z}\hat{L}_y^2 = \\ &\overset{Eq.18}{=} 2i\hbar(\hat{L}_z\hat{y}\hat{L}_y - \hat{z}\hat{L}_y^2)\end{aligned} \tag{22}$$

For the second one:

$$\begin{aligned}-\left[\hat{\boldsymbol{L}}^2, \hat{y}\right]\hat{L}_x &= -2i\hbar(\hat{z}\hat{L}_z - \hat{x}\hat{L}_z - i\hbar\hat{y})\hat{L}_x = 2i\hbar(-\hat{z}\hat{L}_x^2 + \hat{x}\hat{L}_z\hat{L}_x + i\hbar\hat{y}\hat{L}_x) = \\ &\overset{Eq.18}{=} 2i\hbar(\hat{L}_z\hat{x}\hat{L}_x - \hat{z}\hat{L}_x^2)\end{aligned} \tag{23}$$

Adding them:

$$\left[\hat{\boldsymbol{L}}^2, \hat{x}\right]\hat{L}_y - \left[\hat{\boldsymbol{L}}^2, \hat{y}\right]\hat{L}_x = 2i\hbar(\hat{L}_z(\hat{x}\hat{L}_x + \hat{y}\hat{L}_y) - \hat{z}(\hat{L}_x^2 + \hat{L}_y^2)) = 2i\hbar(\hat{L}_z\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{L}} - \hat{z}\hat{L}_z^2 - \hat{z}\hat{\boldsymbol{L}}^2 + \hat{z}\hat{L}_z^2) \tag{24}$$

But we have that:

$$\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{L}} = \hat{x}_i\hat{L}_i = \epsilon_{ijk}\hat{x}_i\hat{x}_j\hat{p}_k = 0 \tag{25}$$

Therefore

$$\left[\hat{\boldsymbol{L}}^2, \hat{x}\right]\hat{L}_y - \left[\hat{\boldsymbol{L}}^2, \hat{y}\right]\hat{L}_x = -2i\hbar\hat{z}\hat{\boldsymbol{L}}^2 \tag{26}$$

Combining this result with

$$-i\hbar\left[\hat{\boldsymbol{L}}^2, \hat{z}\right] = -i\hbar(\hat{\boldsymbol{L}}^2\hat{z} - \hat{z}\hat{\boldsymbol{L}}^2) \tag{27}$$

we get from Equation 21 the final result:

$$\left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{z}\right]\right] = 2\hbar^2(\hat{z}\hat{\boldsymbol{L}}^2 + \hat{\boldsymbol{L}}^2\hat{z}) \tag{28}$$

Therefore we can conclude in general that:

$$\left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{r}}\right]\right] = 2\hbar^2(\hat{\boldsymbol{r}}\hat{\boldsymbol{L}}^2 + \hat{\boldsymbol{L}}^2\hat{\boldsymbol{r}}) \tag{29}$$

Note: If we consider instead of $\hat{\boldsymbol{r}}$ an operator $\hat{\boldsymbol{v}}$ such that

$$\left[\hat{L}_i, \hat{v}_j\right] = i\hbar\epsilon_{ijk}\hat{v}_k \tag{30}$$

then the proof is almost identical: the only difference is that in general Equation 25 is not true anymore. This would lead to an extra term proportional to $\hat{\boldsymbol{L}}(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{L}})$. Keeping into account the coefficient, the result reads

$$\left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{v}}\right]\right] = 2\hbar^2(\hat{\boldsymbol{v}}\hat{\boldsymbol{L}}^2 + \hat{\boldsymbol{L}}^2\hat{\boldsymbol{v}}) - 4\hbar^2\hat{\boldsymbol{L}}(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{L}}) \tag{31}$$

(vi) We can now use this result to derive the other selection rules.

$$\begin{aligned}\Big\langle \underbrace{n', l', m'}_{\psi_b} \Big| \left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{r}}\right]\right] \Big| \underbrace{n, l, m}_{\psi_a} \Big\rangle &= 2\hbar^2 \left\langle n', l', m' \left| \hat{\boldsymbol{r}}\hat{\boldsymbol{L}}^2 + \hat{\boldsymbol{L}}^2\hat{\boldsymbol{r}} \right| n, l, m \right\rangle = \\ &= 2\hbar^4 \left[l'(l'+1) + l(l+1)\right] \langle n', l', m' \,|\, \hat{\boldsymbol{r}} \,|\, n, l, m\rangle\end{aligned} \tag{32}$$

But on the other side:

$$\begin{aligned}\left\langle n', l', m' \left| \left[\hat{\boldsymbol{L}}^2, \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{r}}\right]\right] \right| n, l, m \right\rangle &= \left\langle n', l', m' \left| \hat{\boldsymbol{L}}^2 \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{r}}\right] - \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{r}}\right] \hat{\boldsymbol{L}}^2 \right| n, l, m \right\rangle = \\ &= \hbar^2 \left[l'(l'+1) - l(l+1)\right] \left\langle n', l', m' \left| \left[\hat{\boldsymbol{L}}^2, \hat{\boldsymbol{r}}\right] \right| n, l, m \right\rangle = \\ &= \hbar^4 \left[l'(l'+1) - l(l+1)\right]^2 \langle n', l', m' \,|\, \hat{\boldsymbol{r}} \,|\, n, l, m\rangle\end{aligned} \tag{33}$$

Therefore the condition to have a non-zero dipole moment is:

$$2\left[l'(l'+1) + l(l+1)\right] = \left[l'(l'+1) - l(l+1)\right]^2 \tag{34}$$

The term in the bracket on the right can be written as

$$l'(l'+1) - l(l+1) = (l'+l+1)(l'-l) \tag{35}$$

The one on the left instead:

$$2\left[l'(l'+1) + l(l+1)\right] = (l'+l+1)^2 + (l'-l)^2 - 1 \tag{36}$$

Therefore the condition in Equation 34 can be written as:

$$\left[(l'+l+1)^2 - 1\right]\left[(l'-l)^2 - 1\right] = 0 \tag{37}$$

The second term of the product is 0 only if:

$$l' - l = \pm 1 \tag{38}$$

The first term is 0 only if $l' = l = 0$; the corresponding matrix element is in principle not zero but direct computation of $\langle n', 0, 0 \,|\, \hat{\boldsymbol{r}} \,|\, n, 0, 0\rangle$ shows that this matrix element is indeed vanishing. In fact, the radial part of the associated wave function does not depend on the angle, while the spherical harmonics $Y_0^0(\theta, \phi)$ is constant: therefore the integration of $\boldsymbol{r}$ on the solid angle gives a vanishing result.

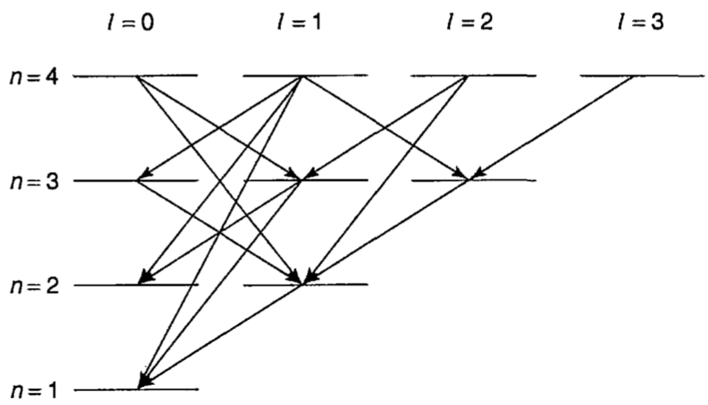

Abbildung 1: Possible transitions.

(vii) The selection rules are

$$\begin{cases} \Delta m = 0, \pm 1 \\ \Delta l = \pm 1 \end{cases} \tag{39}$$

The condition on Δl has an important physical reason. The transition between two energy level happens through the absorption/emission of a photon which is a spin-1 particle. Therefore the selection rule on l just reflects angular momentum conservation.

Exercise 65

Show that the life time τ of an atom in an excited state is inversely proportional to the Einstein-coefficient A of spontaneous emission.

Solution The definition of Einstein coefficient is

$$\frac{\mathrm{d}\,N_2}{\mathrm{d}t} = -A_{21}N_2 \tag{40}$$

where N_2 is the number of atoms in the excited level and A_{21} is the related Einstein coefficient. The solution is

$$N_2(t) = N_2(0)e^{-A_{21}t} \tag{41}$$

Therefore the lifetime is

$$\tau = \frac{1}{A_{21}} \tag{42}$$

Exercise 66

Consider a two-level system as in the lecture. Write the equations for the occupation number of the lower level, $\frac{dN_a}{dt}$, and upper level, $\frac{dN_b}{dt}$. Using these, show that $N_a + N_b = const.$

Solution The equations for the occupation numbers of the two levels are given by

$$\begin{aligned} \frac{dN_b}{dt} &= -N_b B_{ba}\rho + N_a B_{ab}\rho - N_b A_{ab} \\ \frac{dN_a}{dt} &= N_b B_{ba}\rho - N_a B_{ab}\rho + N_b A_{ab} \end{aligned} \tag{43}$$

By summing these two equations we obtain

$$\frac{d}{dt}(N_a + N_b) = 0 \implies N_a + N_b = const. \tag{44}$$

Exercise 67

Consider the general Schrödinger equation

$$i\hbar\frac{\partial\psi(q,t)}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2\psi(q,t)}{\partial q^2} + V(q)\psi(q,t) \tag{45}$$

where V is at most quadratic in q. Validate that the Ansatz

$$\psi(q,t) = \frac{1}{N}\exp\left[\alpha(t) + \frac{i}{\hbar}p_{cl}(t)(q - q_{cl}(t)) - \frac{(q - q_{cl}(t))^2}{2\sigma^2(t)}\right] \tag{46}$$

where $p_{cl} = m\frac{dq_{cl}}{dt}$ leads to an equation of the form

$$F_1(t) + F_2(t)(q - q_{cl}(t)) + F_3(t)(q - q_{cl}(t))^2 = 0 \tag{47}$$

Show that

$$\begin{aligned} F_1 = 0 &\equiv \frac{d\alpha}{dt} = \frac{i}{\hbar}(\frac{p_{cl}^2}{2m} - V(q_{cl})) - \frac{i\hbar}{2m\sigma^2(t)} \\ F_2 = 0 &\equiv \frac{dp_{cl}}{dt} = -\frac{\partial V}{\partial q}(q_{cl}) \\ F_3 = 0 &\equiv \frac{d\sigma^2}{dt} = \frac{i\hbar}{m} - \frac{i}{\hbar}\frac{\partial^2 V}{\partial q^2}\sigma^4 \end{aligned} \tag{48}$$

Solution Substituting the ansatz for $\psi(q,t)$ into (43) and using $p_{cl}\frac{dq_{cl}}{dt} = \frac{p_{cl}}{m}$ one obtains

$$0 = \frac{\psi(q,t)}{2m\sigma^4}\left[-\hbar^2\sigma^2 + p_{cl}^2\sigma^4 - 2mV(q)\sigma^4 + 2i\hbar m\frac{d\alpha}{dt}\sigma^4 - 2m\sigma^4\frac{dp_{cl}}{dt}(q-q_{cl}) + (\hbar^2 + 2i\hbar m\sigma\frac{d\sigma}{dt})(q-q_{cl})^2\right] \tag{49}$$

Since $\frac{\psi(q,t)}{2m\sigma^4} \neq 0$, it follows

$$-\hbar^2\sigma^2 + p_{cl}^2\sigma^4 - 2mV(q)\sigma^4 + 2i\hbar m\frac{d\alpha}{dt}\sigma^4 - 2m\sigma^4\frac{dp_{cl}}{dt}(q-q_{cl}) + (\hbar^2 + 2i\hbar m\sigma\frac{d\sigma}{dt})(q-q_{cl})^2 = 0 \tag{50}$$

Expanding $V(q)$ around q_{cl} we obtain

$$V(q) = V(q_{cl}) + \frac{dV}{dq}(q-q_{cl}) + \frac{1}{2}\frac{d^2V}{dq^2}(q-q_{cl})^2 \tag{51}$$

Note that all higher derivatives of V vanish since V is at most quadratic in q. Following this, let's collect all the terms

$$\begin{aligned} F_1(t) &= -\hbar^2\sigma^2 + p_{cl}^2\sigma^4 - 2mV(q_{cl})\sigma^4 + 2i\hbar m\frac{d\alpha}{dt}\sigma^4 \\ F_2(t) &= -2m\sigma^4\frac{dp_{cl}}{dt} - 2m\sigma^4\frac{dV}{dq} \\ F_3(t) &= \hbar^2 + 2i\hbar m\sigma\frac{d\sigma}{dt} - m\sigma^4\frac{d^2V}{dq^2} \end{aligned} \tag{52}$$

Then, the equation (50) becomes

$$F_1(t) + F_2(t)(q-q_{cl}(t)) + F_3(t)(q-q_{cl}(t))^2 = 0 \tag{53}$$

Now from the condition $F_1 = F_2 = F_3 = 0$ the proposition given by (48) follows.

General information

The *lecture* takes place on:
Monday at 10:00 - 12:00 c.t. in B 052 (Theresienstraße 37)
Friday at 10:00 - 12:00 c.t. in B 052 (Theresienstraße 37)
The *central tutorial* takes place on Monday at 12:00 - 14:00 c.t. in B 139 (Theresienstraße 37)
The *webpage* for the lecture and exercises can be found at

`https://www.physik.uni-muenchen.de/lehre/vorlesungen/wise_19_20/T_M1_TV_-Quantum-Mechanics-II`