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Exercises for Conformal Field Theory (MD4)

Problem set 8, due December 18, 2019

If you have questions write an E-mail to: mtraube@mpp.mpg.de

1 $c = 1$ theories: Bosonization & Fermionization and their moduli space

A) N free fermions transforming in the vector represenation of $SO(N)$ obey the $\widehat{\mathfrak{so}}(N)_1$ Kac-Moody algebra and have $c = \frac{N}{2}$. Compute their partition function.

Hint: How does the Fock-space change when going from one to N ***free*** *fermions?*

B) In the lecture we had another bosonic theory, the free boson on the orbifold $S^1/\mathbb{Z}_2$. At radius one this theory can be fermionized as well, giving a different theory of fermions. Interpret the $S^1/\mathbb{Z}_2$ theory in terms of fermionic theories by writing the partition functions in terms of θ functions.

C) (Optional) The moduli space of $c = 1$ theories has an ADE classification with three exceptional theories. The A series was the free boson on a circle, the D series its $\mathbb{Z}_2$ orbifold. Show that the A and D series intersect in one point. For this show $\mathcal{Z}_{\mathrm{S}^1}(R = 2\sqrt{2}) = \mathcal{Z}_{\mathrm{S}^1/\mathbb{Z}_2}(R = \sqrt{2})$.

2 Free boson on S^1 as orbifold

Recompute the partition function of the free boson on a S^1 as orbifold $\mathbb{R}/g$ where $g(x) = x + 2\pi R$. Use the general formula for an orbifold partition function. Compute the effect of the projector and determine the twisted sector and its mode expansion.

3 Shift orbifold

We consider a free boson on an orbifold $S^1/(-1)^{\mathcal{P}}$. The $\mathbb{Z}_2$ operator $(-1)^{\mathcal{P}}$ acts on a state with momentum m and winding n as $(-1)^{\mathcal{P}}|m, n\rangle = (-1)^m|m, n\rangle$. Compute the partition function of this orbifold in the following way:

A) Determine the action of the $\mathbb{Z}_2$ onto the coordinate X.

Hint: Recall sheet 4. The result explains the name "shift orbifold".

B) Formulate the twisted boundary conditions and read off the mode expansion of the twisted sector.

C) Compute the partition function for the twisted sector.