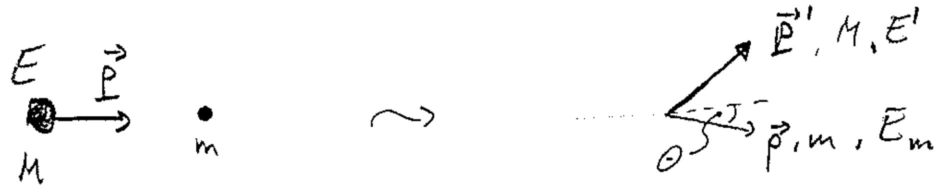
(max.) Energieübertrag im relativist. Stoß ($T_{max} \equiv E_{kin}$)



$$\vec{P} = \vec{P}' + \vec{p} \quad \rightsquigarrow \quad \vec{P}' = \vec{P} - \vec{p}$$

$$E + m = E' + E_m \;; \qquad E_m = E_{kin} + m \;; \quad E' = \sqrt{\vec{P}'^2 + M^2}$$

$$\Rightarrow \left.\begin{aligned} E'^2 &= (E - E_m + m)^2 = (E - E_{kin})^2 \\ &= \vec{P}'^2 + M^2 = (\vec{P} - \vec{p})^2 + M^2 \end{aligned}\right\} \rightarrow (\vec{P} - \vec{p})^2 + M^2 = (E - E_{kin})^2$$

$$\Rightarrow \vec{P}^2 + \vec{p}^2 - 2|\vec{P}||\vec{p}|\cos\theta + M^2 = E^2 - 2EE_{kin} + E_{kin}^2$$

$$\left.M^2 = E^2 - \vec{P}^2\right\} \Rightarrow \quad \vec{p}^2 - 2|\vec{P}||\vec{p}|\cos\theta = E_{kin}^2 - 2EE_{kin}$$

$$(*)\; \vec{p}^2 = E_m^2 - m^2 = (E_{kin} + m)^2 - m^2 = E_{kin}^2 + 2mE_{kin}$$

$$(*) \Rightarrow (E_{kin}^2 + 2mE_{kin}) - 2|\vec{P}||\vec{p}|\cos\theta = E_{kin}^2 - 2E_{kin}$$

$$\Rightarrow |\vec{P}||\vec{p}|\cos\theta = E_{kin}(E+m) \quad \rightsquigarrow \quad |\vec{p}| = \frac{E_{kin}\cdot(E+m)}{|\vec{P}|\cdot\cos\theta} \quad (***)$$

$$\Rightarrow \vec{P}^2\,\vec{p}^2\cos^2\theta = E_{kin}^2(E+m)^2$$

$$(*) \Rightarrow \vec{P}^2\cos^2\theta\cdot(E_{kin}^2 + 2mE_{kin}) = E_{kin}^2(E+m)^2$$

$$\Rightarrow E_{kin}\cdot\left((E+m)^2 - \vec{P}^2\cos^2\theta\right) = \vec{P}^2\cos^2\theta\cdot 2m$$

$$\Rightarrow \boxed{E_{kin} = \frac{2m\cdot\vec{P}^2\cos^2\theta}{(E+m)^2 - \vec{P}^2\cos^2\theta} \;\underset{\text{zentraler Stoß}}{\overset{\theta=0}{=}}\; \frac{2m\vec{P}^2}{(E+m)^2 - \vec{P}^2}} \quad (**)$$

$$(E+m)^2 - \vec{P}^2 = E^2 + 2Em + m^2 - \vec{P}^2 = M^2 + 2Em + m^2 \;; \quad \vec{P} = \beta\gamma Mc \;; \gamma = \frac{E}{M}$$

$$\Rightarrow \boxed{E_{kin} = \frac{2m\beta^2\gamma^2c^2}{1 + 2\gamma\frac{m}{M} + \left(\frac{m}{M}\right)^2}} = \frac{\vec{p}^2}{E_{kin} + 2m} \qquad \left(E_{kin} = \frac{\vec{p}^2}{m(\gamma_m + 1)} \;;\gamma_m = \frac{E_m}{m}\right)$$

$$(***) \text{ u. } (**) \Rightarrow \boxed{|\vec{p}| = \frac{2m|\vec{P}|(E+m)}{(E+m)^2 - \vec{P}^2} = \frac{2m\beta\gamma c(E+m)/M}{1 + 2\gamma\frac{m}{M} + (m/M)^2} = 2m\beta\gamma c\frac{\gamma + m/M}{1 + 2\gamma\frac{m}{M} + (m/M)^2}}$$