Sheet 05

Discussion: Thursday 13.06.2024

Exercise 1 Green's functions and residue theorem

In chapter 28.4.3 we solve the inhomogeneous diffusion equation

$$\partial_t f - D\Delta f = s. \tag{1}$$

For simplicity, we focus on $x \in \mathbb{R}^1$.

- What are f, D and s?
- For the homogeneous equation $s = 0$: What is the difference to wave equations?
- Write down the (inhomogeneous) solution of Eq. (1) in terms of the causal Green's function $G(x, t)$.
- What does G satisfy for delta-like sources at $x = 0$ and $t = 0$? Fourier transform the condition to fourier space by using

  $$\delta(x)\delta(t) = \int \frac{\mathrm{d}k}{2\pi} \int \frac{\mathrm{d}\omega}{2\pi} e^{ikx - i\omega t}. \tag{2}$$

  You should arrive at

  $$G(k, \omega) = \frac{1}{Dk^2 - i\omega}. \tag{3}$$

- Now we want to obtain $G(x, t)$ from $G(k, \omega)$.
  1. Integration w.r.t. ω:
     One can use the residue theorem when integrating over ω. How? In which plane do we have to integrate? Consider $t > 0$ and $t < 0$ seperately.
  2. Integrate w.r.t. k to obtain $G(x, t)$.

Exercise 2 State equations

A gas has the following equations of state:

$$P = E/V \tag{4}$$

and

$$T = 3B\left(\frac{E^2}{NV}\right)^{1/3} \tag{5}$$

where B is a positive constant. The system satisfies the third law of thermodynamics, so $S \to 0$ as $T \to 0$. Initially, the gas is at temperature T_i and pressure P_i, and it is then pushed through a porous membrane. The expansion is isenthalpic. Calculate the final temperature T_f as a function of the pressure P_f.