# TMP-TC2: Cosmology

**Problem Set 10** 25 & 27 June 2024

## 1. MOND

Assume that Newton's second law is modified as

$$m\ a_0\ f\left(\frac{a}{a_0}\right) = F\ ,$$

where the function $f(x)$ has the following properties :

$$f(x) \sim \begin{cases} x, \text{ for } x \gg 1 \\ x^2, \text{ for } x \ll 1\ . \end{cases}$$

1. Show that such a modification of Newtonian dynamics would explain the rotation curves of the galaxies.
2. Estimate the value of $a_0$ keeping in mind that for the Milky Way the circular velocity at large radii is approximately 200 km/sec and the enclosed mass is roughly $10^{11}\ M_\odot$.

## 2. Baryon Asymmetry of the Universe

At temperatures $T \gg m_p$ the universe had an excess of protons of the order of

$$\frac{n_p - n_{\bar{p}}}{n_p + n_{\bar{p}}} = 10^{-10}$$

This number changes due to annihilation and pair creation of protons and anti-protons until decoupling happens. Estimate the decoupling temperature and give a comment on the existence of anti-protons today.

## 3. Magnetic Monopole Problem

Domain walls are not the only type of objects that can be created during phase transitions. In this problem, we will focus on the so-called magnetic monopoles.
During the phase transition, there were many causally disconnected regions. Each such region can obtain a different vacuum expectation value (in the case of a degenerate vacuum). Let us assume that there was one magnetic monopole created per causally disconnected region of radius $r_H \sim \frac{M_{\rm Pl}}{T_{\rm GUT}^2}$, with $T_{\rm GUT} \sim 10^{16}$GeV the temperature at which the monopole of mass $m_M \sim 10^{17}$GeV was created.
Assuming that the monopole was not destroyed and also no more monopoles were created until now, what would be the contribution to the energy density today ? Why is this a problem ?