08 Problem Sheet

Samstag, 8. Juni 2024 12:16

Problem 1:

- $W \longrightarrow e\bar{\nu}$

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L = \frac{g}{\sqrt{2}} W_\mu^- \bar{e} \gamma^\mu \frac{1}{2}(1+\gamma_5)\nu$$

- Where is term coming from?

$$D_\mu = \partial_\mu - ig \hat{A}_\mu^a \tau^a - i g' \hat{Y} B_\mu$$

With definitions from sheet 6, we can rewrite this to

$$D_\mu = \partial_\mu - i g (W_\mu^+ \hat{\tau}^+ + W_\mu^- \hat{\tau}^-) - i \frac{g}{\cos\Theta} Z_\mu (\hat{\tau}^3 - \sin^2\Theta\, \hat{Q}) - ie A_\mu \hat{Q}$$

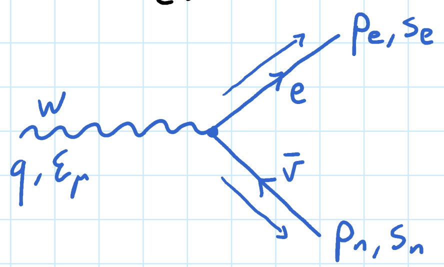
$$\mathcal{L} \supset \bar{E}_L i \cancel{D} E_L \quad \text{with} \quad E_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$\supset \bar{E}_L \frac{g}{\sqrt{2}} \gamma^\mu W_\mu^- \hat{\tau}^- E_L$$

$$= \frac{g}{\sqrt{2}} W_\mu^- \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}^T \gamma^\mu \underbrace{(\hat{\tau}_1 - i\hat{\tau}_2)}_{\frac{1}{2}\begin{pmatrix} 0 & 1-1 \\ 1+1 & 0 \end{pmatrix}} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L$$

(1)

- $W \longrightarrow e\bar{\nu}$



with $\varepsilon^\mu = \varepsilon_T^\mu(+) = (0, +1, +i, 0)/\sqrt{2}$

- $i\mathcal{M} = \bar{u}_{s_e}(p_e) \frac{g}{\sqrt{2}} \gamma^\mu \frac{(1+\gamma_5)}{2} v_{s_n}(p_n)\, \varepsilon_\mu(q)$

$$\bullet\ i\mathcal{M} = \bar{u}_{s_e}(p_e)\, \frac{g}{\sqrt{2}}\, \gamma^\mu\, \frac{(1+\gamma_5)}{2}\, v_{s_n}(p_n)\, \varepsilon_\mu(q)$$

$$|\tilde{\mathcal{M}}|^2 = \sum_{s_n, s_e} \varepsilon^*_\mu(q)\, \varepsilon_\nu(q)\, \frac{g^2}{8}$$

$\gamma^0\gamma^0$ $\gamma^0\gamma^0$

$$\cdot\ v^\dagger_{s_n}(p_n)\, (1+\gamma_5)^\dagger (\gamma^\mu)^\dagger\, \gamma^0\, u_{s_e}(p_e)$$

$\gamma^0\gamma^5\gamma^0 = -\gamma^5$

$\gamma^0\gamma^\mu\gamma^0 = (\gamma^\mu)^\dagger$

$$\cdot\ \bar{u}_{s_e}(p_e)\, \gamma^\nu (1+\gamma_5)\, v_{s_n}(p_n)$$

$$= \sum_{s_n, s_e} \varepsilon^*_\mu(q)\, \varepsilon_\nu(q)\, \frac{g^2}{8}$$

$$\cdot\ \bar{v}_{s_n}(p_n)_a \left[(1-\gamma_5)\gamma^\mu\right]_{ab} u_{s_e}(p_e)_b\, \bar{u}_{s_e}(p_e)_c \left[\gamma^\nu(1+\gamma_5)\right]_{cd} v_{s_n}(p_n)_d$$

$$= \varepsilon^*_\mu(q)\, \varepsilon_\nu(q)\, \frac{g^2}{8}$$

we assume massless

$$\cdot\ \mathrm{Tr}\left[(\not{p}_n - \cancel{m_n})\, (1-\gamma_5)\, \gamma^\mu\, (\not{p}_e + \cancel{m_e})\, \gamma^\nu\, (1+\gamma_5)\right]$$

where we used

$$\sum_s u_s(p)\, \bar{u}_s(p) = \not{p} + m \quad , \quad \sum_s v_s(p)\, \bar{v}_s(p) = \not{p} - m$$

$$|\tilde{\mathcal{M}}|^2 = \varepsilon^*_\mu(q)\, \varepsilon_\nu(q)\, \frac{g^2}{8} \cdot p_{n\alpha} \cdot p_{e\beta}$$

$$\cdot\ \mathrm{Tr}\left[\gamma^\alpha (1-\gamma_5)\, \gamma^\mu \gamma^\beta \gamma^\nu (1+\gamma_5)\right]$$

$$= \varepsilon^*_\mu(q)\, \varepsilon_\nu(q)\, \frac{g^2}{8}\, p_{n\alpha} \cdot p_{e\beta} \cdot 2$$

$$\mathrm{Tr}\left[\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu (1+\gamma_5)\right]$$

$$= \varepsilon^*_\mu(q)\, \varepsilon_\nu(q)\, \frac{g^2}{4}\, p_{n\alpha} \cdot p_{e\beta}$$

$$\left[4\left(\eta^{\alpha\mu}\eta^{\beta\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - \eta^{\alpha\beta}\eta^{\mu\nu}\right) + 4i\, \varepsilon^{\alpha\mu\beta\nu}\right]$$

$$= g^2 \Big[(\varepsilon^* \cdot p_n)(\varepsilon \cdot p_e) + (\varepsilon^* \cdot p_e)(\varepsilon \cdot p_n) - (p_e \cdot p_n) \underbrace{(\varepsilon^* \cdot \varepsilon)}_{=-1}$$

$$+ \underbrace{i\, \varepsilon^{\alpha\mu\beta\nu}\, \varepsilon^*_\mu \varepsilon_\nu\, p_{e\alpha} p_{n\beta}} \Big]$$

$$= i p_{e\alpha} p_{n\beta} \left(\varepsilon_1^* \varepsilon_2\, \varepsilon^{\alpha 1 \beta 2} + \varepsilon_2^* \varepsilon_1\, \varepsilon^{\alpha 2 \beta 1} \right)$$

$$= \frac{1}{2} i p_{e\alpha} p_{n\beta} \left(1 \cdot i \cdot \varepsilon^{\alpha 1 \beta 2} - (-i) \cdot 1 \cdot \varepsilon^{\alpha 1 \beta 2} \right)$$

$$= - p_{e\alpha} p_{n\beta}\, \varepsilon^{\alpha 1 \beta 2}$$

$$= - p_e^3 p_n^0\, \varepsilon_{3102} - p_e^0 p_n^3\, \varepsilon_{0132}$$

$$= g^2 \Big[(\varepsilon^* \cdot p_n)(\varepsilon \cdot p_e) + (\varepsilon^* \cdot p_e)(\varepsilon \cdot p_n) + (p_e \cdot p_n) - p_e^3 p_n^0 + p_e^0 p_n^3 \Big]$$

- $q^2 = M_W^2$ and $q^0 = p_e^0 + p_n^0 = |\vec{p}_e| + |\vec{p}_n|$ with $\vec{p}_e = -\vec{p}_n$

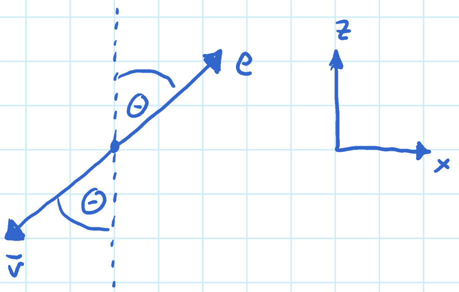
$$\rightarrow |\vec{p}_e| = \frac{M_W}{2}$$

$$\rightarrow p_e^0 = p_n^0 = \frac{M_W}{2}$$

- Let us choose the coordinate system

$$p_e^\mu = \frac{M_W}{2} (1, \sin\Theta, 0, \cos\Theta)$$

$$p_n^\mu = \frac{M_W}{2} (1, -\sin\Theta, 0, -\cos\Theta)$$



$$\varepsilon^* \cdot p_e = - \frac{M_W}{2} \frac{1}{\sqrt{2}} \sin\Theta = \varepsilon \cdot p_e$$

$$\varepsilon^* \cdot p_n = \frac{M_W}{2} \frac{1}{\sqrt{2}} \sin\Theta = \varepsilon \cdot p_n$$

- $$|\tilde{M}|^2 = g^2\Big(-\frac{M_W^2}{8}\cancel{\sin^2}\Theta\cdot 2 + \frac{M_W^2}{4}(1+\cancel{\sin^2}\Theta + \cos^2\Theta) - \frac{M_W^2}{4}\cos\Theta\cdot 2\Big)$$

$$= \frac{g^2 M_W^2}{4}(1+\cos^2\Theta - 2\cos\Theta) = \frac{g^2M_W^2}{4}(1-\cos\Theta)^2$$

- decay rate: (Peskin, Schroeder p. 107)

$$\Gamma = \frac{1}{2q^0}\int\frac{d^3p_e}{(2\pi)^3\,2p_e^0}\int\frac{d^3p_n}{(2\pi)^3\,2p_n^0}\;|\tilde{M}|^2\,(2\pi)^4\,\delta^{(4)}(q-p_e-p_n)$$

$$= \frac{1}{8M_W}\frac{1}{(2\pi)^2}\int d|\vec{p}_e|\,d\Omega\;\cancel{|\vec{p}_e|^2}\cdot\frac{1}{\cancel{p_e^0}\cdot\cancel{p_n^0}}\;\underbrace{\delta(M_W-2|\vec{p}_e|)}_{=\frac{1}{2}\delta\left(\frac{M_W}{2}-|\vec{p}_e|\right)}$$

$$= \frac{1}{64\pi^2 M_W}\int d\Omega\,|\tilde{M}|^2$$

$$\rightarrow \frac{d\Gamma}{d\Omega} = \frac{g^2M_W}{256\pi^2}(1-\cos\Theta)^2$$

- total decay rate:

$$\Gamma = \frac{g^2M_W}{256\pi^2}\int_0^{2\pi}d\phi\int_{-1}^{1}d\cos\Theta\,(1-\cos\Theta)^2$$

$$= \frac{g^2 M_W}{48\pi}$$

(2)

- $$\varepsilon^*\cdot p_e = -\frac{M_W}{2}\frac{1}{\sqrt{2}}\sin\Theta = \varepsilon\cdot p_e$$

$$\varepsilon^*\cdot p_n = \frac{M_W}{2}\frac{1}{\sqrt{2}}\sin\Theta = \varepsilon\cdot p_n$$

$$\varepsilon^* \cdot p_n = -\frac{M_W}{2}\frac{1}{\sqrt{2}}\sin\Theta = \varepsilon \cdot p_n$$

$\rightarrow$ same as before

- but

$$\underbrace{i\,\varepsilon^{\alpha\mu\beta\nu}\,\varepsilon_\mu^*\,\varepsilon_\nu\,p_{e\alpha}\,p_{n\beta}}$$

$$= i\,p_{e\alpha}\,p_{n\beta}\left(\varepsilon_1^*\varepsilon_2\,\varepsilon^{\alpha 1\beta 2} + \varepsilon_2^*\varepsilon_1\,\varepsilon^{\alpha 2\beta 1}\right)$$

$$= \frac{1}{2}\,i\,p_{e\alpha}\,p_{n\beta}\left(1\cdot \overset{\cdot(-1)}{i}\cdot\varepsilon^{\alpha 1\beta 2} - (-i)\overset{\cdot(-1)}{\cdot}1\cdot\varepsilon^{\alpha 1\beta 2}\right)$$

$$= -p_{e\alpha}\,p_{n\beta}\,\varepsilon^{\alpha 1\beta 2}\cdot(-1)$$

$$= \left(-p_e^3\,p_n^0\,\varepsilon_{3102} - p_e^0\,p_n^3\,\varepsilon_{0123}\right)\cdot(-1)$$

$$\rightarrow \frac{d\Gamma}{d\Omega} = \frac{g^2 M_W}{256\pi^2}(1+\cos\Theta)^2$$

- $\Gamma = \dfrac{g^2 M_W^2}{48\pi}$ is the same as before.

This makes of course sense, because a change of the polarisation can be 'compensated' by a change of the axes. The choice of the axes shouldn't change physics.

(3)

- $p_e \cdot \varepsilon = \dfrac{M_W}{2}\cos\Theta$

$p_n \cdot \varepsilon = -\dfrac{M_W}{2}\cos\Theta$

$i\,\varepsilon^{\alpha\mu\beta\nu}\,\varepsilon_\mu^*\,\varepsilon_\nu\,p_{e\alpha}\,p_{n\beta} = 0$

- $|\tilde{M}|^2 = g^2\left[(\varepsilon^*\cdot p_n)(\varepsilon\cdot p_e) + (\varepsilon^*\cdot p_e)(\varepsilon\cdot p_n) + (p_e\cdot p_n)\right]$

$$= g^2\left[\ldots 2\cdot\frac{M_W^2}{\ldots}\ldots^2\Theta + \frac{M_W^2}{\ldots}\left(1 + \sin^2\Theta + \ldots^2\Theta\right)\right]$$

$$= g^2\left[-2\cdot\frac{M_W^2}{4}\cos^2\Theta + \frac{M_W^2}{4}\left(1+\sin^2\Theta+\cos^2\Theta\right)\right]$$

$$= \frac{g^2 M_W^2}{2}\left(1-\cos^2\Theta\right) = \frac{g^2 M_W^2}{2}\sin^2\Theta$$

- $\frac{d\Gamma}{d\Omega} = \frac{g^2 M_W}{128\pi^2}\sin^2\Theta \rightarrow \Gamma = \frac{g^2 M_W}{48\pi}$

$\rightarrow$ same as before

(4)

- $|\tilde{M}|^2 \overset{\text{averaging}}{=} \frac{1}{3}\left[\frac{g^2 M_W^2}{4}(1-\cos\Theta)^2 + \frac{g^2 M_W^2}{4}(1+\cos\Theta)^2 + \frac{g^2 M_W^2}{2}\sin^2\Theta\right]$

$$= \frac{g^2 M_W^2}{3}\left[\frac{1}{4}\left(1-\cancel{2\cos\Theta}+\underline{\cos^2\Theta}\right) + \frac{1}{4}\left(1+\cancel{2\cos\Theta}+\underline{\cos^2\Theta}\right) + \frac{1}{2}\underline{\sin^2\Theta}\right]$$

$$= \frac{g^2 M_W^2}{3}$$

- $\frac{d\Gamma}{d\Omega} = \frac{g^2 M_W}{192\pi^2} \rightarrow \Gamma = \frac{g^2 M_W}{48\pi}$

$\rightarrow$ the same as before

(5)

- decay into top quark is not possible, because the top quark is to heavy

- leptons: e, μ, τ

quark pairs with charge -1: $\underbrace{d\bar{u}, s\bar{u}, b\bar{u}}_{\overset{\text{color}}{3}\cdot 1\cdot\Gamma_1}, \underbrace{d\bar{c}, s\bar{c}, b\bar{c}}_{\overset{\text{color}}{3}\cdot 1\cdot\Gamma_1}$

$\rightarrow \Gamma = (6+3)\,\Gamma_1$

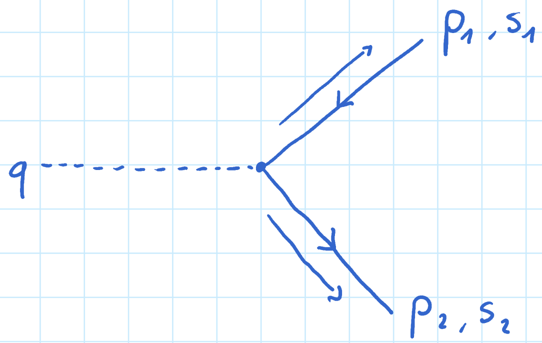
$$= 9\cdot\frac{g^2 M_W}{48\pi} = 9\cdot\frac{1}{12}\cdot\frac{1}{30}\cdot 80\,\text{GeV} = 2\,\text{GeV}$$

particle data group: $\Gamma = 2{,}085 \text{ GeV}$

$\rightarrow$ fits quite well although we did some approximations

Problem 2:

- $$\mathcal{L}_Y = \frac{g}{2} \frac{m_f}{M_W} h \bar{f} f$$



- $$i\mathcal{M} = \bar{u}_{s_2}(p_2) \left(i \frac{g}{2} \frac{m_f}{M_W} \right) v_{s_1}(p_1)$$

$$|\tilde{\mathcal{M}}|^2 = \frac{g^2 m_f^2}{4M_W^2} \sum_{s_1, s_2} \bar{v}_{s_1}(p_1) u_{s_2}(p_2) \bar{u}_{s_2}(p_2) v_{s_1}(p_1)$$

$$= \frac{g^2 m_f^2}{4M_W^2} \mathrm{Tr}((\not{p}_1 - m_f)(\not{p}_2 + m_f))$$

$$= \frac{g^2 m_f^2}{M_W^2} \left[(p_1 \cdot p_2) - m_f^2 \right]$$

- $$\Gamma = \frac{1}{2m_h} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta(m_h - E_1 - E_2) \cdot \delta(\vec{p}_1 - \vec{p}_2) |\tilde{\mathcal{M}}|^2$$

$$= \frac{1}{32\pi^2 m_h} \int d\Omega \int d|\vec{p}_1| \, |\vec{p}_1|^2 \frac{1}{E_1^2} \delta\left(m_h - 2\sqrt{|\vec{p}_1|^2 + m_f^2}\right) |\tilde{\mathcal{M}}|^2$$

$$= \frac{1}{8\pi m_h} \frac{|\vec{p}_1|^2}{E_1^2} \left| -2 \frac{|\vec{p}_1|}{E_1} \right|^{-1} \frac{g^2 m_f^2}{M_W} \left[\underbrace{(p_1 \cdot p_2)}_{= E_1^2 + |\vec{p}_1|^2} - m_f^2 \right]$$

$g^2 m_f^2$ $|\vec{p}_1|$

$$= \frac{g^2 m_f^2}{16\pi m_h M_W} \quad \frac{|\vec{p}_1|}{E_1} \quad \left(E_1^2 + |\vec{p}_1|^2 - m_f^2\right)$$

- With $E_1 = \frac{m_h}{2}$ and $|\vec{p}_1| = \sqrt{\frac{m_h^2}{4} - m_f^2}$

$$\Gamma = \frac{g^2 m_f^2}{16\pi m_h M_W} \sqrt{1 - 4\frac{m_f^2}{m_h^2}} \left(\frac{m_h^2}{4} + \frac{m_h^2}{4} - 2m_f^2\right)$$

$$= \frac{g^2 m_f^2 m_h}{32\pi M_W^2} \left(1 - 4\frac{m_f^2}{m_h^2}\right)^{3/2}$$

- In the quark case we get a factor of three, because of color.