Standard Model and QCD

Problem Sheet 8 18 June 2024

1. W boson decay

In this exercise you will compute the decay rate for the process $W \to e\bar{\nu}$. The relevant interaction term is

$$\frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L = \frac{g}{\sqrt{2}} W_\mu^- \, \bar{e} \gamma^\mu \frac{1}{2}(1+\gamma_5)\nu \ .$$

In what follows, the electron and neutrino should be taken as unpolarized, i.e. you should sum over all polarizations. For simplicity, take the W at rest. You can assume that the electron and neutrino are massless (their masses are negligible compared to their energies since $M_W \simeq 80$ GeV).
W boson polarization vectors :

- *positive z direction* $\epsilon_T^\mu(+) = \frac{1}{\sqrt{2}}(0,1,i,0)$
- *negative z direction :* $\epsilon_T^\mu(-) = \frac{1}{\sqrt{2}}(0,1,-i,0)$
- *Longitudinally polarized :* $\epsilon_L^\mu(0) = (0,0,0,1)$

1. Compute the partial differential decay rate and the total decay rate for the $W \to e\bar{\nu}$ process, with W polarized in the positive z direction.
2. Repeat the same for the W boson polarized in the negative z direction.
   Is the total decay rate the same as before ? Why ?
3. Now, compute the same for the longitudinally polarized W boson (zero spin component in the z direction). Is the total rate the same as before ?
4. Compute the total decay rate for an unpolarized W. Is it the same as before ?
5. Compute the total decay rate of W into all SM mesons and leptons (assume that all are massless for simplicity). Take three generations of fermions and recall that the top quark is very heavy - heavier than the W boson. Take $g^2/4\pi = 1/30$ and $M_W = 80\,GeV$.

   PDG experimental result : $\Lambda \approx 2.085 GeV$
   We expect a small error since the τ lepton and b quark masses are not completely negligible.

2. Higgs boson decay to fermions

The Higgs Yukawa coupling to fermions is given by

$$\mathcal{L}_Y = \frac{g}{2}\frac{m_f}{M_W} h \bar{f} f \ .$$

Find the decay rate for the Higgs into a fermion-antifermion pair. Account for the situation $f =$ quark.