Standard Model and QCD

Problem Sheet 7 11 June 2024

The low-energy description of weak interactions I : The charged sector of the Fermi theory

In the previous Exercise sheet we discussed the Higgs phenomenon in a gauged SU(2)×U(1) theory. Actually, this is the part of the Standard Model (SM) describing the dynamics of the electroweak interactions. Yet, the way for SM has been paved by an effective description, called *Fermi theory*, whose phenomenology of weak decays gives very accurate insights. In this exercise, we will show how to derive the model and to estimate its relevant parameters.

a) Add the fermionic sector to the SU(2)×U(1) theory from the previous Exercise sheet. You can ignore the fermion masses and their interactions with the Higgs field (don't add the Yukawa sector).

b) Experiments show that the theory involves vector and axial currents. They are related to global U(1) symmetries, which act on a Dirac bispinor as

$$\psi \;\rightarrow\; \psi' = e^{i\alpha}\psi \;, \tag{1}$$
$$\psi \;\rightarrow\; \psi' = e^{i\gamma_5\beta}\psi \;, \tag{2}$$

respectively. Construct a combination of the resulting Noether currents (take the Dirac field to be free and massless), in such a way that the low-energy theory reflects the coupling of weak interactions only to left-handed fermions at high energies.

c) When the Higgs field acquires a VEV, the charged current sector of the electroweak Lagrangian can be written as

$$\mathcal{L}_{\rm CC} \;=\; \frac{g}{\sqrt{2}}\left(W^{+\mu}J^{+}_{\mu} + W^{-\mu}J^{-}_{\mu}\right) \;. \tag{3}$$

What are the currents $J^{\pm}_{\mu}$, respecting the prescription of the previous point ?

d) Consider the vector bosons $W^{\pm}$ only as internal states and integrate them out, requiring that $E \ll m_W$, where E is the energy scale of the system. Write down the local interaction term of the 4−Fermi Lagrangian density.

e) Perform naïve power counting on the charged current interactions. Are they renormalizable ?

f) Compute the decay rate for the muon

$$\mu \longrightarrow e^- + \nu_\mu + \bar{\nu}_e \tag{4}$$

in this effective theory, considering the electron ultra-relativistic. Given the measured decay time, $\tau_\mu = 2.197\ \mu$s, estimate the value of the VEV of the electroweak sector, v. Is it in agreement with its experimental measure ?