Standard Model and QCD

Problem Sheet 6 4 June 2024

1. Higgs phenomenon in SU(2)×U(1)

Consider the following Lagrangian invariant under a gauged SU(2)×U(1) symmetry

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu\,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger D^\mu H - \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2 ,$$

where

$$\begin{aligned} W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu \,, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,, \end{aligned}$$

and the covariant derivative of the complex doublet field $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$, $H_{1,2} \in \mathbb{C}$, is given by

$$D_\mu H = \partial_\mu H - igW^a_\mu \tau^a H - i\frac{g'}{2}B_\mu H \,.$$

In the above τ^a, $a = 1, 2, 3$, are the SU(2) generators and g, g' are the gauge couplings associated with the SU(2) and U(1) groups, respectively.

1. Minimize the potential and identify the vacuum manifold.
2. Let us chose the vacuum to be

   $$\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

   Write down the unbroken generators, if there are any. What is the unbroken subgroup ? How many massive gauge fields do you expect ?
3. Write the potential around the minimum, identify the Higgs mass m_h and write the terms in the potential (quadratic, cubic and quartic) as functions of m_h and v.

   Hint : Work in the unitary gauge, meaning that you use the gauge redundancy to absorb the would-be Nambu-Goldstone bosons in the gauge fields, and use the convention

   $$H \,=\, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \,,$$

   with h a real scalar field.
4. Expand the kinetic term of H around the vacuum to determine how many gauge bosons acquire masses and how many remain massless. Does that agree with your expectations from part 2 ? Explain.

   Hint : Define the physical gauge bosons by

   $$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}} \,, \quad Z_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2 + g'^2}} \,, \quad A_\mu = \frac{gB_\mu + g'W^3_\mu}{\sqrt{g^2 + g'^2}} \,.$$

5. Let us define the angle θ by

$$\tan\theta = \frac{g'}{g}.$$

   Show that the electric charge is given by $e = g\sin\theta$. What are the electric charges of the massive gauge bosons ?

6. What are the decay channels for the Higgs boson in this theory ? Consider only decays that can appear at tree-level.