Standard Model and QCD

Problem Sheet 4 — 14 May 2024

1. An example of UV completion of gauge theories : the Abelian Higgs mechanism for Proca fields

Let us consider the so-called Proca Lagrangian

$$\mathcal{L}[\tilde{A}_\mu] \,=\, -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{m^2}{2}\tilde{A}_\mu\tilde{A}^\mu + \frac{\xi^4}{4}\left(\tilde{A}_\mu\tilde{A}^\mu\right)^2 \ , \qquad \tilde{F}_{\mu\nu} \equiv 2\partial_{[\mu}\tilde{A}_{\nu]} \ , \tag{1}$$

and provide a UV completion by integrating in an additional degree of freedom.

1. Check that the Stueckelberg decomposition[1] $\tilde{A}_\mu = A_\mu + \frac{1}{m}\partial_\mu\theta$ implies a redundancy of the form

$$\begin{cases} A_\mu & \to \ A'_\mu = A_\mu + \frac{1}{m}\partial_\mu\chi \\ \theta & \to \ \theta' = \theta - \chi \end{cases} \tag{2}$$

   Consider the free theory (*i.e.* $\xi = 0$), for the sake of simplicity, and compute the number of propagating degrees of freedom of a massive vector field.

2. Compute the propagator of the theory in the parametrization given by (1) in momentum space. Sandwich it between two sources j_μ and investigate the behavior when $m \to 0$.

3. In the interacting case, show that at large momenta unitarity is lost . You can argue this by analyzing the 2→2 scattering amplitude at tree level.

   (**Hint :** It is useful to rewrite the Lagrangian in terms of A_μ and θ instead of $\tilde{A}_\mu$.)

4. Unitarity is restored by integrating in a new degree of freedom $H = \rho(x)e^{i\theta(x)/v}$ and realising the Abelian Higgs model

$$\mathcal{L}[A_\mu,\, H] \,=\, -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(D_\mu H\right)^\dagger D^\mu H - \frac{\lambda^2}{2}\left(H^\dagger H - v^2\right)^2 \ , \tag{3}$$

   where $F_{\mu\nu} \equiv 2\partial_{[\mu}A_{\nu]}$ and $D_\mu = \partial_\mu + igA_\mu$. Check that the conditions, similar to Eq. (2),

$$\begin{cases} A_\mu & \to \ A'_\mu = A_\mu + \frac{1}{g}\partial_\mu\alpha \\ H & \to \ H' = e^{-i\alpha}H \end{cases} \tag{4}$$

   still leave the Lagrangian invariant. Moreover, assume that the Higgs has a VEV, i.e. expand the theory around the vacuum according to the prescription

$$\rho(x) = v + \frac{1}{\sqrt{2}}h(x)$$

   and identify the massive modes.

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1. Here, A_μ is the transverse part of the Proca field $\tilde{A}_\mu$.

5. Assuming that the Higgs is heavier than the vector, that is $m \lesssim E \ll m_H$, one can integrate the former out. Show that in this limit one recovers the Proca Lagrangian. Identify the coupling ξ in terms of the parameters in Eq. (3).

   (**Hint :** Consider the EOM for h limited to the lowest order interaction in h.)