Sheet 07

Discussion: Thursday 06.07.23

Exercise 1 Throttling I

A gas has the following equations of state

$$P = \frac{E}{V} \qquad \text{and} \qquad T = 3B\left(\frac{E^2}{NV}\right)^{1/3}, \tag{1}$$

where B is a positive constant. The system satisfies the third law, so that $S \to 0$ if $T \to 0$. Initially, the gas is at temperature T_i and pressure P_i, and is then forced through a porous membrane. Let the expansion proceed like the Joule-Thomson process, i.e. isenthalp. Calculate the final temperature T_f, depending on the pressure P_f.

Exercise 2 Throttling $\mathrm{CO_2}$

Show that the molar enthalpy h of a van der Waals gas can be expressed as

$$h = -\frac{2a}{v} + RT\left(\xi + \frac{v}{v-b}\right). \tag{2}$$

Now let such a gas be forced through a porous membrane so that it expands from v_i to v_f. Calculate the final temperature T_f depending on T_i.

Use this to calculate the temperature difference for $\mathrm{CO_2}$. Let the mean temperature be $0^\circ, \mathrm{C}$, the mean pressure $10^7\,\mathrm{Pa}$, and the pressure difference $10^6\,\mathrm{Pa}$. Let the heat capacity c_P of $\mathrm{CO_2}$ for this pressure and temperature range be $29.5\,\mathrm{J/mol\,K}$. Calculate only to the first order of b/v and a/RTv.
Use as vdW constants $a_{\mathrm{CO_2}} = 0.401\mathrm{Pa\,m^6}$ and $b_{\mathrm{CO_2}} = 42.7 \cdot 10^{-6}\mathrm{m^3}$.

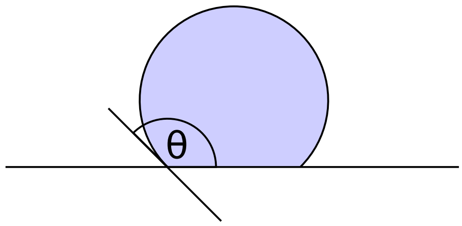
Exercise 3 Interfacial tension

Thermodynamic properties of interfaces between two phases are described by the interfacial tension σ. This is defined by the work $\mathrm{d}W = \sigma\,\mathrm{d}A$, which is required to increase the interfacial tension by $\mathrm{d}A$.

(a) Show that the pressure inside a spherical water droplet of radius R is larger by $2\sigma/R$ than the pressure outside the droplet. Consider the work done against the boundary surface stress for an infinitesimal change of the radius.

(b) A spherical water drop condenses on a solid surface. Three different boundary surface tensions play a role: σ_{aw}, σ_{sw}, σ_{sa}. Here a, s and w represent air, the solid surface and water, respectively. Calculate the contact angle θ between the surface and the water droplet.

Find the condition for the appearance of a liquid film.



Exercise 4 Capillary effect

Use our result from 3 (a) and the Laplace equation

$$\Delta P = \frac{2\sigma}{r}\,, \tag{3}$$

to calculate the rise h of a fluid in a thin tube of radius r_0. For this, note that „inside " always means inside the sphere segment.
Hint: The exact definition of h at the spherical surface, ,as well as the volume change of the reservoir do not matter here. Your result should depend on σ and θ.