Problem 1

$$\Phi \xrightarrow{P} \Phi^+ , \quad \Delta_{LR} \xrightarrow{P} \Delta_{RL}$$

For terms depending only on bi-doublet see previous homework (HW5)

$$V_\Delta = -\mu_\Delta^2 \left[\mathrm{Tr}(\Delta_L \Delta_L^+) + \mathrm{Tr}(\Delta_R \Delta_R^+) \right] + \left[\rho_1 \, \mathrm{Tr}(\Delta_L \Delta_L^+)^2 \right.$$
$$\left. + \rho_2 \, \mathrm{Tr}\, \Delta_L^2 \, \mathrm{Tr}\, \Delta_L^{+2} + L \leftrightarrow R \right] +$$
$$+ \rho_3 \, \mathrm{Tr}\, \Delta_L \Delta_L^+ \, \mathrm{Tr}\, \Delta_R \Delta_R^+ + \rho_4 \left(\mathrm{Tr}\, \Delta_L^+ \Delta_L^+ \, \mathrm{Tr}\, \Delta_R \Delta_R \right.$$
$$\left. + L \leftrightarrow R \right)$$

$$V_{\Delta\Phi} = \alpha_1 \left[\mathrm{Tr}\, \Phi^+ \Phi + \alpha_2 \left(\mathrm{Tr}(\tilde{\Phi} \Phi^+) + h.c. \right) \right] \cdot$$
$$\cdot \left[\mathrm{Tr}\, \Delta_L \Delta_L^+ + \mathrm{Tr}\, \Delta_R \Delta_R^+ \right] + \alpha_3 \left[\mathrm{Tr}(\Phi \Phi^+ \Delta_L \Delta_L^+) + \right.$$
$$\left. + \mathrm{Tr}(\Phi^+ \Phi \Delta_R \Delta_R^+) \right] +$$
$$+ \beta_1 \, \mathrm{Tr}(\Phi \Delta_R \Phi^+ \Delta_L^+) + \beta_2 \, \mathrm{Tr}(\tilde{\Phi} \Delta_R \Phi^+ \Delta_L^+)$$
$$+ \beta_3 \, \mathrm{Tr}(\Phi \Delta_R \tilde{\Phi}^+ \Delta_L^+) + h.c.$$

Invariant under $\Delta_L \to U_L^+ \Delta_L U_L \quad \Delta_R \to U_R^+ \Delta U_R$

$$\Phi \to U_L^+ \Phi U_R$$

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix} \; ; \; \langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{LR} & 0 \end{pmatrix} \; ; \; \langle \Phi \rangle = v \begin{pmatrix} \cos\beta & 0 \\ 0 & e^{-i\alpha} \sin\beta \end{pmatrix}$$

if $\langle \Phi \rangle = 0 \Rightarrow v_L = 0$

But if $\Phi \neq 0$? Tadpole induced VEV v_L from β_i's terms.

$$V \supset v^2 v_L v_R \left(-2\beta_2 \cos\beta^2 + \beta_1 \cos\alpha \sin 2\beta - 2\beta_3 \sin\beta^2 \right)$$

tadpole

$\rightarrow$ implies $v_L \neq 0$

$\beta_i \lesssim 1 \rightsquigarrow$ perturbative unitarity.

Variation $\frac{\partial V}{\partial v_L} = 0 \Rightarrow v_L \sim \frac{v^2 v_R}{\Lambda^2}$

Λ^2 there for dimensionality.

$$\Lambda \sim v_R \Rightarrow v_L \sim \frac{v^2}{v_R} = \frac{v}{v_R} v \ll v$$

must be that or smaller to preserve SM.

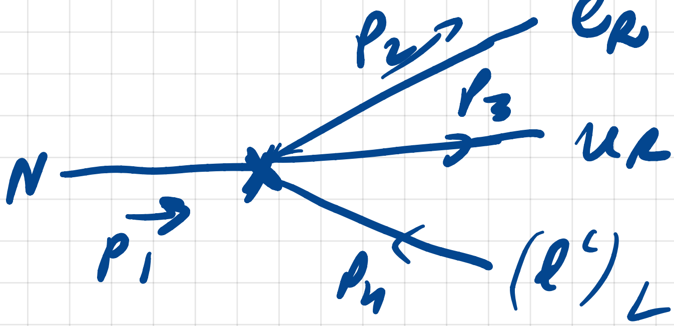
• $N \to e_R + u_R + (d^c)_L$

via W_R exchange.

(take $m_e \simeq m_u \simeq m_d \simeq 0$)

$$\frac{g^2}{2m_{W_R}^2}\, \bar{N} \gamma_\mu e\, \bar{d} \gamma_\mu u$$

$$\left(\# g\, W_R^\mu \bar{N} \gamma_\mu e + \# g\, W_R^\mu \bar{d} \gamma_\mu u\right)$$



$$\mathcal{M} = \frac{g^2}{2m_{W_R}^2}\, \bar{u}_{3R}^{\alpha} \gamma_\mu^{\alpha\alpha'} u_{4R}\, \bar{u}_{2R} \gamma_\mu u_1$$

$$\mathcal{M}^\dagger = \frac{g^2}{2m_{W_R}^2}\, \bar{u}_1 \gamma_\mu u_{2R}\, \bar{u}_{4R} \gamma_\mu u_{3R}$$

$$|\mathcal{M}|^2 = \frac{g^4}{4 m_{W_R}^4} \mathrm{Tr}(\not{p}_1 \gamma_\mu \not{p}_2 \gamma_\nu R)\, \mathrm{Tr}(\not{p}_3 \gamma_\nu \not{p}_4 \gamma_\mu R)$$

$$= \frac{g^4}{4 m_{W_R}^4} \Big[p_1^\alpha p_2^\beta (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\mu\beta}$$

$$- i \epsilon_{\alpha\mu\beta\nu}) \, p_{3\sigma} p_{4\lambda} (g^{\sigma\nu} g^{\lambda\mu} - g^{\sigma\lambda} g^{\mu\nu}$$

$$+ g^{\sigma\mu} g^{\nu\lambda} - i \epsilon^{\sigma\nu\lambda\mu}) \Big]$$

$$= \frac{g^4}{4 m_{W_R}^4} \Big[(\underline{p_{1\mu} p_{2\nu}} - p_1 \cdot p_2 \, g_{\mu\nu} + p_{1\nu} p_{2\mu}$$

$$- i \underline{\epsilon_{\alpha\mu\beta\nu}} \, p_1^\alpha p_2^\beta) (p_3^\nu p_4^\mu - p_3 \cdot p_4 \, g^{\mu\nu}$$

$$+ p_3^\mu p_4^\nu - i \underline{\epsilon^{\sigma\nu\lambda\mu}} \, p_{3\sigma} p_{4\lambda}) \Big]$$

$$- \epsilon^{\alpha\mu\beta\nu} \epsilon_{\sigma\nu\lambda\mu}$$

$$= \ \ldots$$

$$= \frac{g^4}{2 m_{W_R}^4} \Big[(p_1 \cdot p_4)(p_2 \cdot p_3) + (p_1 \cdot p_3)(p_2 \cdot p_4)$$

$$+ (g_{\alpha\sigma} g_{\beta\lambda} - g_{\alpha\lambda} g_{\beta\sigma}) p_1^\alpha p_2^\beta p_3^\sigma p_4^\lambda \Big]$$

$$|\mathcal{M}|^2 = \frac{g^4}{m_{W_R}^4} (p_1 \cdot p_3)(p_2 \cdot p_4)$$

$$d\Gamma = \frac{1}{2m_N} \prod_f \frac{d^3\vec{p}_f}{(2\pi)^3 E_f} |\mathcal{M}|^2$$

(Hint: go to rest frame of N)

$$\rightarrow \Gamma = \frac{1}{6144\pi^3} \frac{g^4}{m_{W_R}^4} m_N^5$$

$$\Gamma \propto \frac{g^4 m_N^5}{m_{W_R}^4}$$

- $\underline{Tr(D_\mu \phi^\dagger D_\mu \phi)}$ $\Big(+ Tr(D_\mu \Delta_L^\dagger) D_\mu \Delta_L + Tr(D_\mu \Delta_R^\dagger) D_\mu \Delta_R \Big)$

$$D_\mu \phi = \partial_\mu \phi + i g \left(\frac{\vec{\tau}}{2} \vec{W}_L \phi - \phi \frac{\vec{\tau}}{2} \vec{W}_R \right)$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$$

$$\left[\langle \Delta_{R,L} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \right]$$

$$\underline{\underline{\mathrm{Tr}\left((D_\mu \varphi)^\dagger D_\mu \varphi\right)}} \supset -g^2 \left(v_1^* v_2 \, W^+_{L\mu} W^-_{R\mu} + h.c. \right)$$

charged gauge bosons

$$-\frac{g^2}{2} \left(|v_1|^2 + |v_2|^2 \right) W_{L3\mu} W_{R3\mu}$$

$$W^\pm_{L,R} = \frac{W^1_{L,R} \mp i \, W^2_{L,R}}{\sqrt{2}}$$

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos\xi & -\sin\xi^* \\ \sin\xi^* & \cos\xi \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}$$

$$\xi = \frac{2|v_1 v_2|}{|v_1|^2 + |v_2|^2} \underline{\underline{\left(\frac{m_{W_L}}{m_{W_R}}\right)^2}} \ll 1,$$

$$\boxed{m_L^2 = \frac{1}{4} g^2 (|v_1|^2 + |v_2|^2)}$$

$$\boxed{m_2^2 \simeq \frac{1}{2} g^2 |v_R|^2 + \mathcal{O}\left(\frac{v_1, v_2}{v_R}\right)}$$

$$\left.\begin{aligned} \underline{W_1} &= \cos\xi\, W_L - \sin\xi\, W_R \\ W_2 &= \sin\xi\, W_L + \cos\xi\, W_R \end{aligned}\right\}$$

" $W_1 \longmapsto W_L$ "

" $W_2 \longmapsto W_R$ "

L

Invert it:

$$W_R^{+} = -\sin\xi \, W_1^{+} + \cos\xi \, W_2^{+} .$$

$$W_{R\mu} J_{R\mu} \supset -\sin\xi \, W_{1\mu} J_{R\mu}$$

$$J_{R\mu} \supset \bar{N} \gamma e$$

$$g \sin\xi \, W_{1\mu} \bar{N} \gamma_\mu e$$

$$\Gamma^{2\,\text{body}\,(\text{I.S.7})} \simeq \frac{g^2}{64\pi} m_\nu \frac{m_N^2}{m_{W_L}^2} + \ldots$$

$$\boxed{\Gamma = \sin^2\xi \, \Gamma^{2\,\text{body}}}$$

$$\frac{g\theta^2}{\sqrt{2}}\left(W_\mu^{L+} J_\mu^L + W_\mu^{R+} J_\mu^R\right) + h.c.$$

$$= W_\mu^{L+}(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu d_L)$$

$$+ W_\mu^{R+}(\bar{N}_R \gamma^\mu e_R + \bar{u}_R \gamma^\mu d_R)$$

- $m_N = \text{TeV}$, $M_{W_R} \simeq 10\ \text{TeV} = 100\, M_{W_L}$

$m_\nu \simeq 0.1\ \text{eV}$.

$$\frac{\Gamma^{3\,\text{body}}}{\Gamma^{2\,\text{body}}} = \frac{\frac{g^2}{64\pi} \cdot \frac{g^2}{96\pi^2} \times \frac{m_N^5}{m_{W_R}^4}}{\frac{g^2}{64\pi}\, m_\nu\, \frac{m_N^2}{m_{W_L}^2}}$$

$$= \frac{g^2}{96\pi^2} \times \frac{m_N^3}{m_\nu} \times \frac{m_{W_L}^2}{m_{W_R}^4}$$

$\rightarrow \quad \dfrac{\Gamma_{3\,body}}{\Gamma_{2\,body}} \simeq 10^{3}$