Problem 1

- M_N symmetric:

$$\nu_R^T \frac{M_N^T}{2} C \nu_R =$$

$$= \nu_{Ri}^T (M_N^\dagger)_{ij} C \nu_{Rj} = - \nu_{Rj} C^T (M_N^*)_{ji} \nu_{Ri}$$

$$= \nu_{Rj} C (M_N^*)_{ji} \nu_{Ri} = \nu_{Ri} C (M_N^*)_{ij} \nu_{Ri}$$

$$\Rightarrow M_N^\dagger = M_N^* \Rightarrow M_N^T = M_N \Rightarrow \text{symmetric matrix}$$

- Show $SU(2) \times U(1)$ SM invariance

$\rightarrow$ ν_R has no quantum #'s under $SU(2)_L \times U(1)$

$$(i\sigma_2 \Phi^*)^\dagger \, l_L = -\Phi^T i \sigma_2^T l_L$$

$$= \Phi^T i \sigma_2 l_L$$

$$U^T i\sigma_2 U = i\sigma_2 \quad \checkmark$$

- $N_{iL} = C\bar{\nu}_{iR}^T$ → drop i index

$$N_L^T C \frac{M_N}{2} N_L = (C\bar{\nu}_R^T)^T C \frac{M_N}{2} C \bar{\nu}_R^T$$

$$= \bar{\nu}_R \underbrace{C^T C}_{1} \frac{M_N}{2} C \bar{\nu}_R^T$$

$\bar{\nu} = \nu^+ \gamma^0$

h.c. ↓

$$(\bar{\nu}_R^T)^+ C^+ \frac{M_N^+}{2} (\bar{\nu}_R)^+ =$$

$$= (\gamma^{0T} \nu_R^*)^+ C^+ \frac{M_N^+}{2} \gamma^{0+} \nu_R$$

$$= \nu_R^T \gamma_0^* C^+ \frac{M_N^+}{2} \gamma^{0+} \nu_R$$

$$= -\nu_R^T \gamma_0^* C \frac{M_N^+}{2} \gamma_0^+ \nu_R$$

$$\gamma_0^* C \gamma_0^+ = \gamma_0 C \gamma_0 = \gamma_0 i \gamma_2 \gamma_0^2 =$$

$$\gamma_0 = \gamma_0^* = \gamma_0^+ = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$= \gamma_0 i \gamma_2$$

$$= -i \gamma_2 \gamma_0$$

$$= \nu_R^T C \frac{M_N^+}{2} \nu_R$$ ✓

other term:

$$\overline{\nu_R}\,\tilde{\Phi}^\dagger Y_D\, l_L \quad , \quad \tilde{\Phi} = i\sigma_2 \Phi^*$$

$$N_L = C\,\overline{\nu_R}^T \Rightarrow -C N_L = \overline{\nu_R}^T$$

$$\Rightarrow \overline{\nu_R} = N_L^T C$$

$$\Rightarrow \overline{\nu_R}\,\tilde{\Phi}^\dagger Y_D l_L = N_L^T C\,\tilde{\Phi}^\dagger Y_D l_L$$

$$= N_L^T C\,\Phi^T i\sigma_2 Y_D l_L \quad \checkmark$$

- $\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$ $\qquad$ "$l = \begin{pmatrix}\nu\\ e\end{pmatrix}$"

$$N_L^T C\,\Phi_0^T i\sigma_2 Y_D l_L = N_L^T C \underbrace{\frac{v Y_D}{\sqrt{2}}}_{\equiv M_D} \nu_L$$

- Consider $N_L^T C M_D \nu_L =$

$$= \frac{1}{2}\left(N_L^T C M_D \nu_L + N_L^T C M_D \nu_L\right)$$

$$= \frac{1}{2}\left(-\nu_L^T M_D^T C^T N_L + N_L^T C M_D \nu_L\right)$$

$$= \frac{1}{2}\left(\nu_L^T M_D^T C N_L + N_L^T C M_D \nu_L\right)$$

Therefore

$$\mathcal{L}_{mass} = \frac{1}{2}\left(\nu_L^T M_D^T C N_L + N_L^T C M_D \nu_L + N_L^T M_N C N_L\right) + h.c. \quad \checkmark$$

$$\Rightarrow \quad M_{\nu N} = \begin{array}{c} \\ \nu \\ N \end{array}\begin{array}{c} \begin{array}{cc} \nu & N \end{array} \\ \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \end{array}$$

- Assume $M_N \gg M_D \Rightarrow U^T M_{\nu N} U = D_{\nu N}$

$$D_{\nu N} \equiv \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$U^T U = \begin{pmatrix} 1 & -\theta^+ \\ \theta & 1 \end{pmatrix}\begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + O(\theta^2) \quad \checkmark$$

$$U^T M_{\nu N} U = \begin{pmatrix} 1 & -\theta^T \\ \theta^* & 1 \end{pmatrix} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\theta^T M_D & M_D^T - \theta^T M_N \\ M_D & \theta^* M_D^T + M_N \end{pmatrix} \begin{pmatrix} 1 & \theta^+ \\ -\theta & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\theta^T M_D & M_D^T - \theta^T M_N \\ M_D - M_N \theta & M_D \theta^+ + \theta^* M_D^T + M_N \end{pmatrix} + O(\theta^2)$$

off diagonal vanish if

$$M_D - M_N \theta = 0 \Rightarrow \theta = M_N^{-1} M_D$$

$$= \begin{pmatrix} -M_D^T (M_N^{-1})^T M_D - M_D^T M_N^{-1} M_D & 0 \\ 0 & M_D M_D^+ (M_N^{-1})^+ + (M_N^{-1})^* M_D^* M_D^T + M_N \end{pmatrix}$$

$M_N = M_N^T$

$$= \begin{pmatrix} -M_D^T M_N^{-1} M_D & 0 \\ 0 & M_N \end{pmatrix} \equiv D_{\nu N}$$

$$M_\nu \equiv -M_D^T M_N^{-1} M_D \, , \quad M_\nu^T = M_\nu$$

- Diagonalize M_ν & M_N:

$$M_\nu = V_{L\nu}^* m_\nu V_{L\nu}^\dagger \quad \rightarrow \text{convention}$$

$$M_N = V_{LN}^* m_N V_{LN}^\dagger$$

reminder
$M_\nu = M_\nu^T$
$M_N = M_N^T$

$$\rightarrow \frac{g}{\sqrt{2}} \overline{\nu_L} \gamma^\mu \nu_L e_L W_\mu^+$$

$$M_\nu = V_L^* m_\nu V_L^\dagger = -M_D^T M_N^{-1} M_D$$

$$\Rightarrow m_\nu = -(V_L^*)^{-1} M_D^T M_N^{-1} M_D V_L =$$

$$= -(M_D V_L)^T M_N^{-1} M_D V_L =$$

$$= -\mathcal{M}^T M_N^{-1} \mathcal{M}$$

$$\Rightarrow M_N^{-1} = -(\mathcal{M}^T)^{-1} m_\nu \mathcal{M}^{-1}$$

$$\Rightarrow M_N = -\mathcal{M} m_\nu^{-1} \mathcal{M}^T$$

Introduce $\nu = \nu_L + C\bar{\nu}_L^T$

$N = \nu_R + C\bar{\nu}_R^T$

Mass Term $\bar{\nu} M_\nu \nu$

$$= (\nu_L^T C M_\nu \nu_L + h.c.)$$

$\nu_L \to V_{L\nu} \nu_L$

$$\Rightarrow = (\nu_L^T C m_\nu \nu_L + h.c.)$$

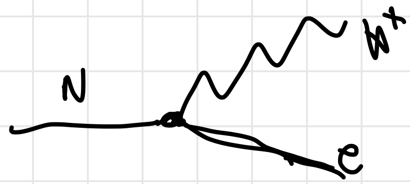
(m_ν → diagonal)

$$\mathcal{L}_{kin} = i\bar{\nu}\gamma^\mu \partial_\mu \nu = 2\bar{\nu}_L \gamma^\mu \partial_\mu \nu_L$$

$$\Rightarrow \mathcal{L}_M = \frac{1}{2}\left[i\bar{\nu}\gamma^\mu\partial_\mu \nu - m_\nu \bar{\nu}\nu\right]$$

($\frac{1}{2}$ → overall factor) same for N

Problem 2



- $N \to W^+ + e$

$$i\mathcal{M} = \frac{ig}{\sqrt{2}}\, \vartheta\, \bar{N}(q)\, \gamma^\mu \left(\frac{1+\gamma_5}{2}\right) e(p)\, \varepsilon^*_\mu(k)$$

$$\mathcal{M}^* = \frac{g\vartheta^*}{\sqrt{2}}\, \varepsilon_\nu(k)\, e^\dagger(p) \left(\frac{1+\gamma_5}{2}\right)^\dagger \gamma^{\nu\dagger}\, \bar{N}^\dagger(q)$$

$$= g\frac{\vartheta^*}{\sqrt{2}}\, \varepsilon_\nu(k)\, e^\dagger(p) \left(\frac{1+\gamma_5}{2}\right)^\dagger \gamma^{\nu\dagger}\, \gamma_0 N(q)$$

$$= g\frac{\vartheta^*}{\sqrt{2}}\, \varepsilon_\nu(k)\, \bar{e}(p) \left(\frac{1-\gamma_5}{2}\right) \gamma^\nu\, N(q)$$

$$|\mathcal{M}|^2 = g^2\frac{|\vartheta|^2}{2}\, \varepsilon_\nu\, \bar{e}\, \gamma^\nu L\, N \bar{N}\, \gamma^\mu L\, e\, \varepsilon^*_\mu$$

$$|\mathcal{M}|^2 = \frac{g^2|\vartheta|^2}{2}\, \bar{e}\, \gamma^\nu L\, N \bar{N}\, \gamma^\mu L\, e \quad \varepsilon_\nu \varepsilon^*_\mu$$

($\bar{e}$ ↑ electron of momentum p; $N\bar{N}$ → anti N of momentum q)

$$\sum_s \varepsilon_\nu \varepsilon^*_\mu = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}$$

$$\frac{1}{2}\sum_{spins} |\mathcal{M}|^2 = \frac{g^2|\vartheta|^2}{4}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right) \mathrm{Tr}\left(\not{p}\,\gamma^\nu L(\not{q} + m_N)\gamma^\mu L\right)$$

(m_N: vanishes $LR=0$)

$$= \frac{g^2 |\theta|^2}{4} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_N^2} \right) \mathrm{Tr} \left(\not{p} \gamma^\nu \not{q} \gamma^\mu \underset{\underset{\frac{1+\gamma_5}{2}}{\shortparallel}}{L} \right)$$

$$\begin{cases} \mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4 \left(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} \right) \\ \mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 4 i \varepsilon^{\mu\nu\rho\sigma} \end{cases}$$

$$= \frac{g^2 |\theta|^2}{2} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_N^2} \right) \left(p_\alpha \, q_\beta \right) \left(\eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\beta}\eta^{\mu\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - + i\varepsilon^{\alpha\mu\beta\nu} \right)$$

$$= \frac{g^2 |\theta|^2}{2} \left(p \cdot q + 2 \frac{(p \cdot k)(q \cdot k)}{m_N^2} \right)$$

Kinematics: $q = (m_N, \underline{0})$.

$$p = (|\underline{p}|, \underline{p}), \quad k = (\underset{\overset{\shortparallel}{\sqrt{p^2 + m_N^2}}}{E_N}, -\underline{p})$$

$$p \cdot q = m_N \underset{\overset{\shortparallel}{E_e}}{|\underline{p}|}$$

$$p \cdot k = |\underline{p}| \sqrt{p^2 + m_N^2} + |\underline{p}|^2$$

$$q \cdot k = E_N m_N$$

$$= \frac{g^2 |\theta|^2}{2} \left(m_N E_e + 2 \frac{\left(E_e E_N + E_e^2 \right) \left(E_N m_N \right)}{m_N^2} \right)$$

$$= \frac{g^2|\theta|^2}{2} \, E_e m_N \left(1 + \frac{2(E_\chi + E_e)(E_\chi)}{m_\chi^2}\right)$$

$$E_e + E_\chi = m_N$$

$$|p_e| + \sqrt{|p_e|^2 + m_\chi^2} = m_N$$

$$\Rightarrow (m_N - |p_e|)^2 = |p_e|^2 + m_\chi^2$$

$$\Rightarrow m_N^2 + \cancel{|p_e|^2} - 2|p_e| m_N = \cancel{|p_e|^2} + m_\chi^2$$

$$\Rightarrow |p_e| = E_e = \frac{m_N^2 - m_\chi^2}{2m_N}$$

$$E_\chi^2 = |p_e|^2 + m_e^2 = \left(\frac{m_N^2 - m_\chi^2}{2m_N}\right)^2 + m_\chi^2$$

$$= \frac{1}{4m_N^2}\left(m_N^4 + m_\chi^4 - 2m_N^2 m_\chi^2 + 4m_N^2 m_\chi^2\right)$$

$$= \frac{1}{4m_N^2}\left(m_N^2 + m_\chi^2\right)^2 \Rightarrow E_\chi = \frac{1}{2m_N}\left(m_N^2 + m_\chi^2\right)$$

$$\frac{1}{2}\sum_s |M|^2 = \frac{g^2|\theta|^2}{2}\left(\frac{m_N^2 - m_\chi^2}{2\cancel{m_N}}\right)\cancel{m_N}\left(1 + \frac{\cancel{2}}{m_\chi^2}\cancel{m_N} \cdot \frac{1}{\cancel{2}m_N}\left(m_N^2 + m_\chi^2\right)\right)$$

$$= \frac{g^2|\theta|^2}{4} \frac{m_N^2}{m_\chi^2}\left(1 - \frac{m_\chi^2}{m_N^2}\right)\left(2m_\chi^2 + m_N^2\right)$$

$$= \frac{g^2|\theta|^2}{4} \frac{m_N^4}{m_\chi^2}\left(1 - \frac{m_\chi^2}{m_N^2}\right)\left(1 + 2\frac{m_\chi^2}{m_N^2}\right)$$

$$\frac{d\Gamma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{m_N} |M|^2$$

$$= \frac{1}{64\pi^2} \frac{g^2|\theta|^2}{4} \frac{m_N^3}{m_W^2} \left(1 - \frac{m_W^2}{m_N^2}\right)\left(1+2\frac{m_W^2}{m_N^2}\right)$$

$$|\theta|^2 = \frac{m_\nu}{m_N}$$

ii) $g \to 0$ or $v \to 0$ $\quad m_W = \frac{1}{2} g v$

a) $g \to 0$ ⇝ W decouples ⇝ no gauge symm. $\Gamma = 0$

b) $v \to 0$ ⇝ kinematically forbidden $\quad m_e$ should be taken into account

c) $g \to 0$, $v \to \infty$, m_W finite $\quad \frac{d\Gamma}{d\Omega} \to 0$ ⇝ process doesn't happen

• Verify N can also decay into $W_\mu^- + e^+$
$\quad (e^c)_L$

$$\Gamma(N \to e^- + W^+) = \Gamma(N \to e^+ + W^-)$$

⊛ What about $\Gamma(N \to Z + \nu)$? and $n>3$ body decays?

- $\Gamma^{-1} \sim \tau \sim \frac{10^2}{g^2 m_\nu} \frac{m_\alpha^2}{m_N^2}$ + higher order

$m_\nu \sim eV \sim 10^{-28}$

$g^2 \sim 10^{-1}$

$m_\alpha \sim 10^2 GeV \sim 10^{-17}$

$1 = 10^{19} GeV$

$1 = 10^{-44} s$

$$\tau \sim 10^{32} \left(\frac{m_\alpha^2}{m_N^2}\right) \sim 10^{36} \left(\frac{GeV^2}{m_N^2}\right)$$

$$\sim 10^{-8} s \; GeV^2/m_N^2$$