Neutrino Physics Course

Lecture XXII

11 / 7 / 2023

LMU

January 2023

Neutrino mass: effective $d = 5$ interaction

Fermi $d = 6$

$$
\begin{tikzcd}
J_\mu^{em} \arrow[r, leftrightarrow] & J_\mu^{W} \\
\text{em int.} \arrow[u] & \text{weak int.} \arrow[u]
\end{tikzcd}
$$

$(A_\mu J^\mu)$

$d(J_\mu) = 3$

$$\frac{1}{\Lambda_F^2} J_\mu^{W} \bar{J}^\mu_W \equiv \frac{G_F}{\sqrt{2}} \bar{J}_\mu^{W} J^\mu_W$$

$$\Lambda_F \simeq 300 \text{ GeV}$$

$(\gg E_W \simeq \mathrm{MeV})$

but first, the essence of seesow

- <u>Type I seesow</u>

$\nu_L \longrightarrow \nu_R \quad \left(N = C\overline{\nu_R}^{T}\right)$

singlet

$\Downarrow$

$\rightarrow Y_D\, \ell\, \phi\, N + m_N\, N\, N$

$\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \Downarrow \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

$$m_\nu^I = -y_D^2 \frac{v_{SM}^2}{m_N} = - \frac{m_D^2}{m_N}$$

$$\equiv \frac{v_{SM}^2}{\Lambda_I}$$

$$\Lambda_I \equiv - \frac{m_N}{y_D^2}$$

$$\boxed{m_\nu^I = \frac{v_{SM}^2}{\Lambda_I}}$$

$(\Lambda_I \gg v_{SM})$

natural

$$\boxed{m_f = y_f \, v_{SM}}$$

(charged fermions)

- Type II

$$\cancel{N} \longrightarrow \Delta \to U \Delta U^{+}$$

$$Y(\Delta) = 2$$

$$\Downarrow$$

$$\underbrace{\bar{l}\, \Delta\, y_\Delta\, l + \mu\, \phi\, \Delta^{*} \phi}$$

insert anti-sym. of Lorentz, $SU(2)_L$

$$+ \frac{1}{2} m_\Delta^2 |\Delta|^2$$

$$\Downarrow$$

$$\langle \Delta \rangle = \mu \frac{v_{SM}^2}{m_\Delta^2}$$

$$m_\nu = y_\Delta \langle \Delta \rangle = y_\Delta \mu \frac{v_{SM}^2}{m_\Delta^2}$$

$$\boxed{m_\nu^{II} = \frac{v_{SM}^2}{\Lambda_{II}} \qquad \Lambda_{II} = \frac{m_\Delta^2}{\mu\, y_\Delta}}$$

- Type III

$\ell\, \phi = ?$ Quantum numbers of spin

$$\underbrace{\tfrac{1}{2} \times \tfrac{1}{2}}_{\text{spin}} = \underbrace{1 \oplus 0}_{\text{spin}}$$

$$\underbrace{2 \times 2}_{\text{doublets}} = \underbrace{3}_{\text{triplet}} \otimes \underbrace{1}_{\text{singlet}}$$

$$\Downarrow$$

$$\rightarrow Y_T \, \underline{l} \, \phi \, \underline{T_F} \leftarrow \text{fermion triplet}$$

$$+ \, m_T \, T_F \, T_F$$

$\Downarrow$

$N \longleftrightarrow T_0$ analogy

$$m_\nu = y_T^2 \frac{v_{SM}^2}{m_T}$$

$$\boxed{m_\nu^{III} = \frac{v_{SM}^2}{\Lambda_{III}} \qquad \Lambda_{III} \equiv \frac{m_T}{y_T}}$$

the same fund. expressions

$$\boxed{m_\nu \propto \frac{1}{\Lambda_{new}}}$$

$\uparrow$ new physics

$$\boxed{\begin{array}{ccc} \Lambda_{new} \to \infty & \Rightarrow & m_\nu \to 0 \\ \text{(SM)} & & \text{(SM)} \end{array}}$$

$\Downarrow$

dimensional grounds

$$\boxed{m_\nu \propto \frac{v_{SM}^2}{\Lambda_{new}}} \quad \text{(at least)}$$

because, we could have:

$$m_\nu \propto \frac{v_{SM}^3}{\Lambda_{new}^2}$$

$$m_\nu \propto \frac{v_{SM}^4}{\Lambda_{new}^3}$$

⋮

digression

Goldbach's conjecture:

even number =

= sum of 2 prime numbers

$12 = 7 + 5$

$11 + 1$

$14 = 13 + 1$

$7 + 7$

Uncle Petros and Goldbach conjecture

Weinberg 1979

Theorem

$$m_\nu = \frac{v_{SM}^2}{\Lambda_\nu}$$

Proof:

(SM)

$$\frac{1}{\Lambda_\nu} \boxed{\nu_L^T \, C \, \nu_L} \; \varphi_0 \, \varphi_0 \qquad (d=5)$$

$$T_3 \quad \frac{1}{2} + \frac{1}{2} + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right) = 0$$

$SU(2)_L$

SM symmetry

$$\Updownarrow$$

$$\boxed{\begin{array}{l} \overline{e_L} \quad e_R \quad \varphi_0 \longrightarrow \overline{e_L}\, e_R\, v_{SM} \\[1em] T_3 : \frac{1}{2} + 0 + \left(-\frac{1}{2}\right) = 0 \end{array}}$$

$$\Downarrow \qquad \varphi_0 \longrightarrow \langle \varphi_0 \rangle \equiv v_{SM}$$

$$\Delta \mathcal{L}_\nu = \frac{1}{\Lambda_\nu} \nu \nu \, \varphi_0^{\,2}$$

$$\Downarrow$$

$$\boxed{m_\nu = \frac{v_{SM}^{\,2}}{\Lambda_\nu} \equiv \frac{v_{SM}^{\,2}}{\Lambda_{new}}}$$

$\Big\downarrow$ SM invariant manner

$$\nu \longrightarrow l$$

$$\varphi_0 \longrightarrow \phi = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix}$$

$$\Downarrow$$

$$\Delta \mathcal{L}_\nu = f(l, \phi)$$

Decoupling theorem

new physics $\longleftrightarrow$ Λ_{new}

$$\boxed{\Lambda_{new} \gg M_W}$$

$$A_{physical} = A_{SM} + \frac{1}{\Lambda^n_{new}}$$

$$\therefore$$

$$\boxed{A_{physical} = A_{SM} \qquad \Lambda_{new} \to \infty}$$

decoupling

$d=5$ Weinberg effective operator (s?)

$$\nu\,\nu \quad \phi_0\,\phi_0$$

$$\downarrow$$

$$\underbrace{\underbrace{l\,l \quad \phi\,\phi}_{\text{Lorentz},\ SU(2)_L \times U(1)_Y \text{ inv.}}}$$

$$\left.\begin{array}{l} l\,l \longrightarrow l^T C\, i\sigma_2\, l \\ \phi\,\phi \longrightarrow \phi^T i\sigma_2\, \phi \end{array}\right\} \text{not } U(1) \text{ inv.}$$

$\downarrow$ but

$$\to \underbrace{\ell^T C\, i\sigma_2\, \ell \quad \phi^T i\sigma_2\, \phi}_{\text{all symm. inv.!}} \leftarrow$$

$(\varepsilon \equiv i\sigma_2)$

. $\phi^T i\sigma_2\, \phi = SU(2)$ inv.

but, it is zero!

$\|$

$$\phi_i\, \varepsilon_{ij}\, \phi_j = \phi_j\, \varepsilon_{ij}\, \phi_i$$

$$= \phi_j\, (-\varepsilon_{ji})\, \phi_i =$$

$$= \phi_i\, (-\varepsilon_{ij})\, \phi_j$$

$$\Downarrow$$

$$\phi^T i\sigma_2 \phi = - \phi^T i\sigma_2 \phi$$

$$\Downarrow$$

$$\phi^T i\sigma_2 \phi = 0 \; !$$

$$\boxed{\ell^T c\, i\sigma_2\, \ell = 0 \quad \text{Prove!}}$$

$$\longrightarrow (\ell^T c\, \ell)$$

<u>Goal</u>: $\frac{1}{\Lambda}\, \ell\, \ell\, \phi\, \phi =$

$$= \frac{1}{\Lambda}\, \ell\phi\, \phi\ell$$

$$\Downarrow \qquad f = (f^\alpha) \leftarrow \text{spinor}$$

$= \text{fermion}$

$$\mathcal{L}_5^{(I)} = \frac{1}{\Lambda} \underbrace{(\ell^T i\sigma_2 \phi)}^{\text{fermion} = \nu} C \underbrace{(\phi^T i\sigma_2 \ell)}$$

$SU(2)_L \times U(1)$ inv.

$$C \equiv i\gamma_2\gamma_0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

unitary gauge $\quad \phi_{un} = \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}$

$$\ell^T i\sigma_2 \phi = (\nu^T e^T) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}$$

$$= \nu^T \phi_0$$

$\Downarrow$

$$\mathcal{L}_5 = \frac{1}{\Lambda} \nu^T \phi_0 \, C \, \phi_0 \nu$$

$$= \frac{1}{\Lambda} \underbrace{(\nu^T C \nu)}_{\text{Lorentz inv.}} \phi_0^2$$

W

$$\underbrace{\underbrace{\ell^T i\sigma_2 \phi}_{SU(2)\ \text{singlet}} \; C \; \underbrace{(\phi^T i \sigma_2 \ell)}_{SU(2)\ \text{singlet}}}_{SU2\ \text{singlet}}$$

$$\ell \; \phi = 2 \times 2 = 3 + 1$$

$$\ell\,\phi \;\longrightarrow\; \ell^T i\sigma_2\, \phi \quad (1)$$

$$\longrightarrow\; \ell^T i\sigma_2\, \vec{\sigma}\, \phi$$

$$f^T C f = \underbrace{f^\alpha C_{\alpha\beta} f^\beta}$$

α, β = spinor indices

Reminder

(a) $D_1, D_2 = 2$ (SU(2) doublets)

$D_1^\dagger D_2$ = singlet ←

$$D_1^\dagger \vec{\sigma} D_2 = \vec{V} \text{ (triplet)}$$

(ii) $$D_1^T i\sigma_2 D_2 = S \text{ (SU(2) singlet)} +$$

$$D_1^T i\sigma_2 \vec{\sigma} D_2 = \vec{V}' \text{ (triplet)}$$

$\boxed{\text{PROVE!}}$

$$\mathcal{L}_5^{(III)} = \frac{1}{\Lambda} \underbrace{(l^T i \sigma_2 \vec{\sigma} \phi)}_{\text{Fermion, SU(2) triplet}} C (\phi^T i \sigma_2 \vec{\sigma} l)$$

$$= \vec{T}_F$$

$$\phi \longrightarrow \phi_{un} = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$$

$$\mathcal{L}_5^{(III)} = \frac{1}{\Lambda} \Big[(l^T i \sigma_2 \sigma_1 \phi) C (\phi^T i \sigma_2 \sigma_1 l)$$

$$+ (l^T i \sigma_2 \sigma_2 \phi) C (\qquad \sigma_2 \quad)$$

$$+ (l^T i \sigma_2 \sigma_3 \phi) C (\qquad \sigma_3 \quad) \Big]$$

$$= \frac{1}{\Lambda} \Big[(l^T \sigma_3 \phi) C (\phi^T \sigma_3 l)$$

$$- (\ell^T \phi) C (\phi^T \ell)$$

$$+ (\ell^T \sigma_1 \phi) C (\phi^T \sigma_1 \ell)]$$

$$\boxed{\begin{aligned} &\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}, \quad \phi = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \\ &\ell^T \sigma_3 \phi = e \phi_0 \end{aligned}}$$

$$\Rightarrow \mathcal{L}_5^{III} = \frac{1}{\Lambda} \Big[e^T C e \, \phi_0^2 - e^T C e \phi_0^2$$

$$+ \nu^T C \nu \, \phi_0^2 \Big]$$

$$\ell^T \sigma_1 \phi = (\nu^T e^T) \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix} = \nu^T \phi_0$$

$$\Downarrow$$

$$\mathcal{L}_5^{(III)} = \frac{1}{\Lambda} \, \nu^T C \nu \, \phi_0^2 = \mathcal{L}_5^{(I)}$$

equal

Claim $\Rightarrow$ $\mathcal{L}_5^{(II)}$

just one more $d=5$ operator

* Prove! (Find it)