Neutrino Physics Course

Lecture XXI

7/7/2023

LMU

Summer 2023

Type II seesaw: a variation on a theme

Aim: a theory of neutrino mass

- in SM $m_\nu = 0$
- in SM we know the origin of charged fermion masses: Higgs mechanism

- question of m_ν is a window into BSM

$$\boxed{\text{theory of mass} = \text{= mass related to new physical processes}}$$

$\Downarrow$

<u>Example: LRSM</u>

self-contained, predictive theory of both origin and nature of m_ν

$$\nu_L \xleftrightarrow{LR} \nu_R$$

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \xleftrightarrow{LR} \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$\Downarrow$$

$$m_\nu \neq 0$$

$$N_L \equiv C \bar{\nu}_R^{\,T} \quad \therefore$$

$$m_N \propto M_R \quad (\text{heavy})$$

with

$$M_\nu = -M_D^T \frac{1}{M_N} M_D$$

Seesaw mechanism : Type I

LRSM :

$$M_D = M_D^T \quad (LR = C)$$

$$M_D = M_D^\dagger \quad (LR = P)$$

$$\Downarrow$$

(*) M_D = determined as a f-n of M_ν, M_N

$$M_N = V_R^T m_N V_R$$

$$M_\nu = \underbrace{V_L^* \, m_\nu \, V_L^\dagger}_{\nu \text{ oscillations},\ 0\nu 2\beta,\ \ldots}$$

$$M_N \longleftarrow \text{LHC}$$

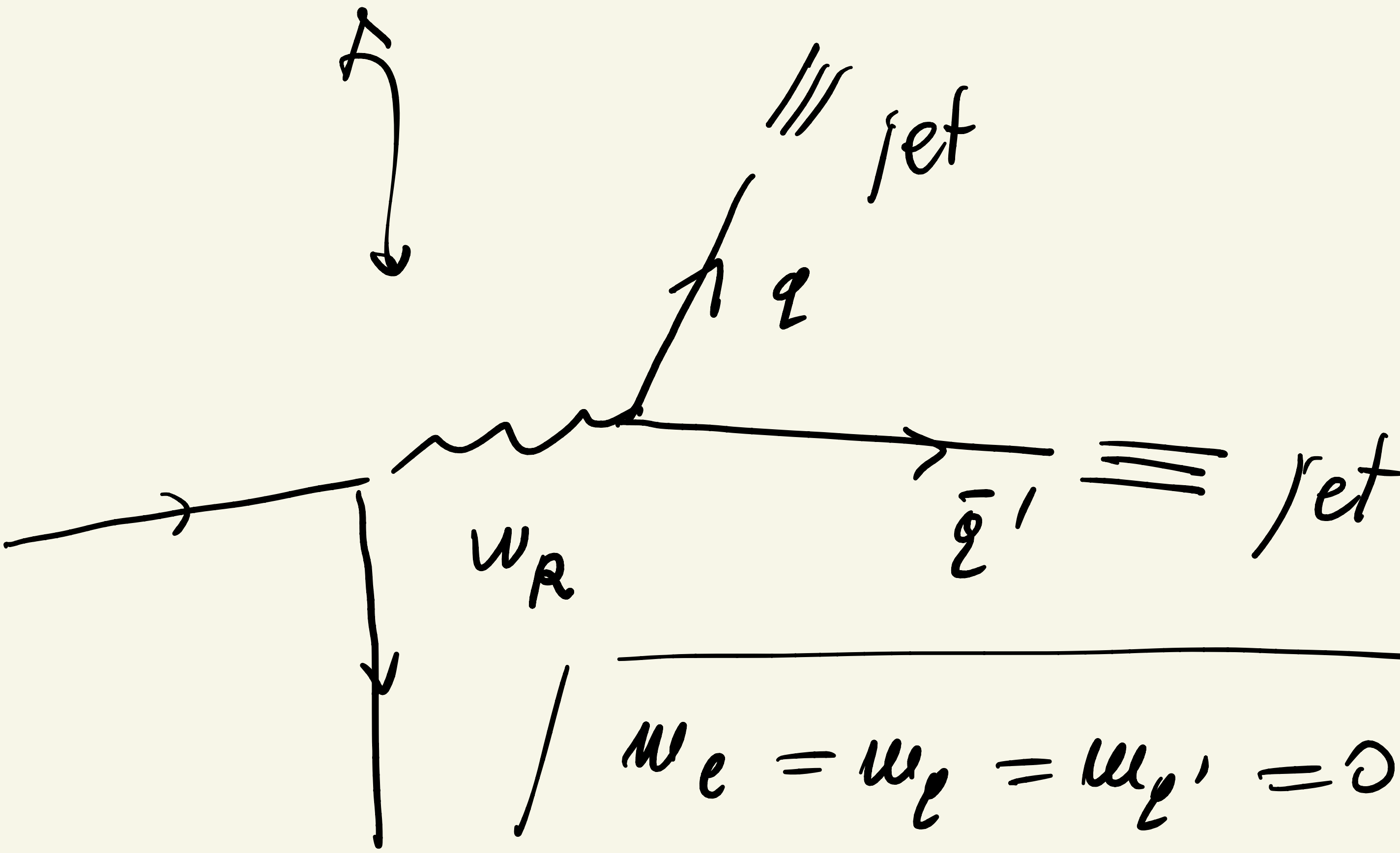
$$\underbrace{\frac{g}{\sqrt{2}} \, \bar{N} \gamma^\mu e \, W_{R\mu}^+}_{\text{produced through } W_R}$$

and N decays:

discussed

- $N \to e_L \, W_L^+$ $(\theta_{\nu N})$

- $N \to e_R \, jj$ (W_R)



$$\boxed{m_e = m_q = m_{q'} = 0}$$

- $N \to e_R \, W_L^+$ ($W_L - W_R$ mixing)

$$\frac{1}{2}|D_\mu \Phi|^2$$

$$\left.\begin{aligned} N &\to e + W^+ \\ N^c &\to \bar{e} + W^- \end{aligned}\right\} \quad N = N^c \text{ Majorana}$$

$$\boxed{\begin{aligned} N &= N_L + C\bar{N}_L^T \\ &= N_L + (N^c)_R \end{aligned}}$$

Majorana : $\boxed{N = N^c}$

$$\Downarrow$$

$$N \to e + W^+$$
$$\to \bar{e} + W^-$$

equal rates = Majoranity

Keung, G.S. 1983

(*) $LR = C$

$$M_D = i\, M_N \sqrt{1/M_N \, M_\nu}$$

$$\theta_{\nu N} = i \sqrt{1/M_N \, M_\nu}$$

$$\Downarrow$$

$N \to e_L + W^+$ predicted

- - - - - - - - - - - - - - -

ignore LRSM,
but keep N

$\underbrace{}$

seesaw (type I)

or: I don't care about
the theory of ν mass

$\downarrow$ instead

$$\boxed{\text{SM} + \text{stuff} = \\ = \text{neutrino mass}}$$

$$\boxed{\text{stuff} = ?}$$

(1) stuff $= N$

- one $N \Rightarrow$ one massive ν
- two $N \Rightarrow$ two massive ν

minimum

Glashow,
Yanagida, ...
('80s)

how to produce N?

$\downarrow$ through $\theta_{\nu N}$

- seesaw: $m_D \ll m_N$

$$\Rightarrow \quad \theta_{\nu N} = m_D / m_N \ll 1$$

$$\Downarrow$$

no way of producing N

- $m_D = i\sqrt{m_N}\ O\ \sqrt{m_\nu}$

not predicted (O = arbitrary)

$\Downarrow$

we should do better
= verifiable scenario
(model)

$\Downarrow$

- stuff = scalar

couples directly to ν

$SM: \; l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \quad e_R$

$\Downarrow$

$$Y_S \underbrace{\nu_L^T C \nu_L} S$$

new scalar

quantum numbers of S ???

$\Rightarrow$ S = $SU(2)_L$ triplet

$$\nu_L \quad C \quad \nu_L \qquad S \qquad Y_S$$

$$T_3: \underbrace{\frac{1}{2} + \frac{1}{2}}_{1} \qquad -1$$

$$Y(S) = +2, \text{ since}$$

$$Y(\nu_L) = Y(e_L) = -1$$

$$\Uparrow$$

$$Q = T_3 + Y/2$$

$$\Downarrow$$

$$\boxed{S \subseteq \Delta \; (Y = 2 \text{ triplet})}$$

$$\|$$

$$\{\Delta_L \text{ of LRSM}\}$$

$\downarrow$ $\boxed{\text{Type II seesaw}}$

to be explained

$$\Delta \mathcal{L}_Y(\Delta) = l_L^T C' \Delta Y_\Delta l_L$$

$$\downarrow$$

$$l_L^T C Y_\Delta U^T \underbrace{U \Delta U^+}_{\text{adjoint}} U l_L$$

$\neq$ inv.

as expected!

correct $\Downarrow$ anti-symmetrise

$$\boxed{\Delta \mathcal{L}_Y(\Delta) = l_L^T y_\Delta C\, i\sigma_2\, \Delta\, l_L}$$

$$\to l_L^T y_\Delta C\, U^T i\sigma_2 U\, \Delta\, \underbrace{U^\dagger U}_{1}\, l_L$$

$$= l_L^T y_\Delta C\, i\sigma_2\, U^\dagger U\, \Delta\, U^\dagger U\, l_L$$

$$= l_L^T y_\Delta C\, i\sigma_2\, \Delta\, l_L = \text{inv.} \checkmark$$

Q. E. D.

$$\Delta = \begin{pmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{pmatrix}$$

Exp. (LHC) : $\boxed{m_{\delta^{++}} \gtrsim 400 \text{ GeV}}$

but:

split of masses in $\Delta \simeq O(M_W)$

$\Downarrow$

$m_{\delta^+}, m_{\delta^0} \gtrsim 400 \text{ GeV}$

$\Downarrow$

$\boxed{\Delta \neq \text{Higgs field}}$

$$\Downarrow$$

$$V_\Delta = \frac{1}{2} m_\Delta^2 \, \mathrm{Tr}\, \Delta^+ \Delta \; + $$

$$+ \frac{1}{4} \lambda_\Delta \left(\mathrm{Tr}\, \Delta^+ \Delta \right)^2 + \cdots$$

$$\Uparrow$$

$$\left(\lambda_\Delta > 0 \right.$$

$$\boxed{m_\Delta^2 > 0}$$

$$\Downarrow$$

$$\left. V = \text{bounded} \right)$$

$$\Downarrow$$

$$\langle \Delta \rangle = 0 \; !$$

$$\frac{\partial V}{\partial \langle \Delta \rangle} = m_\Delta^2 \langle \Delta \rangle + \lambda_\Delta \langle \Delta \rangle^3 = 0$$

$$\parallel$$

$$= \langle \Delta \rangle \left(m_\Delta^2 + \lambda_\Delta \langle \Delta \rangle^2 \right)$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{>0}$$

$$\Rightarrow \langle \Delta \rangle = 0$$

$$\Rightarrow m_\nu = 0 \;!!\;??$$

but

$$V = V_\phi + V_\Delta + V_{\phi\Delta}$$

$$\boxed{V_{\phi\Delta} = \mu\, \phi^T i\sigma_2\, \Delta^+ \phi}$$

$$Y(\phi) = +1$$

$$\begin{pmatrix} \Delta \longrightarrow U \Delta U^{+} \\ \Delta^{+} \longrightarrow U \Delta U^{+} \end{pmatrix}$$

$\Downarrow$ next step

$$\langle \Delta \rangle = ?$$

$$\Delta = \begin{pmatrix} \delta^{+} & \delta^{++} \\ \delta^{0} & -\delta^{+} \end{pmatrix}$$

$$\Delta^{+} = \begin{pmatrix} \delta^{-} & \delta^{0*} \\ \delta^{--} & -\delta^{-} \end{pmatrix}$$

$$\Downarrow$$

$$\langle \Delta^+ \rangle = \begin{pmatrix} 0 & v_\Delta \\ 0 & 0 \end{pmatrix}$$

$$\langle V_{\phi\Delta} \rangle = \mu \langle \phi \rangle^T i\sigma_2 \langle \Delta^+ \rangle \langle \phi \rangle$$

$$= \mu \begin{pmatrix} 0 & v_{SM} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & v_\Delta \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v_{SM} \end{pmatrix}$$

$$= \mu \begin{pmatrix} 0 & v_{SM} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -v_\Delta & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v_{SM} \end{pmatrix}$$

$$= -\mu v_{SM}^2 v_\Delta$$

$$\Downarrow$$

$$\langle V \rangle = \langle V_{\phi\Delta} \rangle + \langle V_{\phi} \rangle$$

$$+ \langle V_{\Delta} \rangle$$

$$\Rightarrow \langle V \rangle = \cdots - \mu \, v_{SM}^2 \, v_{\Delta}$$

$$+ \frac{1}{2} m_{\Delta}^2 v_{\Delta}^2 + \frac{\lambda}{4} v_{\Delta}^4$$

$$\frac{\partial \langle V \rangle}{\partial v_{\Delta}} = -\mu^2 v_{SM}^2 + m_{\Delta}^2 v_{\Delta} +$$

$$+ \lambda v_{\Delta}^3 = 0$$

$$\Downarrow$$

$$\boxed{v_{\Delta} \neq 0}$$ (we made it)

linear in Δ term

$\phi\,\phi\,\Delta$

tadpole

$\Downarrow$

(i) $v_\Delta \neq 0$

(ii) $v_\Delta \ll v_{SM}$ $\quad (m_\Delta^2 \gg m_h^2)$

$$M_W^2 = \frac{1}{4} g^2 v_{SM}^2 + g^2 v_\Delta^2$$

$$M_Z^2 = \frac{1}{4}(g^2+g'^2)\, v_{SM}^2 + 2(g^2+g'^2)\, v_\Delta^2$$

but

$$M_Z^2 \simeq \frac{g^2+g'^2}{g^2} M_W^2 \quad (\text{exp.})$$

$$\Rightarrow \boxed{v_\Delta \ll v_{SM}}$$

$$\boxed{v_\Delta \lesssim \text{GeV}} \quad \boxed{v_{SM} \simeq 200\ \text{GeV}}$$

$\downarrow$ compute v_Δ

$$-\mu v_{SM}^2 + m_\Delta^2 v_\Delta + \underbrace{\lambda_\Delta v_\Delta^3}_{\text{neglect}} = 0$$

$$\left(\begin{array}{l} \lambda_\Delta \lesssim 1 \Rightarrow \\ \lambda v_\Delta^3 \lesssim v_\Delta^3 \ll m_\Delta^2 v_\Delta \end{array} \right)$$

$$\Downarrow$$

$$\boxed{v_\Delta \simeq \mu \frac{v_{SM}^2}{m_\Delta^2}}$$

- $\mu \simeq m_\Delta$ for illustration

$$v_\Delta = \frac{v_{SM}^2}{m_\Delta}$$

$$\Downarrow$$

$$\boxed{m_\nu = y_\Delta v_\Delta \simeq y_\Delta \frac{v_{SM}^2}{m_\Delta}}$$

$$m_\Delta \to \infty \Rightarrow m_\nu \to 0$$

$\Rightarrow$ Type II seesaw

analog $\Updownarrow$ Type I seesaw

$$\boxed{m_\nu = \frac{m_D^2}{m_N} \simeq Y_D^2 \frac{v_{SM}^2}{m_N}}$$

$$(m_D = Y_D v_{SM})$$

$\downarrow$ more generations

$$\boxed{M_\nu = Y_\Delta v_\Delta}$$

$\Uparrow$

$$\Delta \mathcal{L}_Y (\Delta) =$$

$$(\nu^T e^T)_L\, C\, y_\Delta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \delta^+ & \delta^{++} \\ +\delta^0 & -\delta^+ \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$= (\nu^T e^T)\, C\, y_\Delta \begin{pmatrix} \delta^0 & -\delta^+ \\ -\delta^+ & -\delta^{++} \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$= (\nu^T e^T)\, C\, y_\Delta \begin{pmatrix} \delta^0 \nu - \delta^+ e \\ -\delta^+ \nu - \delta^{++} \end{pmatrix}_L$$

$$= \nu_L^T C\, y_\Delta\, \nu_L \delta^0 - \nu_L^T C\, y_\Delta\, e_L \delta^+ 2$$

$$-\, e_L^T C\, y_\Delta\, \delta^{++} e_L$$

$$\Downarrow$$

$$\mathcal{M}_\nu = Y_\Delta v_\Delta \quad +$$

$$- e_L^T C \frac{\mathcal{M}_\nu}{v_\Delta} e_L \delta^{++}$$

neutrino mass matrix

$$\mathcal{M}_\nu = V_\ell^* m_\nu V_\ell^\dagger = \text{measured}$$

$$\Downarrow$$

$$\delta^{--} \longrightarrow l_i + l_j \quad (\mathcal{M}_\nu)_{ij}$$

charged leptons

overall decay rate depends on v_Δ

but

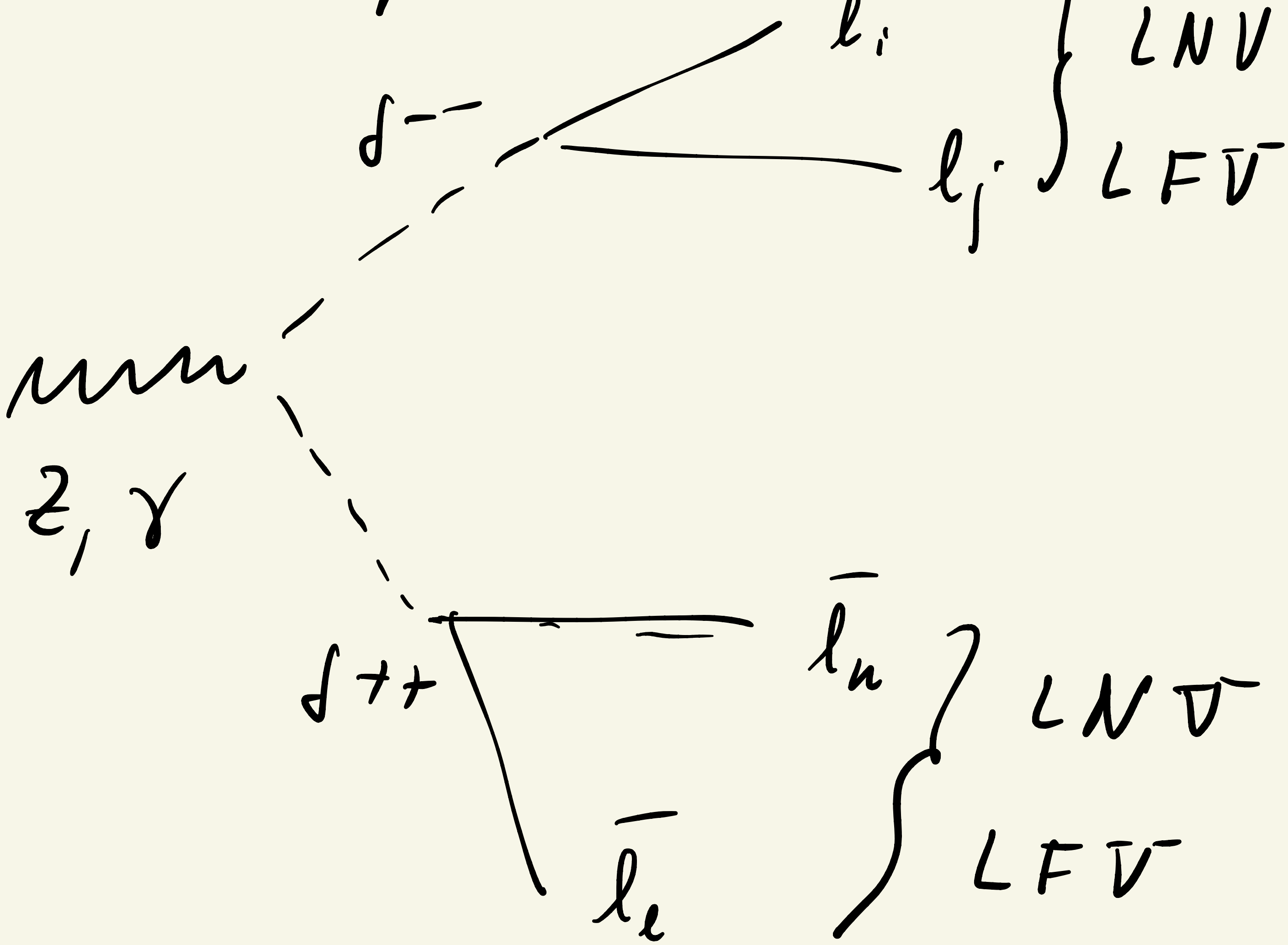
branching ratios = determined

$$\delta^{--} \longrightarrow l_i + l_j$$

Schwetz, ? 2008

Type II seesaw = 1980 - 1981

(Magg, Wetterich
Lazarides et al
Mohapatra, G.S.)



on mixings

quarks: V_{CKM}

$$\theta_{12}^{q}, \quad \theta_{13}^{q}, \quad \theta_{23}^{q}$$
$$\quad \| \qquad\quad \| \qquad\quad \|$$
$$13^\circ \quad 10^{-3} \quad 10^{-2}$$

leptons: V_{PMNS}

$$\theta_{12}^{l}, \quad \theta_{13}^{l}, \quad \theta_{23}^{l}$$
$$\quad \| \qquad\quad \| \qquad\quad \|$$
$$30^\circ \quad 10^\circ \quad 45^\circ$$

$$M_\nu = V_{PMNS}^{*} \, m_\nu \, V_{PMNS}^{\dagger}$$

LFV = large

μ vs m_Δ ?

$$M_\nu = Y_\Delta v_\Delta$$

$$\begin{cases} v_\Delta = \mu \left(\frac{v_{SM}}{m_\Delta} \right)^2 \leq \mu / 4 \\ (Y_\Delta \leq O(1)) \end{cases}$$

$$m_\nu \leq v_\Delta \Rightarrow$$

$$10^{-1} eV \leq v_\Delta \leq GeV$$

$$\boxed{\Delta + \text{Higgs}}$$

<u>Higgs</u> ϕ

$$V = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

$$\Rightarrow \langle\phi\rangle = \mu^2/\lambda$$

$$m_\phi^2 = 2\lambda\langle\phi\rangle^2$$

$$\boxed{m_\phi \lesssim \langle\phi\rangle}$$

• Δ : $\boxed{m_\Delta \gtrsim 400 \text{ GeV}}$

$\boxed{v_\Delta \leq \text{GeV}}$

$\Downarrow$

$\Delta \neq$ Higgs

$\Downarrow$

$$\langle \Delta \rangle = \mu \frac{v_{SM}^2}{m_\Delta^2}$$

— — — — — —

"Higgsing" Δ

$\downarrow$

$$V_\Delta = -\mu_\Delta^2 \Delta^2 + \lambda_\Delta \Delta^4$$

$$\Downarrow$$

$$m_\Delta \simeq v_\Delta \leq O(\text{GeV})$$

$$\Rightarrow \quad m_{\delta^{++}} \leq \text{GeV}$$

exp.

wrong