Neutrino Physics Course

Lecture XX

4/7/2023

LMU

Summer 2023

Probing the origin of ν mass: untangling the seesaw

$$\underline{M}_\nu = - \underline{M}_D^T \frac{1}{\underline{M}_N} \underline{M}_D \quad (1)$$

measure at low E (oscillations, ...)

measure at high E (LHC)

a reminder!

SM

$$\underline{M}_f = Y_f \, v_{SM}$$

|| (matrix) //

$$\bar{f}_L^{\,0} \, \underline{M}_f \, f_R^{\,0}$$

$\underbrace{f_L^0, f_R^0}_{\text{weak basis states}} \neq$ physical fields

weak basis states

↕

$$\frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L^{\,0} \gamma^\mu d_L^{\,0} + \bar{\nu}_L^{\,0} \gamma^\mu e_L^{\,0} \right)$$

$$f^0_{L,R} \longrightarrow \boxed{U^{\dagger}_{L,R}\, f^0_{L,R} = f_{L,R}} \quad (2)$$

$f_{L,R}$ = physical states

$\therefore$

$$\bar{f}^0_L\, M_f\, f^0_R = \bar{f}_L \underbrace{U_{Lf}\, M_f\, V^{\dagger}_R}_{m_f \,=\, \text{diagonal}} f_R$$

$$= \bar{f}_L\, m_f\, f_R$$

$\Downarrow$

$$\frac{g}{\sqrt{2}}\, \bar{u}_L \underbrace{U^{\dagger}_{Lu}\, U_{Ld}}_{V_{CKM} \,=\, V_{quark}} \gamma^\mu d_L\, W^+_\mu$$

$$+ \frac{g}{\sqrt{2}} \, \bar{\nu}_L \underbrace{U_{L\nu}^{+} U_{Le}}_{V_{PMNS}^{+}} \gamma^\mu e_L W_\mu^{+} \quad (3)$$

$$V_{PMNS}^{+} = V_{lepton}^{+}$$

$$\boxed{V_{CKM} = U_{Lu}^{+} U_{Ld}}$$

(i) basis: $U_{Ld} = \mathbb{1}$

(ii) basis: $U_{Lu} = \mathbb{1}$

(i) basis: $U_{Le} = \mathbb{1} \Leftrightarrow$

charged lepton mass matrix = diagonal

above: $u = \begin{pmatrix} u \\ c \\ s \end{pmatrix}$

$$d = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$e = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$m_d = \mathrm{diag}\,(m_d, m_s, m_b)$$

- - - - - -

$\underline{\underline{M}}_u, \; \underline{\underline{M}}_d$ = independent

in SM

$\Updownarrow$

Y_u, Y_d = — " —

$\Downarrow$

$U_{Lu}, \; U_{Ld}$ = — " —

$U_{Ru}, \; U_{Rd}$ = — " —

$\Downarrow$ { no information on U_{Rf}

at the end of the day:

m_f , $V_{CKM} = V_{quark}$

$V_{PMNS} = V_{lepton}$

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seesaw $\Leftarrow$

neutrinos (ν, N) = Majorana fermions

$f^T C M_f^M f \Leftarrow$

$(M_f^M)^T = (M_f^M)$

Majorana mass matrices

= symmetric

$\Downarrow$ (in the diagonal charged fermion basis)

$$(\nu_L^0)^T \underline{M}_\nu \nu_L^0 =$$

$$= \nu_L^T \underbrace{V_\ell^* \underline{M}_\nu V_\ell^+}_{m_\nu} \nu_L$$

$$= \nu_L^T m_\nu \nu_L$$

$$= \nu_{1L}^T m_{\nu_1} \nu_{1L} + \nu_{2L}^T m_{\nu_2} \nu_{2L} + \cdots$$

$\Uparrow$

a) $H = H^\dagger \Rightarrow U H U^\dagger =$ diagonal

b) $M = M^T \Rightarrow V^* M V^\dagger =$ — " —

$\Updownarrow$

$$(V^* M V^\dagger)^T = V^* M^T V^\dagger$$

$$= V^* M V^\dagger$$

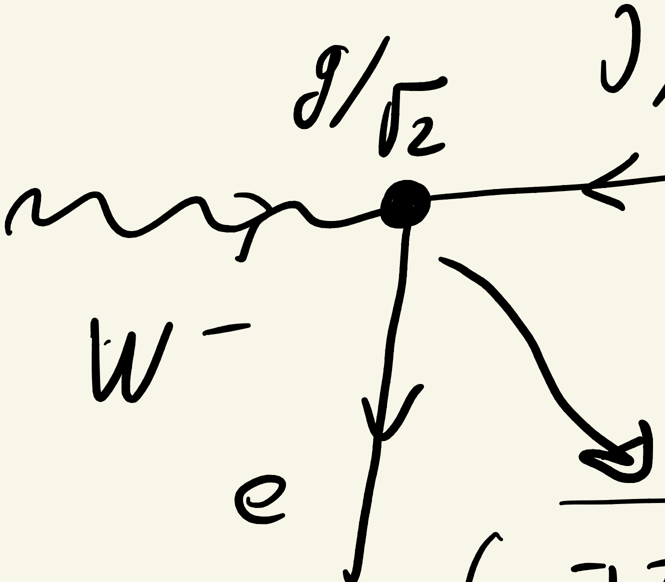
$\Downarrow$ bottom line

$$M_\nu = V_\ell^* m_\nu V_e^\dagger$$

"

↑ diagonal matrix

oscillations



$(i = 1,2,3)$

$$\bar V_{ei} = \bar V_{1i}$$

⇕

measure $\bar M_\nu$ = measure m_ν and $\bar V_{\text{leptonic}} \equiv \bar V_\ell$

↓

3 mixing angles + phases

θ_{12}, θ_{23}, θ_{13}

1-2 gen, 2-3 gen, 1-3 gen

$$\theta_{12} \simeq 30^\circ, \quad \theta_{23} \simeq 45^\circ$$

$$\theta_{13} \simeq 10^\circ$$

↑ oscillations

$$\underline{M}_\nu = \underline{M}_{\nu_L} = V_L^* m_{\nu_L} V_L^\dagger$$

$$\underline{M}_{\nu_R} = V_R^* m_{\nu_R} V_R^\dagger$$

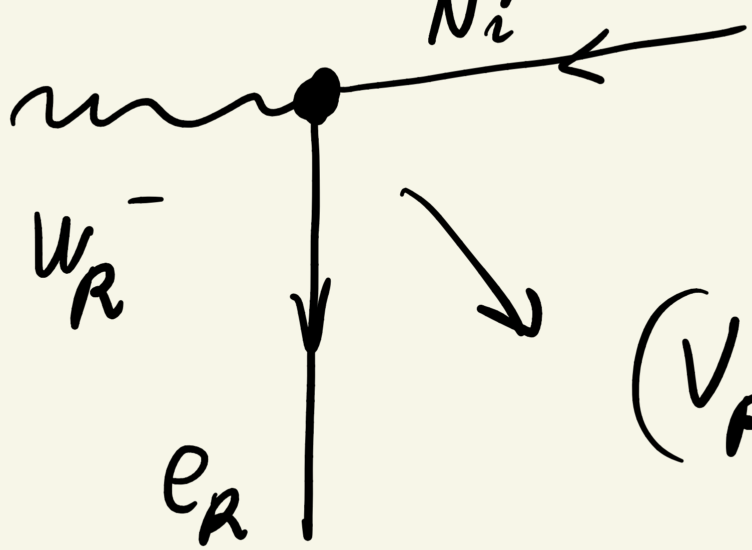
$$\underline{M}_N = \underline{M}_{\nu_R}^*$$

$$\left(N_L = \bar{\nu}_R^T \propto \nu_R^* \right)$$

$$\Downarrow$$

$$\boxed{\underline{M}_N = V_R \, m_N \, V_R^T}$$

measure at LHC



$$(V_R)_{1i}$$

$$e_R \to \mu_R \Rightarrow (V_R)_{2i}$$

$\Downarrow$

input in (1) = M_ν, M_N

$\Downarrow$ untangle seesaw

$$\underline{M}_D = f(\underline{M}_\nu, \underline{M}_N)$$

seesaw:

$$\underline{M}_D = i\sqrt{\underline{M}_N}\; O \sqrt{\underline{M}_\nu}$$

arbitrary $O = O^T$
complex matrix

$\Rightarrow$ seesaw fails as a theory of origin of ν mass

$\Downarrow$

Seesaw in LRSM

$$\mathcal{L}_Y(\Phi, \tilde{\Phi}) \longrightarrow$$

$$\bar{f}_L M_f f_R + h.c.$$

$$= \bar{f}_L M_f f_R + \bar{f}_R M_f^{+} f_L$$

$\Downarrow$

$$\boxed{M_f = M_f^{+}}$$

where we used:

$$f_L \xrightarrow{P} f_R$$

$$\boxed{P = LR \text{ symmetry}}$$

but there is another LR symmetry!

$$C: \; f \longrightarrow C \bar{f}^T$$

$$(f^c)_L \equiv C \bar{f_R}^T$$

$$= i \gamma_2 f_R^*$$

$$C: \; f_L \longrightarrow f_R^{*}$$

$$\Downarrow$$

if $M = M^{+}$ for $LR = P$

$\Rightarrow$ $M = M^{T}$ for $LR = C$

<u>Proof:</u>

$$(M^{*})^{*} = M^{T}$$

Q.E.D.

comment.

$$CP = C \times P$$

$$\shortparallel$$

$$\sim \text{good} \ (\sim 10^{-3}) \iff$$

$$C \simeq P$$

$$\Downarrow$$

C = maximally broken

in SM

just like P

• $LR = C$ assume

$\Downarrow$

$$\boxed{\underline{M}_D^T = \underline{M}_D}$$

back to (1):

$$\underline{M}_\nu = - \underline{M}_D^T \frac{1}{\underline{M}_N} \underline{M}_D$$

$\Downarrow$

$$\underline{M}_\nu = - \underline{M}_D \frac{1}{\underline{M}_N} \underline{M}_D \quad \cancel{\mu_N}$$

[Nemevšek, G.S., Tello 2012]

$\Downarrow$

$$\frac{1}{M_N} M_\nu = -\left(\frac{1}{M_N} M_D\right)\left(\frac{1}{M_N} M_D\right)$$

$$= -\left(\frac{1}{M_N} M_D\right)^2$$

$\Downarrow$

$$\frac{1}{M_N} M_D = i\sqrt{\frac{1}{M_N} M_\nu}$$

$\Downarrow$

$$\boxed{\underline{M}_D = i\, m_N \sqrt{\frac{1}{\underline{M}_N}\, \underline{M}_\nu}}$$

determined

$\Updownarrow$

seesaw = untangled

$\Updownarrow$

O = fixed

$$\underline{M}_D = i \sqrt{m_N}\; O \sqrt{\underline{M}_\nu}$$

$$= i \, M_N \sqrt{{}^1/_{M_N} \, M_\nu}$$

$$\Downarrow$$

$$\sqrt{M_N} \, O \sqrt{M_\nu} = M_N \sqrt{{}^1/_{M_N} \, M_\nu}$$

$$\Downarrow$$

$$O = \frac{1}{\sqrt{M_N}} M_N \sqrt{{}^1/_{M_N} \, M_\nu} \, \frac{1}{\sqrt{M_N}}$$

$$\Downarrow$$

$$\boxed{O = \sqrt{M_N} \sqrt{{}^1/_{M_N} \, M_\nu} \, \frac{1}{\sqrt{M_\nu}}}$$

↑ [fixed]

- ## $\nu - N$ mixing

$$\Theta_{\nu N} = \frac{1}{M_N} M_D$$

$$\Updownarrow$$

$$M_{\nu N} = \begin{matrix} \\ \nu \\ N \end{matrix} \begin{matrix} \nu & N \\ \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} \end{matrix}$$

$$W \bar{\nu} \, l_L \xrightarrow{\text{implies}} \bar{N} \, \Theta_{\nu N}^{+} \, l_L W \quad (*)$$

$N \longrightarrow l \, W^{+}$ via $\Theta_{\nu N}$

Majorana: $\bar{l} \, W^{-}$ — " —

$$\theta_{\nu N} = \frac{1}{M_N} M_D$$

$$= i \sqrt{1/M_N \, M_\nu}$$

LHC:

(i) direct LNV

(ii) Majorana nature of N

(iii) $\theta_{\nu N}$ ← probes ν mass

(iv) establish C as LR

$$\mathcal{M}_\nu,\ \mathcal{M}_N \longrightarrow \mathcal{M}_D$$

$$\theta_{\nu N},\ M_N \longrightarrow \mathcal{M}_D$$

$$\Downarrow \ \text{LRSM}$$

$$m_\nu = - m_D^2 / m_N \quad (1-1)$$

$$\Downarrow$$

$$m_D = i\sqrt{m_\nu m_N}$$

- $m_d = m_e \simeq \text{MeV}$

$$|m_\nu| \simeq \frac{m_e^2}{m_N} = \frac{10^{-6}\ \text{GeV}^2}{m_N}$$

so for $m_N \simeq \text{TeV} \simeq 10^3\ \text{GeV}$

$$\Downarrow$$

$$\boxed{m_\nu \simeq \text{eV}}$$

- $LR = P$

$$\Rightarrow \quad M_D = M_D^\dagger$$

$$\Downarrow$$

$$M_\nu = - M_D^* \frac{1}{M_N} M_D$$

$$\Downarrow$$

no tuned solution

Tello, G.S.

2012 - 2016

$$\Downarrow$$

M_D = determined

LNV = crucial

$\updownarrow$

$0\nu 2\beta$: $e = e_R$

$\Downarrow$

$(W_R, N) \Rightarrow M_{W_R} \lesssim 10\,\text{TeV}$

- $\bar{N}\, \Theta_{\nu N}^{+}\, l\, W^{+}$ $\leftarrow$ important

$\Downarrow$

$$N_i \to \ell_j \, W^+$$

$$\Gamma(N_i \to \ell_j) \propto (\Theta_{\nu\nu})_{ij}^2$$

and many more decays in the Higgs sector

Tello, G.S. 2018

(*)

$$\frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L^0 \gamma^\mu e_L^0 =$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ \bar{\bar{\nu}}_L^0 \gamma^\mu e_L =$$

$$= g/\sqrt{2}\; W_\mu^+ \overline{\nu_L + \Theta_{\nu N} N_L} \gamma^\mu e_L$$

$$= g/\sqrt{2}\; W_\mu^+ \left(\bar{\nu}_L \gamma^\mu e_L + \bar{N}_L \Theta_{\nu N}^+ \gamma^\mu e_L \right)$$

$\Uparrow$

$$\underline{M}_{\nu N} = \begin{pmatrix} 0 & m_D^T \\ m_D & m_N \end{pmatrix}$$

$\Rightarrow$ a) $\Theta_{\nu N} \propto \underline{M}_D$

b) $\Theta_{\nu N} \xrightarrow[m_N \to \infty]{} 0$

$\Downarrow$

$$\boxed{\Theta_{\nu N} = \frac{1}{\underline{M}_N} \underline{M}_D}$$