Neutrino Physics Course

Lecture XVIII

27/6/2023

LMU

Summer 2023

SM Higgs in LRSM

$$G_{LR} \xrightarrow[\substack{\parallel\!\!\!\!\equiv \\ M_R}]{\langle \Delta_R \rangle \equiv v_R} G_{SM} \xrightarrow[\substack{\parallel\!\!\!\!\equiv \\ M_W \; (M_L)}]{\langle \Phi \rangle} U_{em}(1)$$

$$M_R \gg M_L \; (M_W)$$

$$V = \underbrace{V_\Delta + V_\Phi + V_{\Phi\Delta}}$$

$$\Downarrow$$

mass spectrum of Φ

(leading terms)

$$V_{\Phi\Delta} = \alpha_{1/2}\left(\mathrm{Tr}\, \Delta_R^\dagger \Delta_L + R\to L\right)\mathrm{Tr}\,\Phi^\dagger\Phi$$

$$+ \alpha_{2/2}\left(\mathrm{Tr}\, \Delta_R^\dagger \Delta_R + R\to L\right)\left(\mathrm{Tr}\,\tilde{\Phi}^\dagger \Phi + h.c.\right)$$

$$+ \alpha_{3/2}\left(\mathrm{Tr}\ \Delta_R^\dagger \Phi^\dagger \Phi\, \Delta_R + R\to L\right)$$

$$\Phi \to U_L\, \Phi\, U_R^\dagger\,, \quad \tilde{\Phi} \to U_L\, \tilde{\Phi}\, U_R^\dagger$$

$$\Phi^\dagger \to U_R\, \Phi^\dagger U_L^\dagger \quad \left(\tilde{\Phi} \equiv \sigma_2\, \Phi^* \sigma_2\right)$$

What about $\underline{\mathrm{Tr}\ \Delta_R^\dagger \tilde{\Phi}^\dagger \Phi\, \Delta_R + h.c}$

$+ \ R \to L$

$$\tilde{\Phi}^\dagger \Phi = \left(\frac{1}{2}\mathrm{Tr}\,\tilde{\Phi}^\dagger\Phi\right)\mathbb{1}$$

$$\Downarrow$$

$$\mathrm{Tr}\, \Delta_R^\dagger \tilde{\Phi}^\dagger \Phi \Delta_R = \frac{1}{2} \underbrace{\mathrm{Tr}\, \Delta_R^\dagger \Delta_R \,\mathrm{Tr}\, \tilde{\Phi}^\dagger \Phi}_{}$$

α_2 term

$$\Downarrow$$

$$V_{\Phi\Delta} = \text{complete!}$$

$$\Big\downarrow \text{ mass spectrum of } \Phi$$

mass terms + quartics $\Delta_R \to \langle \Delta_R \rangle$

$$\Downarrow \quad \boxed{\mathrm{Tr}\, \tilde{\Phi}^\dagger \tilde{\Phi} = \mathrm{Tr}\, \Phi^\dagger \Phi}$$

$$V_{mass}(\Phi) = -\frac{\mu_0^2}{2} Tr\, \Phi^+ \Phi - \frac{\bar{\mu}_0^2}{2} Tr\, \tilde{\Phi}^+ \Phi$$

$(+h.c.)$

$$+ \frac{\alpha_1}{2} v_R^2\, Tr\, \Phi^+ \Phi + \frac{\alpha_2}{2} v_R^2\, Tr\, \tilde{\Phi}^+ \Phi$$

$$+ \frac{\alpha_3}{2}\, Tr \langle \Delta_R \rangle^+ \Phi^+ \Phi \langle \Delta_R \rangle$$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$$\Phi = (\tilde{\phi}_1 \;\; \phi_2) = \begin{pmatrix} \phi_1^{0*} & \phi_2^+ \\ -\phi_1^- & \phi_2^0 \end{pmatrix}$$

$$\Downarrow$$

$$\mathrm{Tr}\,\langle\Delta_R\rangle^\dagger\,\Phi^\dagger\Phi\,\langle\Delta_R\rangle =$$

$$= \mathrm{Tr}\begin{pmatrix}0 & v_R\\ 0 & 0\end{pmatrix}\left(\Phi^\dagger\Phi\right)\begin{pmatrix}0 & 0\\ v_R & 0\end{pmatrix}$$

$$= \left(\Phi^\dagger\Phi\right)_{22}$$

$$= \begin{pmatrix}\varphi_1^0 & -\varphi_1^+\\ \varphi_2^- & \varphi_2^{0*}\end{pmatrix}\begin{pmatrix}\varphi_1^{0*} & \varphi_2^+\\ -\varphi_1^- & \varphi_2^0\end{pmatrix}$$

$$= \left(\varphi_2^-\varphi_2^+ + |\varphi_2^0|^2\right) = \underbrace{\phi_2^\dagger\,\phi_2}_{SU(2)_L\ \text{inv.}}$$

$$\phi_i = \begin{pmatrix}\varphi_i^+\\ \varphi_i^0\end{pmatrix}$$

$$\mu^2 = \mu_0^2 + \alpha_1 v_R^2$$

$$\bar{\mu}^2 = \bar{\mu}_0^2 + \alpha_2 v_R^2$$

$$\boxed{\mathrm{Tr}\,\Phi^\dagger \Phi = \underbrace{\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2}_{\text{(some masses)}}}$$

$$\mathrm{Tr}\,\tilde{\Phi}^\dagger \Phi = \phi_1^\dagger \phi_2 \; (\phi_2^\dagger \phi_1)$$

$$\Uparrow$$

$SU(2)_L$ point of view:

$$\phi_1^\dagger \phi_1, \quad \phi_2^\dagger \phi_2, \quad \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1$$

$\Downarrow$

$$\boxed{V_{mass}(\Phi) = -\frac{\mu^2}{2}\left(\phi_1^+\phi_1 + \phi_2^+\phi_2\right) - \frac{\bar{\mu}^2}{2}\left(\phi_1^+\phi_2 + \phi_2^+\phi_1\right) + \frac{\alpha_3}{2} v_R^2 \, \phi_2^+ \phi_2}$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$$

(i) $v_2 = 0$, $v_1 \equiv v$

$\Downarrow$

$$\Phi = \begin{pmatrix} v + h + i G^0 & \phi_2^+ \\ - G^- & \phi_2^0 \end{pmatrix}$$

$$\Big\downarrow \text{ mass analysis}$$

$$\left(\mu^2 \simeq \bar{\mu}^2 \simeq M_W^2 \; (M_L^2) \right)$$

$$\Downarrow$$

$$m_h^2 \simeq M_W^2 , \quad (G^-, G = \text{eaten})$$

$$\underbrace{\boxed{m_{\phi_2}^2 \cong \alpha_3 v_R^2}}$$

ϕ_2 = heavy scalar doublet

$\Downarrow$

only SM Higgs = "light"

Q. Was this automatic?

ϕ_1 = light , ϕ_2 = heavy

$(\sim M_L)$ $\qquad$ $(\sim M_R)$

A. $\boxed{\text{NO}}$

$$\mu^2 = \mu_0^2 + \underbrace{\alpha_1 v_R^2}_{\text{large } (\sim v_R)} \simeq M_L^2$$

$M_L << M_R \longrightarrow$ limit : $M_L = 0$

illustration

$\Downarrow$

$\mu^2 = 0 \implies$

$$\boxed{\mu_0^2 + \alpha_1 v_R^2 = 0}$$

$$\Rightarrow \underbrace{\boxed{\mu_0^2 = -\alpha_1 v_R^2 \text{ (huge)}}}_{\Downarrow}$$

Fine-Tuning (FT)

$\uparrow$

$\downarrow$

Hierarchy Problem

$M_L \ll M_R$ (?)

$\Uparrow$

$\underbrace{\phi^\dagger \phi}$ $\quad$ $\underbrace{T, \Delta^\dagger \Delta}_{\text{any invariant}}$

<u>limit</u>: $m_\phi \to 0 \Longrightarrow$

<u>no</u> new symmetry

$$\Updownarrow$$

$\phi^+\phi$ = invariant under

$$\phi \to U\phi$$

$$\Updownarrow$$

$$\boxed{\text{scalar mass} \neq \text{protected by a symmetry}}$$

$$\Downarrow$$

$$\boxed{\text{completely different from fermion mass}}$$

Fermion mass

$$m_f \bar{f} f = m_f f^+ \gamma_0 f$$

$\Downarrow$ $\qquad\qquad\qquad\qquad$ $\uparrow$

$\qquad\qquad\qquad\qquad$ gauge changes

$$\bar{f} f = f^+ \gamma_0 f = \bar{f_L} f_R + \bar{f_R} f_L$$

- $m_f = 0$ limit

$$\Rightarrow \mathcal{L}_D = i\,\bar{f_L} \gamma^\mu \partial_\mu f_L + i\,\bar{f_R} \gamma^\mu \partial_\mu f_R$$

$$\underbrace{f_L \to e^{i\alpha} f_L, \quad f_R \to e^{i\beta} f_R}_{\text{chiral symmetry}}$$

- $m_f \neq 0$

$$\bar{f}_L f_R \longrightarrow \bar{f}_L f_R \Leftrightarrow \alpha = \beta$$

$$\Updownarrow$$

$$f_{L,R} \to f_{L,R}\, e^{i\alpha}$$

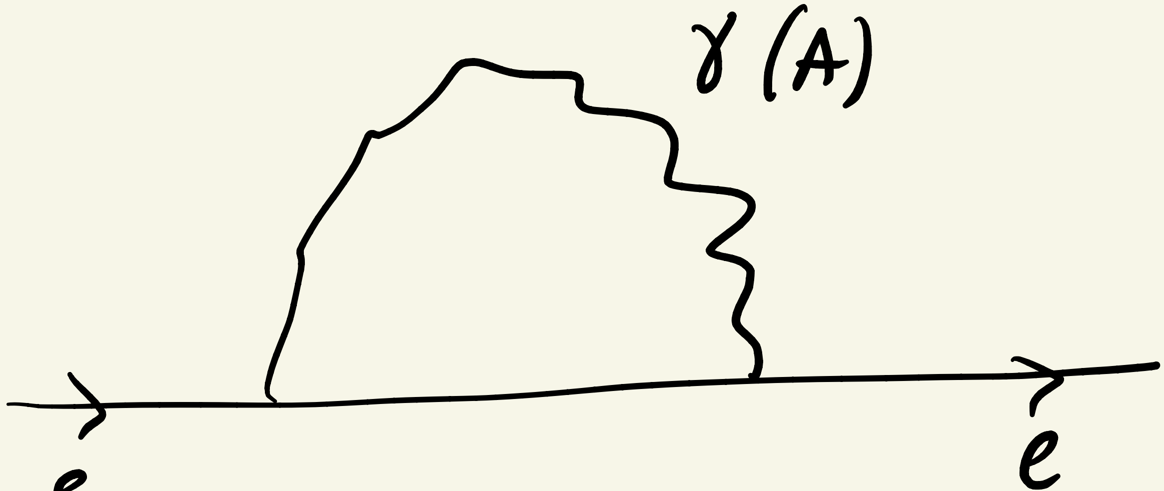
<u>no</u> chiral symmetry

$$\Downarrow$$

$$\boxed{\text{fermion mass = protected by chiral symmetry}}$$

Veiskopf 1930

loop correction to e mass



$\Downarrow \qquad m_e^0(\bar{e}_L e_R + h.c.)$

$$m_e = m_e^0 \underbrace{\left[1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{m_e^0} \right]}_{\text{1 loop}}$$

<u>145 th loop</u>

$\Downarrow$ claim

$$m_e = m_e^0 \left[1 + \left(\frac{\alpha}{\pi}\right)^{145} \ln \frac{\Lambda}{m_e^0} \right]$$

<u>1 - loop</u>

$$m_e = m_e^0 \left[1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{m_e^0} \right]$$

and not

$$m_e = m_e^0 + \frac{\alpha}{\pi}\Lambda$$

$\boxed{\text{cannot be}}$

as in the scalar

once again: $m_e^0 \to 0$

$$\Downarrow$$

chiral symmetry! $f_R \to e^{i\varphi} f_R$
$f_L \to f_L$

$$\Downarrow$$

pert. theory (to all orders)

must have: $\psi_R \to e^{i\beta}\psi_R$, $\psi_L \to \psi_L$

$\Downarrow$

$$\boxed{m_e \text{ (to all orders)} = 0}$$

$\Rightarrow$ $m_e \propto m_e^0$!!!

$\Downarrow$

$$m_e = m_e^0 \left[1 + \left(\frac{\alpha}{\pi} + \left(\frac{\alpha}{\pi}\right)^2 \cdots \right) \ln \frac{1}{m_e^0} \right]$$

chiral symmetry =

= protective symmetry

<u>Scalars</u>

$$\mu^2 = \mu_0^2 + \frac{\alpha}{\pi}\Lambda^2 + \cdots$$

$$\left(\mu^2 = \mu_0^2 + \alpha_1 v_R^2 \right)$$

LRSM

$\Uparrow$

FT = ugly, but <u>not</u> a problem

• log. "infinity" = small

$$\Lambda \leq M_{pl} = G_N^{-1/2}$$

↑ string gravity

$$\left(\frac{\alpha}{4\pi} \ln \frac{M_p}{m_e} \ll 1 \right)$$

— "naturalness" (technical sense)

↕ 't Hooft '70,

$\Theta \rightarrow 0 \Rightarrow$ more symmetry a parameter

$$\Theta^{\text{loop}} = \Theta_0 \left[1 + - - - \right]$$

- fermion mass = "naturally" small (protected)
- scalar mass $\neq$ naturally small

$\Longleftrightarrow$ <u>not</u> protected

Q. Go back to Φ

$v_2 = 0 \Leftrightarrow \phi_2 \neq$ Higgs

$$\Downarrow$$

$$m_{\phi_2}^2 \simeq v_R^2$$

but:

$\alpha_3 v_R^2 \, \phi_2^\dagger \phi_2$ and ϕ_2 is

in general a part of Higgs ($v_2 \neq 0$)

$$\Big\downarrow \; v_2 \neq 0$$

$$\phi_1, \ \phi_2 \longrightarrow \left(\langle \phi_i \rangle = v_i \right)$$

$$h = \cos\beta\, \phi_1 + \sin\beta\, \phi_2$$

$$H = -\sin\beta\, \phi_1 + \cos\beta\, \phi_2$$

$$\tan\beta \equiv v_2/v_1 \quad \therefore$$

$$\langle h \rangle = \sqrt{v_1^2 + v_2^2} \quad (\text{Higgs})$$

$$\langle H \rangle = 0 \quad (\text{scalar doublet})$$

$$\Rightarrow \boxed{m_H^2 \simeq \alpha_3\, v_R^2\, \frac{1}{\cos 2\beta}}$$

$m_h^2 \stackrel{!}{=}$ small by FT

Fine Tuning = necessary