Neutrino Physics Course

Lecture XVI

20/6/2023

LMU

Summer 2023

LRSM: completing SSB

- $G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$$Q_{em} = T_{3L} + \underbrace{T_{3R} + \frac{B-L}{2}}_{\frac{Y}{2}}$$

- <u>matter</u>

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R = q_R$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R = l_R$$

- <u>Higgs</u>

$$G_{LR} \longrightarrow G_{SM}$$
$$\langle \Delta_R \rangle = M_R$$

$$\Delta_L \xleftrightarrow{LR = P} \Delta_R$$

$$\Delta_{L,R} \longrightarrow U_{L,R}\, \Delta_{L,R}\, U_{L,R}^{\dagger}$$

(symb)

$$V = -\frac{\mu_\Delta^2}{2} \underbrace{\left(\Delta_L^2 + \Delta_R^2\right)} +$$

$$\underbrace{+ \frac{\lambda}{4}\left(\Delta_L^4 + \Delta_R^4\right) + \frac{\lambda'}{2} \Delta_L^2 \Delta_R^2}$$

$$\frac{\lambda}{4} \underbrace{\left(\Delta_L^2 + \Delta_R^2\right)^2} + \frac{\lambda' - \lambda}{2} \Delta_L^2 \Delta_R^2$$

decides who gets a vev

if $\lambda' - \lambda > 0$

$$\Downarrow$$

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = v_R \; (M_R)$$

$$\Downarrow$$

forget (for now) Δ_L

$$\Downarrow$$

$$V_R = -\frac{\mu_\Delta^2}{2} \, \mathrm{Tr}\, \Delta_R^\dagger \Delta_R \quad \leftarrow \text{mass term}$$

$$+\underbrace{\frac{\lambda}{4}\left(\underline{\mathrm{Tr}\,\Delta_R^+\Delta_R}\right)^2+\frac{\lambda'}{2}\mathrm{Tr}\,\Delta_R^2\,\mathrm{Tr}(\Delta_R^+)^2}_{\text{interaction}}$$

$$\Uparrow$$

$$\boxed{\begin{aligned}\mathrm{Tr}\,\Delta_R^+\Delta_R\Delta_R^+\Delta_R &= a\left(\mathrm{Tr}\,\Delta_R^+\Delta_R\right)^2\\ (**)\qquad &+ b\,\mathrm{Tr}\,\Delta_R^2\,\mathrm{Tr}(\Delta_R^+)^2\end{aligned}}$$

$$\Delta_R=\left(\begin{array}{c|c}\delta_R^+ & \delta_R^{++}\\ \hline v_R+H_R+iG_R & -\delta_R^+\end{array}\right)$$

$\int \delta_R^+,\ \delta_R^-$: eaten by $W_R^+,\ W_R^-$

G_R : eaten by Z_R

$$\boxed{W_R^+ ,\ W_R^- ,\ Z_R : \text{ new heavy gauge bosons, } m \propto M_R}$$

$$\Updownarrow$$

$$\boxed{m_{\delta_R^+} = m_{G_R} = 0 \text{ in } V_R}$$

- $m_{H_R}^2 = 2 \lambda v_R^2 \quad (\lambda > 0)$
- $m_{\delta_R^{++}}^2 \propto \lambda' v_R^2 \quad (m_{\delta_R^{++}} \equiv m_{++})$

$$\Uparrow$$

if $\lambda' = 0 \Rightarrow V_R$ has $SO(6)$ global symm.

$$\mathrm{Tr}\, \Delta_R^\dagger \Delta_R = \sum_{i=1}^{6} \phi_i^2$$

component: $$\boxed{H_R,\ G_R,\ \overbrace{R_+,\ I_+}^{\delta_R^+},\ \underbrace{R_{++},\ I_{++}}_{\delta_R^{++}}}$$

$$\Downarrow$$

$$SO(6) \xrightarrow{\langle \Delta_R \rangle} SO(5)$$

$$\underbrace{\frac{6\cdot 5}{2} - \frac{5\cdot 4}{2} = 5}_{}$$

\# of broken gen.

= \# of NG massless bosons

$\Downarrow$

if $\lambda' \neq 0 \Rightarrow \underline{\underline{m_{++}^2 \propto \lambda' v_R^2}}$

$\Downarrow$

$$\boxed{\lambda > 0, \quad \lambda' > 0}$$

$$\Updownarrow$$

$$\boxed{\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \text{ is a local minimum}}$$

. comment : $\lambda > 0 \iff$

V_R = bounded from below

• Local min = global min

<u>Proof:</u>

$$(B-L)\,\Delta_{L,R} = 2\,\Delta_{L,R}$$

<u>not</u> hermitian

$$\Downarrow$$

$(\Delta_R \equiv \Delta)$ complex, un-

$$\Delta = \Delta_1 + i\,\Delta_2$$

$$\Delta_i^{\dagger} = \Delta_i \quad , \quad i = 1, 2$$

$$\left(\Delta_1 = \frac{\Delta + \Delta^+}{2} \qquad \Delta_2 = \frac{\Delta - \Delta^+}{2i} \right)$$

$$\Rightarrow \quad \langle \Delta \rangle = \langle \Delta_1 \rangle + i \langle \Delta_2 \rangle$$

$$U H U^+ = \mathrm{diag}(h_1, h_2, \dots)$$

$$(H = H^+) \qquad h_i \in \mathbb{R}$$

$$\mathrm{Tr}\, \Delta_1 = 0$$

$$\Rightarrow \quad \langle \Delta_1 \rangle = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \quad a \in \mathbb{R}$$

$$(\quad U_1 \langle \Delta_1 \rangle U_1^+ = \langle \Delta_1 \rangle \qquad ,$$

if $U_1 = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}$

$$\langle \Delta_2 \rangle = \begin{pmatrix} b & r e^{i\beta} \\ r e^{-i\beta} & -b \end{pmatrix}$$

$$b, r \in \mathbb{R}$$

$$U_1 \langle \Delta_2 \rangle U_1^{+} = \begin{pmatrix} b & r e^{i(2\alpha+\beta)} \\ r e^{-i(2\alpha+\beta)} & -b \end{pmatrix}$$

Prove!!

$$\Rightarrow \langle \Delta_2 \rangle = \begin{pmatrix} b & r \\ r & -b \end{pmatrix}$$

$\therefore \quad \alpha = -\beta/2$

$$\Downarrow$$

$$\langle \Delta \rangle = \langle \Delta_1 \rangle + i \langle \Delta_2 \rangle$$

$$= \begin{pmatrix} a+ib & i\gamma \\ i\gamma & -(a+ib) \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\langle \Delta \rangle = \begin{pmatrix} z & i\gamma \\ i\gamma & -z \end{pmatrix}}$$

$$\Downarrow$$

Prove

$$\mathrm{Tr}\, \Delta^{+} \Delta = (|z|^2 + \gamma^2)\, 2$$

$$\mathrm{Tr}\, \Delta^2 = (z^2 - \gamma^2)\, 2$$

$$\Downarrow \qquad \overbrace{\qquad\qquad\qquad\qquad}^{\mu_\Delta^2,\ \lambda}$$

$$\boxed{\begin{aligned}\langle V\rangle &= f\left(|z|^2+v^2\right)\\ &\quad + 2\lambda'\left(z^2-v^2\right)\left(z^{*2}-v^2\right)\end{aligned}}$$

- $\boxed{\lambda'>0}$ $\leftarrow$ established:

$$\langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix} \text{ is local minimum}$$

$$\Downarrow$$

$$\boxed{z^2 = v^2} \Rightarrow \boxed{z = v}$$

$$\boxed{\langle \Delta_R \rangle = v \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}}$$

but

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} = v_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

<u>a must !</u> ? ? ? ?

These vevs look very different !

Exercise

$$U \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} U^{\dagger} \propto \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$$

$$\Updownarrow$$

$$U^{\dagger} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} U \propto \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{\text{physical vev}}$$

$$\Downarrow$$

$$\boxed{\lambda' > 0 \Rightarrow \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \text{ is a unique minimum}}$$

• $\lambda' < 0$ – what happens?

$$\underbrace{\lambda' (z^2 - \gamma^2)(z^{*2} - \gamma^2)}_{\text{maximal}}$$

what is the residual symmetry?

• gauge boson masses

$$\mathcal{L}_{kin} = \frac{1}{2} \mathrm{Tr}(D_\mu \Delta_R^+)(D^\mu \Delta_R)$$

$$D_\mu \Delta_R = \partial_\mu - i g \hat{T}_a^R A_\mu^R -$$

$$- i \bar{g} \frac{B-L}{2} \bar{B}_\mu$$

$$\Delta_R \longrightarrow U_R \Delta_R U_R^+$$

$$U_R = e^{i T_a^R \theta_a^R}$$

$$T_a^R = \frac{\sigma_a}{2}$$

$$\Downarrow$$

$$\boxed{\hat{T}_a^R \Delta_R = [T_a^R, \Delta_R]}$$

$$\Downarrow$$

$$\underbrace{M_{W_R} = g\, v_R\,, \quad M_{Z_R} = \;?\; M_{W_R}}_{\text{HW (posted)}}$$

$$\boxed{\sin^2\theta_W \simeq \frac{1}{4}} \Rightarrow \boxed{M_{Z_R} = \sqrt{3}\, M_{W_R}}$$

$$M_{W_R} \gtrsim 4 \text{ TeV} \leftarrow \text{LHC}$$

$$\Rightarrow \underbrace{M_{Z_R} \gtrsim 7 \text{ TeV}}_{\text{cannot be seen @ LHC}}$$

· HW problem

$$\mathrm{Tr}\,\Delta^+\Delta\Delta^+\Delta = a\,(\mathrm{Tr}\,\Delta^+\Delta)^2 + b\,\mathrm{Tr}\,\Delta^2\,\mathrm{Tr}(\Delta^+)^2$$

Hint.

$$\Delta \longrightarrow \begin{pmatrix} \varphi & i\chi \\ i\chi & -\varphi \end{pmatrix}$$

$$\varphi, \chi \subseteq \mathbb{R}$$

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digression

SM: $\tan\theta_w = g'/g$

$e = \sin\theta_W\, g$

unbroken

$Q_{em} = T_{3L} + Y/2$

$\Leftrightarrow \quad \boxed{\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}}$

$\Downarrow$

LRSM: $\left|\ \frac{1}{g'^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2}\ \right|$

$Y \qquad SU(2)_R \qquad B-L$

$Y/2 = T_{3R} + \frac{B-L}{2}$

$$\tan\theta_R \equiv \bar{g}/g$$

$$\tan\theta_W \equiv g'/g$$

$$g' = \frac{g\bar{g}}{\sqrt{g^2+\bar{g}^2}} \Longleftrightarrow \frac{1}{g'^2} = \frac{1}{g} + \frac{1}{\bar{g}^2}$$

$$\tan\theta_W = \frac{\cancel{g}\,\bar{g}}{\cancel{g}\sqrt{g^2+\bar{g}^2}} = \frac{\bar{g}}{\sqrt{g^2+\bar{g}^2}}$$

$$\Rightarrow \boxed{\tan\theta_W = \sin\theta_R}$$

Hint:

SM: $$M_Z^2 = \frac{g^2+g'^2}{g^2} M_W^2$$

LRSM: $$M_{Z_R}^2 = 2 \underbrace{\frac{g^2+\tilde{g}^2}{g^2}}_{f(\theta_W)} M_{W_R}^2$$