

# Neutrino Physics Course

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## Lecture XVI

20/6/2023

LMU

Summer 2023

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# Higgs

$$G_{LR} \rightarrow G_{SM}$$
$$\langle \Delta_R \rangle = M_R$$

$$\Delta_L \quad \xleftrightarrow{LR=P} \quad \Delta_R$$

$$\Delta_{L,R} \rightarrow U_{L,R} \Delta_{L,R} U_{L,R}^\dagger$$

(Symb)

$$V = - \frac{\mu_\Delta^2}{2} \underbrace{(\Delta_L^2 + \Delta_R^2)} +$$

$$+ \frac{\lambda}{4} (\Delta_L^4 + \Delta_R^4) + \frac{\lambda'}{2} \Delta_L^2 \Delta_R^2$$

$$\underbrace{\frac{\lambda}{4} (\Delta_L^2 + \Delta_R^2)^2} + \frac{\lambda' - \lambda}{2} \Delta_L^2 \Delta_R^2$$

decides who gets  
a vev

$$\text{if } \lambda' - \lambda > 0$$



$$\langle \Delta_L \rangle = 0, \quad \langle D_R \rangle = v_R (M_R)$$



forget (for now)  $\Delta_L$



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$$V_R = -\frac{\mu^2}{2} \underbrace{T_\nu \Delta_R^+ D_R}_{\text{mass term}} \leftarrow \text{mass term}$$



$G_R$ : eaten by  $Z_R$

$W_R^+$ ,  $W_R^-$ ,  $Z_R$ : new heavy gauge bosons,  $\mu \propto M_R$



$$\mu_{\delta_R^+} = \mu_{G_R} = 0 \quad \text{in } \overline{V}_R$$

- $\mu_{H_R}^2 = 2 \lambda v_R^2 \quad (\lambda > 0)$
- $\mu_{\delta_R^{++}}^2 \propto \lambda' v_R^2 \quad (\mu_{\delta_R^{++}} \equiv \mu_{++})$



if  $\lambda' = 0 \Rightarrow V_R$  has  $SO(6)$  global sym.

$$T_V \Delta_R^+ \Delta_R = \sum_{i=1}^6 \phi_i^2$$



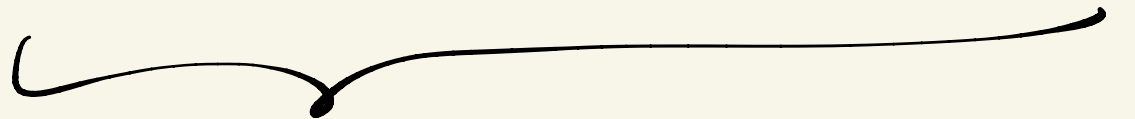
components:

$$\left. \begin{array}{l} H_R, G_R, R_+, I_+, \\ R_{++}, I_{++} \end{array} \right\} \Delta_R^+$$

$$\Delta_R^{++}$$



$$\begin{array}{ccc}
 SO(6) & \longrightarrow & SO(5) \\
 \downarrow & \langle \Delta_R \rangle & \downarrow \\
 \frac{6.5}{2} & - & \frac{5.4}{2} = 5
 \end{array}$$



# of broken gen.

= # of NG massless bosons

$\Downarrow$

if  $\lambda' \neq 0 \Rightarrow \underline{\underline{m_{++}^2 \propto \lambda' v_R^2}}$

$\Downarrow$

$\lambda > 0, \lambda' > 0$





$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ V_R & 0 \end{pmatrix} \text{ is a}$$

local minimum

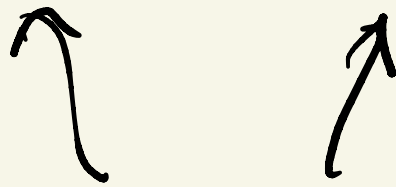
. comment :  $\lambda > 0 \Leftrightarrow$

$V_R =$  bounded from below

• Local min = global min

Proof:

$$(B-L) \Delta_{L,R} = 2 \Delta_{L,R}$$



not hermitian



$(\Delta_R \equiv \Delta)$  complex, un-

$$\Delta = \Delta_1 + i \Delta_2$$

$$\Delta_i^+ = \Delta_i, \quad i=1,2$$

$$\left( \Delta_1 = \frac{\Delta + \Delta^\dagger}{2} \quad \Delta_2 = \frac{\Delta - \Delta^\dagger}{2i} \right)$$

$$\Rightarrow \langle \Delta \rangle = \langle \Delta_1 \rangle + i \langle \Delta_2 \rangle$$

$$U H U^\dagger = \text{diag}(h_1, h_2, \dots)$$

$$(H = H^\dagger)$$

$$h_i \in \mathbb{R}$$

$$T_1 \Delta_1 = 0$$

$$\Rightarrow \langle \Delta_1 \rangle = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \quad a \in \mathbb{R}$$

$$(U_1 \langle \Delta_1 \rangle U_1^\dagger = \langle \Delta_1 \rangle)$$

$$L \quad \text{if } U_1 = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix} |$$

$$\langle \Delta_2 \rangle = \begin{pmatrix} b & r e^{i\beta} \\ r e^{-i\beta} & -b \end{pmatrix}$$

$$b, r \in \mathbb{R}$$

$$U_1 \langle \Delta_2 \rangle U_1^\dagger = \begin{pmatrix} b & r e^{i(2\alpha+\beta)} \\ r e^{-i(2\alpha+\beta)} & -b \end{pmatrix}$$

Prove!!

$$\Rightarrow \left[ \begin{array}{l} \langle \Delta_2 \rangle = \begin{pmatrix} b & r \\ r & -b \end{pmatrix} \\ \therefore \alpha = -\beta/2 \end{array} \right]$$

↓

$$\langle \Delta \rangle = \langle \Delta_1 \rangle + i \langle \Delta_2 \rangle$$

$$= \begin{pmatrix} a+ib & ir \\ ir & -(a+ib) \end{pmatrix}$$

↓

$$\langle \Delta \rangle = \begin{pmatrix} z & ir \\ ir & -z \end{pmatrix}$$

↓

Prove

$$\text{Tr } \Delta^\dagger \Delta = (|z|^2 + r^2) 2$$

$$\text{Tr } \Delta^2 = (z^2 - r^2) 2$$

$\Downarrow \mu_D^2, \lambda$

$$\langle V \rangle = f(|z|^2 + v^2)$$

$$+ 2\lambda' (z^2 - v^2) (z^{*2} - v^2)$$

•  $\lambda' > 0$  ← established ∴

$\langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}$  is local minimum

$\Downarrow$

$$\boxed{z^2 = v^2} \Rightarrow \boxed{z = v}$$



$$\langle \Delta \rangle_R = v \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$$

but

$$\langle \Delta \rangle_R = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} = v_R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

a must!

? ? ? ?  
• • • •

These vevs look very  
different!

## Exercise

$$U \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} U^\dagger \propto \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$$



$$U^\dagger \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} U \propto \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{\text{physical neutrino}}$$

physical neutrino



$$\lambda' > 0 \Rightarrow \begin{pmatrix} 0 & 0 \\ \nu_R & 0 \end{pmatrix} \rightarrow a$$

unique neutrino



- $\lambda' < 0$  - what happens?



$$\lambda' (z^2 - v^2) (z^{*2} - v^2)$$

maximal

what is the residual symmetry?

- 
- gauge boson masses



$$\mathcal{L}_{kin} = \frac{1}{2} \text{Tr} (D_\mu \Delta_R^\dagger) (D^\mu \Delta_R)$$

$$D_\mu \Delta_R = \partial_\mu - ig \hat{T}_a^R A_\mu^R - i\bar{g} \frac{B-L}{2} \bar{B}_\mu$$

$$\Delta_R \rightarrow U_R \Delta_R U_R^\dagger$$

$$U_R = e^{i T_a^R \Theta_a^R}$$

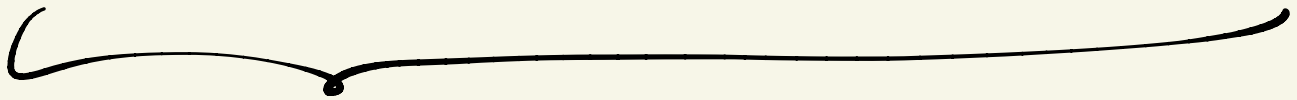
$$T_a^R = \frac{\sigma_a}{2}$$



$$\hat{T}_a^R \Delta_R = [T_a^R, \Delta_R]$$



$$M_{WR} = g v_R, \quad M_{ZR} = \dots M_{WR}$$



H W (putted)

$$\left[ \sin^2 \theta_w \approx \frac{1}{4} \right] \Rightarrow M_{ZR} = \sqrt{3} M_{WR}$$

$$M_{WR} \gtrsim 4 \text{ TeV} \leftarrow \text{LHC}$$

$$\Rightarrow M_{ZR} \gtrsim 7 \text{ TeV}$$



cannot be seen @ LHC

HW problem

$$\begin{aligned} \text{Tr } \Delta^+ \Delta \Delta^+ \Delta &= a (\text{Tr } \Delta^+ \Delta)^2 \\ &+ b \text{Tr } \Delta^2 \text{Tr } (\Delta^+)^2 \end{aligned}$$

Hint:

$$\Delta \rightarrow \begin{pmatrix} \varphi & i\chi \\ i\chi & -\varphi \end{pmatrix}$$

$$\varphi, \chi \in \mathbb{R}$$

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digression

Sol:  $\text{fau} \theta_w = g'/g$



$$\tan \theta_R \equiv \frac{\bar{g}}{g}$$

$$\tan \theta_w \equiv g'/g$$


$$g' = \frac{g\bar{g}}{\sqrt{g^2 + \bar{g}^2}} \Leftrightarrow \frac{1}{g'^2} = \frac{1}{g^2} + \frac{1}{\bar{g}^2}$$

$$\tan \theta_w = \frac{g\bar{g}}{g\sqrt{g^2 + \bar{g}^2}} = \frac{\bar{g}}{\sqrt{g^2 + \bar{g}^2}}$$

$$\Rightarrow \boxed{\tan \theta_w = \tan \theta_R}$$

Hint: SM:  $M_Z^2 = \frac{g^2 + g'^2}{g^2} M_W^2$

LRSM:  $M_{Z_R}^2 = \underbrace{2 \frac{g^2 + \bar{g}^2}{g^2}}_{f(\theta_w)} M_{W_R}^2$