Neutrino Physics Course

Lecture XIX

30/6/2023

LMU

Summer 2023

Seesaw mechanism and LR symmetry

but first

Higgs mass spectrum of the LRSM

$\Delta_L \longleftrightarrow \Delta_R$

Φ

$\downarrow$

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ 0 & v_R \end{pmatrix}, \quad \langle \Delta_L \rangle = 0$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$$

$$SU(2)_R \Longrightarrow \langle \Delta_R \rangle \in \mathbb{R}$$

$$SU(2)_L \Longrightarrow v_1, v_2 \in \mathbb{R}$$

($\kappa = 0$ for simplicity and illustration)

$$\Downarrow$$

$$\Delta_R = \begin{pmatrix} 0 & \delta_R^{++} \\ v_R + H_8 & 0 \end{pmatrix}$$

$$\Delta_L = \begin{pmatrix} \delta_L^{+} & \delta_L^{++} \\ \delta_L^{0} & -\delta_L^{+} \end{pmatrix}$$

$$\Phi = \left(\tilde{\phi}_1 \, , \; \phi_2 \right)$$

$$\Downarrow \qquad \tan\beta \equiv v_2 / v_1$$

$$(0) \quad \left| \begin{aligned} h &= \cos\beta \, \phi_1 + \sin\beta \, \phi_2 \\ H &= -\sin\beta \, \phi_1 + \cos\beta \, \phi_2 \end{aligned} \right\} \text{ 2 SM scalar doublets}$$

$$\Downarrow$$

h = SM Higgs $\Leftarrow \langle h \rangle \neq 0$

$\langle H \rangle = 0 \Leftrightarrow H$ = doublet scalar (heavy)

- $\mu^2, \bar{\mu}^2 \leftarrow$ masses of Φ in V

$$\mu^2 = \mu_0^2 + \alpha_1 v_R^2 \simeq M_W^2$$

$$\bar{\mu}^2 = \mu_0^2 + \alpha_2 v_R^2 \simeq M_W^2$$

$\Downarrow$?

FT

?

↓

<u>not</u> a priori ! $\alpha_1, \alpha_2 \simeq 0$!

(could be)

⇓

$$m_h^2 \simeq \mu^2 \quad (\pm M_W^2)$$

$$m_H^2 \simeq \alpha_3 \frac{v_R^2}{\cos 2\beta}$$

FT ?

Q. $\alpha_3 \simeq 0$?

A. NO, $\alpha_3 \simeq O(1)$

Proof: $q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R = q_R$

$$\mathcal{L}_Y^{\Phi} = \bar{q}_L \left(Y_\Phi \Phi + \tilde{Y}_\Phi \tilde{\Phi} \right) q_R \quad (1)$$
$$+ h.c.$$

$$\rightarrow \bar{q}_L \left(Y_\Phi \langle \Phi \rangle + \tilde{Y}_\Phi \langle \tilde{\Phi} \rangle \right) q_R$$

$$\langle \Phi \rangle = \mathrm{diag}\,(v_1, v_2)$$

$$\langle \tilde{\Phi} \rangle = \mathrm{diag}\,(v_2, v_1)$$

$$\Downarrow$$

$$\boxed{\begin{aligned} M_u &= Y_\Phi v_1 + \tilde{Y}_\Phi v_2 \\ M_d &= Y_\Phi v_2 + \tilde{Y}_\Phi v_1 \end{aligned}}$$

LRSM

$\updownarrow$ compare with SM

$$M_u = Y_u v$$
$$M_d = Y_d v$$

$\Downarrow$

compute couplings with

h, H from (1)

$\uparrow$ (0) $\begin{cases} \phi_1 = \cos\beta\, h + \sin\beta\, H \\ \phi_2 = -\sin\beta\, h + \cos\beta\, H \end{cases}$

$$\mathcal{L}_Y^{\mathrm{II}} = \bar{q}_L \left(Y_\Phi \begin{pmatrix} \phi_1^{0*} & \phi_2^+ \\ -\phi_1^- & \phi_2^0 \end{pmatrix} + \cdots \right) q_R$$

$$+ h.c.$$

$$\Downarrow$$

$$\mathcal{L}_Y(h, H) = \bar{q}_L \underbrace{\left(\frac{m_d}{v}\right)}_{= y_d} h \, d_R + \cdots$$

$$\left(\begin{aligned} m_d &= \mathrm{diag}(m_d, m_s, m_b) \\ v &= \sqrt{v_1^2 + v_2^2} \end{aligned} \right)$$

$$\Updownarrow$$

$h \longleftrightarrow$ usual SM interactions

$$\propto \bar{q}_L \underline{H} M_d d_R + \bar{q}_L \underline{\tilde{H}} \frac{M_u}{v \cos\beta} d_R$$

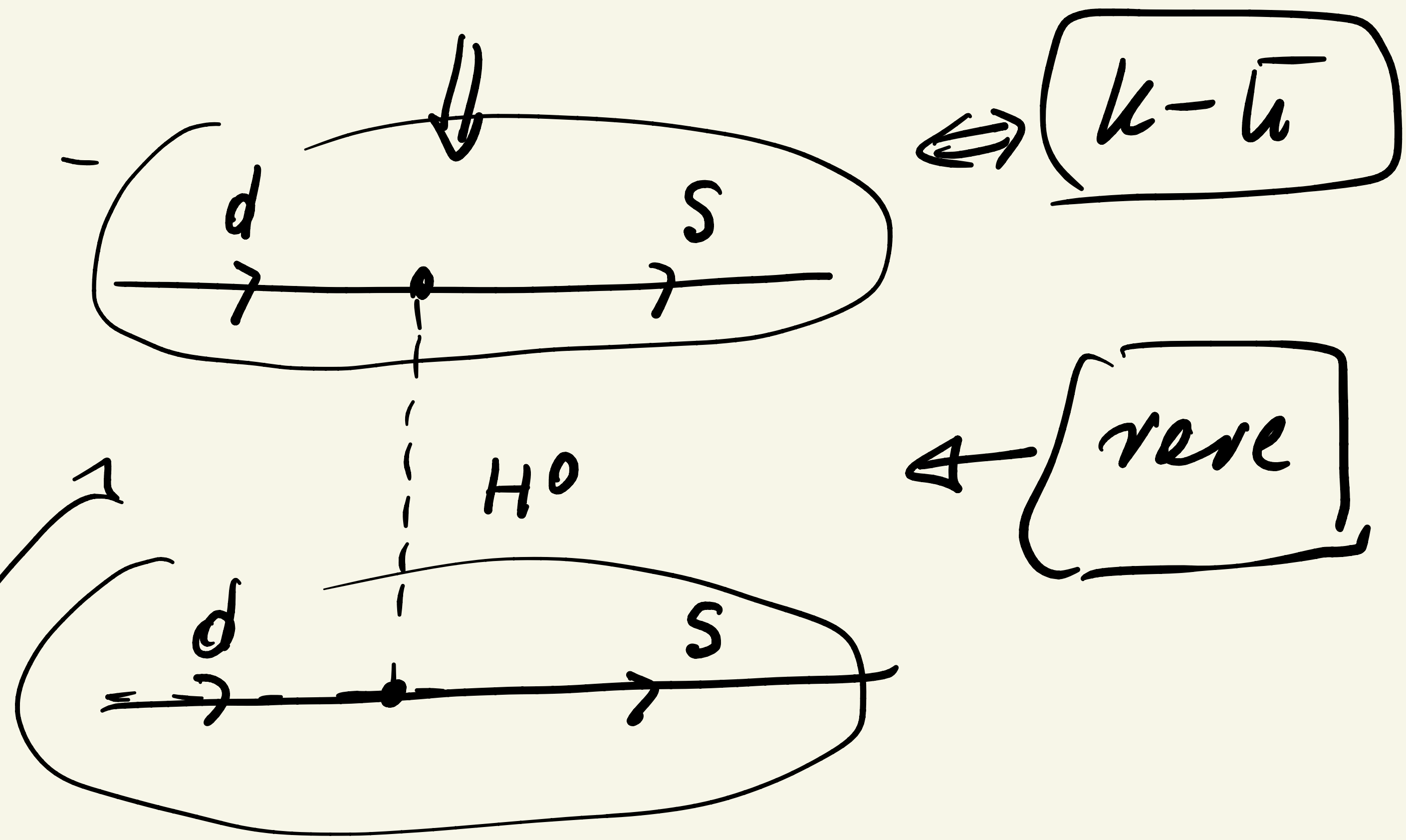
similar to h

new

$\Downarrow$

$$\bar{d}_L V_L^+ \frac{m_u}{v} V_R d_R \tan\beta H^0$$

$$\underline{H} = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$



$$V_L \equiv V_{CKM} = \text{LH quark mixing}$$

$$V_R \equiv \text{RH analog of } V_{CKM} \simeq V_L$$

$$\frac{\Delta m_K}{m_K} \equiv \frac{m_K - m_{\bar{K}}}{m_K} \simeq 10^{-15}$$

$$m_K \simeq m_K \simeq m_{\bar{K}}$$

$$\Downarrow$$

$$\boxed{m_{H^0} > 10\ \text{TeV}} \quad (\sim 20\ \text{TeV?})$$

$$\begin{pmatrix} H^+ \\ h^0 \end{pmatrix} \Rightarrow \underbrace{m_{H^+} - m_{H^0} \leq M_W}_{SU(2)_L \text{ symmetry}}$$

$$\Downarrow$$

$$\boxed{m_H \simeq m_{H^+} \simeq m_{H^0} > 10\ \text{TeV}}$$

but

$$m_H^2 = \alpha_3 \frac{v_R^2}{\cos 2\beta}$$

$\Downarrow$

$\alpha_3 \simeq O(1)$

$\Downarrow$

FT

$\Updownarrow$

LHC:

$m_h \gtrsim 100\ \mathrm{GeV}$

at $E \leq M_W$

$$\mathrm{LRSM} \simeq \mathrm{SM} + O\left(\frac{M_L}{M_R}\right)^2$$

$\underbrace{\qquad}_{\leq 10^{-3}}$

$\updownarrow$ $(M_R > 4\,\text{TeV})$

true for ν mass - - -

$$m_\nu(\text{LRSM}) = \cancel{m_\nu(\text{SM})} + O\left(M_L^2/M_R^2\right)$$

(above the crossed-out term: $\parallel$ 0)

the end of the Higgs story — go back to **neutrino mass**

Probe of origin of neutrino mass = ininterplay seesaw

LR. $\Longrightarrow$ $\exists\, \nu_R \rightarrow N_L = C\bar{\nu}_R^{\,T}$

$\Downarrow$ seesaw

$$M_\nu = - M_D^T \frac{1}{M_N} M_D \quad (*)$$

$$M_N = Y_\Delta v_R$$

$$M_D = Y_D v_{SM}$$

$$m_\nu \simeq M_W^2 / M_R \xrightarrow[M_R \to \infty]{} 0$$

$M_\nu \leftarrow$ low E experiments

(masses and mixing)

$M_N \leftarrow$ high E exp

(LHC)

⊛ formula : M_ν, M_N = input

$\downarrow$ output

M_D

$$Y_D = \frac{M_D}{v}$$

↳ dictates decays

$\Longleftrightarrow$ $\Theta_{\nu N} = \frac{1}{M_N} M_D$ determined

for M_D

$M_D \ (M_\nu, M_N)$

$\Downarrow$

$\Theta_{\nu N} \longleftrightarrow N \to e_L + W^+$

fund. decays

↓ compare with SM for charged f

$$y_f = \frac{m_f}{v}$$

$m_f \longrightarrow y_f \longrightarrow$ decays

($h \to f\bar{f}$)

⇕

probe origin of m_f

SM origin of m_f

$\Updownarrow \quad m_f \to y_f = \frac{m_f}{v}$

LRSM origin of m_ν

if $M_\nu, (M_N) \to M_D$

a must!

$\Updownarrow$

$$\underline{M}_\nu = - M_D^T \frac{1}{\underline{M}_N} \underline{M}_D \quad (*)$$

$$\downarrow$$

$$\underline{M}_D = f(M_\nu, \underline{M}_N)$$

from $(*)$ $\Rightarrow$

$$O O^T = O^T O = 1, \quad O \in \mathbb{C}$$

$$\underline{M}_D = i \sqrt{M_N}\; O \; \sqrt{\underline{M}_\nu}$$

$$M_D^T = i \sqrt{\underline{M}_\nu}\; O^T \sqrt{M_N}$$

$$\Downarrow$$

$$\underline{M}_\nu = -i \cdot i \sqrt{M_\nu} \sqrt{M_N} \frac{1}{M_N} \sqrt{M_N} \sqrt{M_\nu}$$

$$\underbrace{\sqrt{M_N} \frac{1}{M_N} \sqrt{M_N}}_{= 1}$$

$$= (-1)(-1) \sqrt{M_\nu} \sqrt{M_\nu} = \underline{M}_\nu \quad \checkmark$$

$$\underline{M}_\nu = - M_D^T \frac{1}{\underline{M}_N} M_D$$

$$= - i \sqrt{\underline{M}_\nu}\, O^T \sqrt{M_N} \frac{1}{M_N} \times$$

$$\times\, i \sqrt{\underline{M}_N}\, O \sqrt{\underline{M}_\nu}$$

(circled: $\sqrt{M_N} \frac{1}{M_N} \sqrt{\underline{M}_N} = 1$)

$$= (-1)(-1)\sqrt{\underline{M}_\nu}\;\underbrace{O^T O}_{1}\;\sqrt{\underline{M}_\nu}$$

$$\left(\text{used: } \sqrt{M}\,\frac{1}{M}\,\sqrt{M} = \cancel{\sqrt{M}}\,\frac{1}{\cancel{\sqrt{M}}\,\cancel{\sqrt{M}}}\,\cancel{\sqrt{M}} = 1 \right)$$

$$\Downarrow$$

$$\underline{M}_\nu = (+1)\sqrt{\underline{M}_\nu}\;\sqrt{\underline{M}_\nu} \quad \checkmark$$

Q.E.D.

$$\Updownarrow$$

$$M_D = i \sqrt{M_N}\, O \sqrt{M_\nu}$$

Casas, Ibarra '05

ambiguity

O = 3x3 complex orthogonal matrix (ambiguity)

- reminder

(a) $O \in \mathbb{R}$

$$\Rightarrow |O_{ij}| \leq 1$$

$$O_{2\times 2} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

but

(b) $O \in \mathbb{C}$

$\Rightarrow |O_{ij}| =$ can be large

<u>example</u>

$$O_{2\times 2} = \begin{pmatrix} \cosh\eta & i\sinh\eta \\ i\sinh\eta & \cosh\eta \end{pmatrix}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$\Downarrow$

O = impossibility of untangling seesaw

= impossibility of probing origin of ν mass

$$M_D = i\sqrt{M_N}\; O \;\sqrt{M_\nu}$$

measure (→ $\sqrt{M_N}$)

measure (→ $\sqrt{M_\nu}$)

O → arbitrary = independent

: illustration : $M_N \lesssim \text{TeV}$

$M_\nu \simeq 10^{-1}\,\text{eV}$

$$M_D \sim \sqrt{10^{3}\,\text{GeV}}\;|O|\;\sqrt{10^{-10}\,\text{GeV}}$$

$$\simeq \sqrt{10^{-7}\,\text{GeV}^2}\;|O|$$

$$\simeq \underbrace{\left(10^{-3-4}\,\text{GeV}\right)}\;|O| \;\leq M_W \simeq 100\,\text{GeV}$$

due to O:

$$m_D \sim \left(10^{-4} - 100\right)\text{GeV}$$

(i) even if I produce N

$\not\Rightarrow$ predict M_D

(ii) how to produce N ?

$$\underbrace{SM + N}_{\text{seesaw}} = \text{"photon"}$$

only coupling

$$\Theta_{\nu N} = \frac{1}{M_N} M_D$$

but:

seesaw : $M_D \ll M_N$

$$\Downarrow \text{ seesaw}$$

$$\theta_{\nu N} \ll 1$$

$$\Downarrow$$

$$\sigma(N) \propto \theta_{\nu N}^2 \longrightarrow 0$$

practically

- wait : $M_D = \textcircled{0}$

$\theta_{\nu N} = \textcircled{0}$ ambiguity

- seesaw $\stackrel{?}{=}$ independent of LR structure

NO: $LR \Rightarrow M_D$

↓ next lecture

NO Θ ambiguity in LRSM

$\Leftrightarrow \quad M_D = f(M_D, M_N)$

SM

$$M_q = Y_q\, v$$

(M_q: complex; Y_q: complex)

$$M_q = U_{Lq}^{\dagger}\, m_q\, U_{Rq}$$

(m_q = diag (real); U_{Lq}, U_{Rq}: complex)

$$\Downarrow$$

$$\boxed{V_{CKM} = U_{Lu}^{\dagger}\, U_{Ld} \in \mathbb{C}}$$

$$\Updownarrow$$

$$\underbrace{\mathcal{M}_e}_{\text{complex}}, \quad \underbrace{M_D(J)}_{\text{complex}}$$