Neutrino Physics Course

Lecture XII

6/6/2023

LMU

Summer 2023

LRSM: Higgs and neutrino mass

- $\phi_{SM} \subseteq \Phi$ (bi-doublet)

$$\Phi \to U_L \, \Phi \, U_R^{+}$$

$$\Updownarrow$$

$$\mathcal{L}_Y = \overline{f_L} \left(Y_f \, \Phi + \tilde{Y}_f \, \tilde{\Phi} \right) f_R$$

$$\tilde{\Phi} \equiv \sigma_2 \, \Phi^* \sigma_2 \to U_L \, \tilde{\Phi} \, U_R^{+}$$

f_L : q_L, l_L $\qquad$ f_R: q_R, l_R

- $\Phi_{new} = \Delta_L + \Delta_R$

$\therefore$

$$\langle \Delta_L \rangle = 0, \quad \langle \Delta_R \rangle = v_R \neq 0$$

$$\boxed{\begin{array}{ccccc} G_{LR} & \xrightarrow[\langle \Delta_R \rangle]{} & G_{SM} & \xrightarrow[\langle \Phi \rangle]{} & U(1)_{em} \\ & \| & & \| & \\ & M_R & & M_W \ (= M_L) & \end{array}}$$

LHC: $M_R \gtrsim 4\,\text{TeV}$

$\Downarrow$ choice for Δ ?

(a) $\Delta_{L,R}$ = <u>doublets</u>

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \ell_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$m_\nu \simeq m_e$$

(b) $$m_{new} \simeq M_R$$

$$m_{SM} \simeq M_{SM}$$

$$\boxed{m_{\nu_R} \lesssim M_R \quad ?}$$

?

$$\ni \underline{\underline{\nu_R^T \, \boxed{C} \, \nu_R}} \qquad \Delta_R^0 \quad (\langle \Delta_R^0 \rangle = M_R)$$

$$T_{3R} : \frac{1}{2} + \frac{1}{2} = 1 \qquad \Delta \qquad T_{3R} = -1$$

$$\boxed{(B-L)\,\nu_R = -\nu_R}$$

TRIPLET

$$\Updownarrow$$

$$\boxed{(B-L)(\Delta_R) = 2}$$

$$\Delta_R \to U_R \, \Delta_R \, U_R^+$$

$\Downarrow$ l $\quad$ / p

$$\Delta_L \to U_L \, \Delta_L \, U_L^{+}$$

with:

S. $\mathrm{Tr}\, \Delta_{L,R} = 0$

Q. $\Delta_{L,R}^{+} \stackrel{?}{=} \Delta_{L,R}$? ? ??

A. <u>NO</u>

$$\Delta_R \longrightarrow e^{i(B-L)\alpha} \Delta_R = e^{2i\alpha} \Delta_R$$

$$\Delta_R^{+} \longrightarrow e^{-2i\alpha} \Delta_R$$

$$\Downarrow$$

$$\boxed{\Delta_R^{+} \neq \Delta_R}$$ ✓ Q.E.D.

$$\boxed{\begin{gathered}\Delta_{L,R} = (\Delta_1)_{L,R} + i\,(\Delta_2)_{L,R} \\ (\Delta_i^{+})_{L,R} = (\Delta_i)_{L,R}\end{gathered}}$$

Spectrum

- $$\Phi = \begin{pmatrix} \varphi_1^{*0} & \varphi_2^{+} \\ -\varphi_1^{-} & \varphi_2^{0} \end{pmatrix}$$

  $$= (\tilde{\phi}_1, \phi_2)_v$$

- $\Delta_{L,R} = \,?$

$$\Delta \to U \Delta U^{+}$$

$$= e^{i\theta_a T_a} \Delta e^{-i\theta_a T_a}$$

$$= \Delta + i\theta_a [T_a, \Delta] + \dots$$

$$\Downarrow$$

$$\hat{T}_a \Delta = [T_a, \Delta]$$

basc gen. $T_a = \frac{\sigma_a}{2}$

- $Q_{em} = \hat{T}_{3L} + \hat{T}_{3R} + \frac{B-L}{2}$

$$\Leftrightarrow \quad Q_{em} \Delta = \hat{T}_3 + 1$$

$$\boxed{\Delta = \begin{pmatrix} \delta^+ & \delta^{++} \\ \delta^0 & -\delta^+ \end{pmatrix}}$$

shifts Q by 1

PROVE!

$$\Delta_R = \begin{pmatrix} \cancel{\delta^+_R} & \delta^{++}_R \\ v_R + \boxed{H_R} + i\cancel{G_R} & -\delta^+_R \end{pmatrix}$$

eaten by W_R^+

eaten by Z_R

analogy

$$\phi_{SM} = \begin{pmatrix} \varphi^+ \\ v_{SM} + h_{SM} + i\, G_{SM} \end{pmatrix}$$

"eaten" by W^+

"eaten" by Z

$$\left.\begin{array}{l} W^{\pm} \longrightarrow W_L^{\pm} \\ Z^0 \longrightarrow Z_L^0 \end{array}\right\} \text{ LR rotation}$$

$$\updownarrow \text{ analog}$$

$$W_R^{\pm}, \quad Z_R^0 \qquad \text{massive (extremely)}$$

$$\Downarrow$$

$$\Delta_R^{m} \text{ (physical)} = \begin{pmatrix} 0 & \delta_R^{++} \\ v + H_R & 0 \end{pmatrix}$$

by gauge rotation

$$\delta_R = H_R^{++},$$

physical state

looked for @ LHC

$m_{++} \equiv m_{\delta_R^{++}}$ ∴

$$\boxed{m_{++} \gtrsim 400 \text{ GeV}}$$

- SM: $m_h^2 = 2\lambda v^2$

$$V_{SM} = \frac{\lambda}{4}\left(\phi^\dagger \phi - v^2\right)^2$$

$\lambda = ? \implies m_h = ?$

• $\underline{LR}$: $\quad m^2_{H^0_R} \simeq \lambda_\Delta v_R^2$

$$\boxed{\lambda_\Delta = ? \Rightarrow m^2_{H^0_R} = ?}$$

$$\boxed{\text{No prediction of } m_{H_R} !}$$

$$\boxed{\text{Physics of } \Delta\text{'s}}$$

• $\mathcal{L}_Y^{(new)}(\Delta) =$

$$l_R^T C\, i\sigma_2 \Delta_R\, l_R\, Y_\Delta$$

$$\Big\downarrow \quad + \quad R \longrightarrow L \; + h.c.$$

$$l_R^T C\, \underbrace{U_R^T i\sigma_2 U_R}_{\parallel} \,\Delta_R\, \underbrace{U_R^\dagger U_R}_{\parallel \atop \mathbb{1}}\, l_R$$

$$i\sigma_2 \underbrace{U_R^\dagger U_R}_{1} \qquad = \text{inv.}$$

✓

$$\Downarrow$$

$$\mathcal{L}_Y(\Delta) =$$

$$Y_\Delta \left(\nu_R^T\; e_R^T\right) C \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \delta_R^{++} \\ v_R + h_R & 0 \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$+ R \to L$

$$= Y_\Delta \left(\nu_R^T \; e_R^T \right) C \begin{pmatrix} v_R + H_R & 0 \\ 0 & -\delta_R^{++} \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$+ R \to L$

$$= Y_\Delta \left[\nu_R^T C \nu_R (v_R + H_R) - \right.$$

$$\left. - e_R^T C e_R \delta_R^{++} \right] + - -$$

$+ h.c.$

$$\Rightarrow \boxed{m_{\nu_R} = Y_\Delta v_R} \Rightarrow Y_\Delta = \frac{m_{\nu_R}}{v_R}$$

$$+ Y_\Delta H_R (\nu_R^T C \nu_R + h.c.)$$

$$+ y_\Delta \delta_R^{++} e_R^T C e_R + h.c.$$

$$\delta_R^{--} \longrightarrow e_R + e_R$$

$$\delta_R^{++} \longrightarrow e_R^c + e_R^c$$

<u>Nutshell:</u>

$$m_{\nu_R} = y_\Delta v_R \,, \quad \boxed{m_{\nu_L} = 0}$$

$$\updownarrow$$

$$\langle \Delta_L \rangle = 0$$

$\Downarrow$ switch on Φ

$$\mathcal{L}_Y^{(\Phi)} = \bar{l}_L \left(Y_e \Phi + \tilde{Y}_e \tilde{\Phi} \right) l_R + h.c.$$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$$

$$\langle \tilde{\Phi} \rangle = \begin{pmatrix} v_2 e^{-i\alpha} & 0 \\ 0 & v_1 \end{pmatrix} \quad \boxed{\text{check}}$$

$$\mathcal{L}_Y^{(\Phi)} \Rightarrow \left(\bar{\nu}_L \ \bar{e}_L \right) \begin{pmatrix} Y_e v_1 + \tilde{Y}_e v_2 e^{-i\alpha} & 0 \\ 0 & Y_e v_2 e^{i\alpha} + v_1 \tilde{Y}_e \end{pmatrix} \times \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$= \bar{e}_L \, m_e \, e_R \quad +$$

$$+ \bar{\nu}_L \, m_D^{+} \nu_R \qquad + h.c.$$

$$\boxed{m_e = y_e v_2 e^{i\alpha} + \tilde{y}_e v_1}$$

$$\boxed{m_D^{+} = y_e v_1 + \tilde{y}_e v_2 e^{-i\alpha}}$$

<u>notation</u>

$$\boxed{\begin{array}{c} \bar{\nu}_R \, m_D \, \nu_L = \\ = \text{Dirac mass term} \\ \text{for neutrino} \end{array}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \mathcal{L}_{mass}^{(\nu)} &= \overline{\nu_R}\, m_D\, \nu_L + h.c. \\ &\quad + M_{\nu_R}\, \nu_R^T C \nu_R + h.c. \end{aligned}}$$

$$\left(\begin{aligned} M_{\nu_R} &= Y_\Delta\, v_R \\ m_D &= f\,(Y_e\, v_1, v_2, \tilde{Y}_e) \end{aligned} \right)$$

$$f_L \longrightarrow C\, \overline{f_L}^T = (f^c)_R$$

$$f_R \longrightarrow C\, \overline{f_R}^T = (f^c)_L$$

$$\nu_R \longrightarrow C\bar{\nu}_R{}^T = (N)_L$$

heavy neutral lepton

$$N_L = C(\nu_R{}^+\gamma^0)^T = C\gamma_0\nu_R{}^*$$

$$N_L = i\gamma_2\gamma_0\gamma_0\nu_R{}^* = i\gamma_2\nu_R{}^*$$

$$= \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ u_R{}^* \end{pmatrix}$$

$$= \begin{pmatrix} i\sigma_2 u_R{}^* \\ 0 \end{pmatrix} \leftarrow LH$$

• Majorana !

$$M_{\nu_R} \, \nu_R^T C \, \nu_R + M^*_{\nu_R} \, \nu_R^+ C^+ \, \nu_R^*$$

but:

$$N_L^T C N_L = \bar{\nu}_R C^T C \, C \, \bar{\nu}_R^T$$

$$= \nu_R^+ \underbrace{\gamma_0 \, C^T C \, C \, \gamma_0}_{C^+} \nu_R^*$$

Show !

$$\bullet \; \bar{\nu}_R \, m_D \, \nu_L = N_L^T C \, \nu_L$$

Show !