Neutrino Physics Course

Lecture X

26/5/2023

LHU

Spring 2023

Spontaneous $\cancel{P}$:

Left-Right Symmetric Theory

- $G_{SM} = SU(2)_L \times U(1)_Y$ ($\cancel{P}$ max)

- $q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ $\qquad u_R, d_R$

  $\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $\qquad e_R$

- $\Phi \to U \Phi$, $Y(\Phi) = 1$

$$\Downarrow$$

- $m_p \propto \Phi_0$

- $h \, m_p \, p p \, (p\bar{p})$

$$\boxed{\Gamma(h \to ff) \propto m_f^2 \;!}$$

- **P good**

$$\equiv \; SU(2)_V$$

$$G_{SM} = SU(2)_{L+R} \times U(1)$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R = q_R$$

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R = \ell_R$$

$$J_\mu^W \;\cancel{\propto}\; \gamma_\mu (L+R) = \gamma_\mu$$

$$\parallel$$

$V \leftarrow$ vector

$$\gamma_\mu \gamma_5 = A_\alpha \quad \text{axial vector}$$

$$V : \quad \bar{\psi} \gamma^\mu \psi = V^\mu \quad (4\text{-vector})$$

$$A : \quad \bar{\psi} \gamma^\mu \gamma^5 \psi = A^\mu \quad (4\text{- axial -"-})$$

$$\boxed{Y(T) = 0} \Updownarrow \text{quarks} \quad \boxed{\mathrm{Tr}\, T = 0, \; T^\dagger = T \\ T \to U T U^\dagger}$$

$$\mathcal{L}_Y = \bar{q}_L (M + y_T T) q_R + h.c.$$

$$T_0 = \mathrm{diag}(v_T, -v_T)$$

$$\left.\begin{aligned} \underline{M}_u &= M + y_T v \\ M_d &= M - y_T v \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} 2 y_T v &= \\ &= \underline{M}_u - M_d \end{aligned}}$$

mass matrices

$$\underline{M}_u = V_{uL} \, \hat{m}_u \, V_{uR}^{+}$$

$$\underline{M}_d = V_{dL} \, \hat{m}_d \, V_{dR}^{+}$$

$$\hat{m}_d = \mathrm{diag}(m_d, m_s, m_b)$$

$$\hat{m}_u = \mathrm{diag}(m_u, m_c, m_t)$$

$$\mathcal{L}_{Wu} = \frac{g}{\sqrt{2}}\, \bar{u}_L \gamma^\mu d_L \; W_\mu^+ + h.c.$$

$$= \frac{g}{\sqrt{2}} \left(\bar{u}_L \;\; \bar{c}_L \;\; \bar{t}_L \right) \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + h.c.$$

$$\to \frac{g}{\sqrt{2}} \left(\bar{u}_L \;\; \bar{c}_L \;\; \bar{t}_L \right) \underbrace{V_{u_L}^+ \, V_{d_L}}_{= V_{CKM}} \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + h.c.$$

?

<u>flavors</u>

V_{CKM}

<u>digression</u> $\quad H = H^{+} \; (H)$

$$H \to U \, h \, U^{+}$$

$$h = \underbrace{\mathrm{diag}\,(h_1, h_2 \, \ldots\ )}_{\text{eigenvalues}}$$

- $T_0 = \mathrm{diag}\,(v_T, -v_T)$

$$\Downarrow$$

$$\underset{L+R}{SU(2)} \xrightarrow[T_0]{\qquad} \underset{3}{U(1)} = SO(2)$$

$\uparrow$ 3rd direction

$$SU(2) \times U(1) \xrightarrow[T_0]{} U(1) \times U(1)$$

$$\Downarrow$$

$$\boxed{M_A = M_Z = 0}$$

$$\Downarrow$$

nur Higgs!

$$\Downarrow \qquad \Phi_0^W = \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

$$\Phi_W \longrightarrow U\,\Phi_W\,, \quad Y(\Phi_W) = 1$$

$$\Downarrow$$

$$)\; M_W^2 = \left(\frac{g}{2}\right)^2 v_D^2 + g^2 v_T^2 \;($$

$$\left(M_Z^2 = \left(\frac{g}{2}\right)^2 v_D^2 + 0 \right)$$

$$\Downarrow$$

- $\boxed{\text{NO } W-Z \text{ mass relation}}$

- $$\Phi_{cw} = \begin{pmatrix} 0 \\ v_D + h_D \end{pmatrix}$$

$$T_{cw} = \begin{pmatrix} v_T + h_T & \cdots \\ \cdots & -(v_T + h_T) \end{pmatrix}$$

$$\Downarrow$$

2 Higgs bosons : h_D, h_T

$$h = \frac{v_D h_D + v_T h_T}{\sqrt{v_D^2 + v_T^2}}$$

$$h' = \frac{v_D h_T - v_T h_D}{\sqrt{v_D^2 + v_T^2}}$$

$$\Downarrow$$

$$\langle h \rangle = \sqrt{v_D^2 + v_T^2} = v^2 \quad \text{(Higgs)}$$

$$\langle h' \rangle = 0$$

NOT Higgs

$\mathcal{L}_Y \Rightarrow y_T \, h_T \quad (u, d)$ quarks

$$y_T \propto M_u - M_d$$

$$h_T = f(h, h')$$

$\Uparrow$ CURSE

$\boxed{\text{Forget it !}}$

but :

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$\Downarrow$

$$\boxed{m_\nu \neq 0}$$

blessing

$\Downarrow$ $\boxed{\not{P} \text{ spont.}}$

$$G_{LR} \supseteq G_{SM}$$

$$(1) \quad G_{LR} \stackrel{?}{=} SU(2)_L \times SU(2)_R \quad (\times P)$$

see below $\Downarrow$

$$Q_{em} = T_{3L} + T_{3R}$$

f: $\begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R$

$\Downarrow \qquad\qquad \Downarrow$

$T_{3L} = \sigma_3/2 \qquad T_{3R} = \sigma_3/2$

$\boxed{\text{WRONG !}}$

$\Downarrow$

(ii)

$$\boxed{G_{LR} = SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$$

$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \Downarrow \qquad \begin{pmatrix} u \\ d \end{pmatrix}_R = q_R$

$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad\qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R = l_R$

$$Q_{em} = T_{3L} + T_{3R} + \frac{Y'}{2}$$

$$Y' = ?$$

$$\Uparrow$$

$$B(q) = 1/3, \quad B(\ell) = 0$$

$$L(q) = 0, \quad L(\ell) = 1$$

$$Y(q) = \frac{1}{3} = B(q)$$

$$Y(\ell) = -1 = -L(\ell)$$

$$\Downarrow \; Y' = Y(L)$$

$$Y' = \frac{B-L}{2}$$

$$Y'(q_L) = Y'(q_R) = \frac{1}{3} = \frac{B-L}{3}$$

$$Y'(l_L) = Y'(l_R) = -1 = \frac{B-L}{3}$$

$$\Downarrow$$

gauge

$$\boxed{Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}}$$

SM: B, L = accidental global symmetries

$$\left.\begin{array}{l}(Z,\ W)\ \bar{q}q\ ,\ \bar{\ell}\ell \\ h\ \ \bar{q}q,\ \bar{\ell}\ell\end{array}\right\}\ \Delta B = \Delta L = 0$$

$\cdot\ p \to e^+ + \pi^0$

(f) (f)

SM

$\cdot\ p \to e^+ + A$

$\Downarrow$

$$\boxed{B - L = \text{anomaly free}}$$

$\Downarrow$

- $G_{LR} = SU(2)_L \times SU(2)_R \times U_{B-L}(1)$, with $SU(2)_L \xleftrightarrow{P} SU(2)_R$

$$\equiv \underbrace{\underbrace{SU(2)_L}_{g} \times \underbrace{SU(2)_R}_{g} \times \underbrace{U_{B-L}(1)}_{\bar{g}} \times P}_{\text{2 gauge couplings}}$$

$$g \longrightarrow g$$

- $q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}_R = q_R$

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longleftrightarrow \begin{pmatrix} \nu \\ e \end{pmatrix}_R = \ell_R$$

$$\Downarrow$$

$$\boxed{m_\nu \neq 0}$$

- Higgs sector

general feature:

$$G_{new} \xrightarrow[M_{new}]{} G_{SM} \xrightarrow[M_W]{} U(1)_{em}$$

$$M_{new} \gg M_W$$

$$\Downarrow$$

$$\boxed{\exists\ \Phi_{new} \text{ on top of } \Phi_{SM}}$$

$$\Downarrow$$

$\subseteq$ repr. of G_{new}

- $G_{LR} \longrightarrow G_{SM}$

  $\Phi_{new} \leftarrow$?

- $G_{SM} \longrightarrow U(1)$

$$\Phi_{SM} \subseteq \Phi_{LR}(SM)$$

repr. of G_R

contains SM Φ_{SM}

$$\mathcal{L}_y = \bar{q}_L Y_{LR} \Phi_{LR} q_R + h.c.$$

$$q_L \longrightarrow U_L q_L$$

$$q_R \longrightarrow U_R q_R$$

$$\mathcal{L}_y \rightarrow \bar{q}_L\, U_L^{+}\, \Phi'_{LR}\, U_R\, Y_{LR}\, q_R + h.c.$$

$$\Downarrow$$

$$U_L^{+}\, \Phi'_{LR}\, U_R = \Phi_{LR}$$

$$\Downarrow$$

$$\boxed{\Phi'_{LR} = U_L\, \Phi_{LR}\, U_R^{+}} \leftarrow$$

$$\boxed{(B-L)\, \Phi_{LR} = 0} \leftarrow$$

$$\Uparrow$$

$$\bar{q}_L\, \Phi_{LR}\, q_R$$

$$\Rightarrow (B-L)\, \Phi_{LR} = (B-L)_{q_L} - (B-L)_{q_R}$$

$\bullet \quad \Phi \longrightarrow \underbrace{U_L \, \Phi \, U_R^{\dagger}}$

bi - doublet

$\Updownarrow$

2 $SU(2)_L$ doublets

$\Updownarrow$

$$\Phi = \downarrow \overset{\xleftarrow{SU(2)_R}}{\Big(\underbrace{\phi_1 , \quad \phi_2}\Big)} \downarrow SU(2)_L$$

2 LH doublets

SM: $\quad Q_{em} = T_{3L} + Y/2$

1

$$\frac{Y}{2} = T_{3R} + \frac{B-L}{2}$$

$$Y(\phi) = +1 \Rightarrow \boxed{\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}}$$

$$Q_{em} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

LR

$$\Phi \to U_L \, \Phi \, U_R^+$$

$$= \left(1 + i\,\theta_{iL} T_{iL} + \dots\right) \Phi \left(1 - i\,\theta_{iR} T_{iR} + \dots\right)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$Q_{em}\,\Phi = T_{3L}\,\Phi - \Phi\,T_{3R}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\left(\begin{bmatrix} 1\cdot a & 1\cdot b \\ -1\cdot c & -1\cdot d \end{bmatrix} - \begin{bmatrix} 1\cdot a & -1\cdot b \\ c\cdot 1 & -1\cdot d \end{bmatrix}\right)$$

$$= \begin{pmatrix} 0 & +1\cdot b \\ -1\cdot c & 0 \end{pmatrix}.$$

$$\Downarrow$$

$$Q_{em}(a) = 0 \qquad Q_{em}(b) = +1$$

$$Q_{em}(d) = 0 \qquad Q_{em}(c) = -1$$

$$\Downarrow$$

$$\Phi = \begin{pmatrix} \varphi_1^0 & \varphi_2^+ \\ \varphi_1^- & \varphi_2^0 \end{pmatrix}$$

$$\varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}$$

anti - doublet

- $\Phi \longrightarrow U \Phi$ (doublet)

$\Phi^* \longrightarrow U^* \Phi$ (not -"-)

$$\Updownarrow$$

$$\left(\begin{array}{c} \phi_1^T i\sigma_2 \phi_2 = \text{inv.} \\ \phi_i \to U \phi_i \end{array} \right)$$

$$\boxed{\begin{array}{c} \varphi^c = \tilde{\varphi} = i\sigma_2 \varphi^* \ (\text{dublet}) \\ \Uparrow \\ \varphi^c \to U \varphi^c \quad (\tilde{\varphi} \to U \tilde{\varphi}) \end{array}}$$

<u>Verify</u> $\qquad U^* = e^{-i \sigma_i^*/2\, \theta_i}$

$$\tilde{\varphi} \to i\sigma_2 U^* \varphi^* = U i\sigma_2 \varphi^*$$

$$= U \tilde{\varphi} \checkmark$$

$$\tilde{\varphi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \varphi^- \\ \varphi_0^* \end{pmatrix}$$

$$= \begin{pmatrix} \varphi_0^* \\ -\varphi_1^- \end{pmatrix}$$

$$\Downarrow$$

$$\Phi = \begin{pmatrix} \varphi_{10}^* & \varphi_2^+ \\ -\varphi_1^- & \varphi_2^0 \end{pmatrix}$$

$$\Updownarrow$$

$$\boxed{\Phi = \left(\tilde{\varphi}_1 , \ \varphi_2 \right)}$$