Neutrino Physics Course

Lecture VI

12/5/2023

LMU

Spring 2023

SM Higgs mechanism :

the Weinberg model

$$G_{SM} = SU(2)_L \overset{g}{} \times U_Y(1) \overset{g'}{}$$

Glashow '61

$$\Downarrow$$

$$\exists\, A \longleftrightarrow Q_{em} = Q_A \ (e)$$

$$\Downarrow$$

$$\exists\, Z \longleftrightarrow Q_Z = T_3 - \sin^2\theta_W Q_{em}$$

$$\left(\frac{g}{\cos\theta_W} \right)$$

$$e = \sin\theta_W\, g$$

$$\tan\theta_W = g'/g$$

$\Downarrow$ renormalisability

$$\boxed{\text{does not work !}}$$

· $M_A \neq 0$

$\Downarrow$

$$\Delta_{\mu\nu}(A) = \frac{-i}{q^2 - M_A^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_A^2}\right]$$

BAD

(divergencies)

Renormalisability

$$\|$$

finite amplitudes

$$\Downarrow$$

① $\underset{\text{propagator}}{D(p)} \underset{p\to\infty}{\sim} \dfrac{1}{p^n}$ $\quad \begin{pmatrix} n = 1 - f \\ n = 2 - b \end{pmatrix}$

② $d(\mathcal{L}_{int}) \leq 4$

Example:

U(1) gauge theory

Goldstone '61

$$\mathcal{L} = \frac{1}{2}(D_\mu \phi)^*(D^\mu \phi) - V(\phi)$$

$$\phi \to e^{i\alpha Q}\phi = e^{i\alpha}\phi$$

$(\alpha = \alpha(x))$

$$D_\mu = \partial_\mu - igQA_\mu$$

(source of mass)

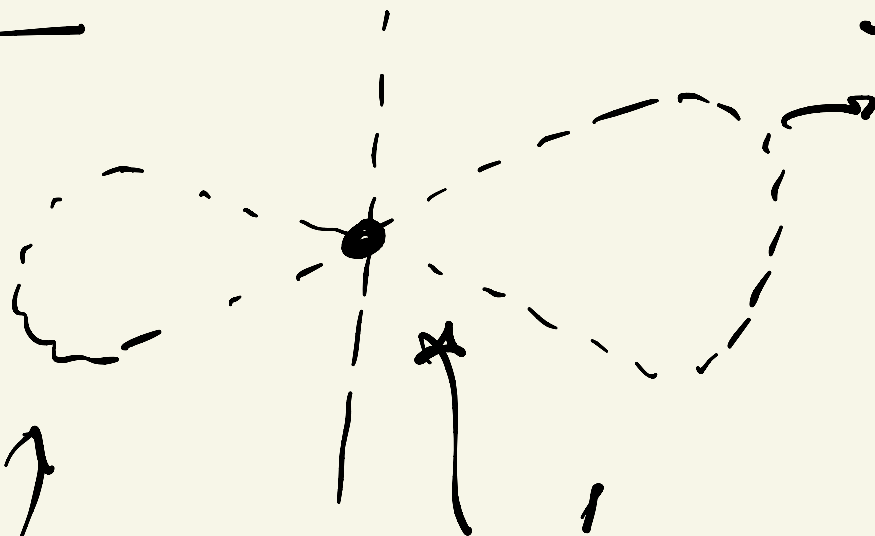
$$\boxed{V = \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2} \quad (1)$$

$$+ \frac{(\phi^*\phi)^3}{M^2} \; (?)$$

Q. why not?

A. Lose renormalisability.

<u>Proof:</u>



$$\int^{\Lambda} d^4k \, \frac{1}{k^2} \propto \Lambda^2$$

$$\frac{1}{M^2}$$

$$\int^{\Lambda} d^4p \, \frac{1}{p^2} \propto \Lambda^2$$

$$\Downarrow$$

$$A(2) \propto \frac{\Lambda^4}{M^2}$$

Q.E.D.

$$\Downarrow$$

$$\mathcal{M}_0 = \{ \phi_0 \therefore V = V_{min} = 0 \}$$

$\parallel$ vacuum

$$= \{ |\phi_0|^2 = v^2 \} = S_1$$

(1) $\phi_0 = v$

(2) $\phi = v + h + i G$

$$\Rightarrow \boxed{\begin{aligned} m_h^2 &= 2 \lambda v^2 \\ m_G &= 0 \end{aligned}}$$

Nambu – Goldstone boson

$$V(h, \sigma) = \cdots + \frac{\lambda}{4}\left(h^4 + \sigma^4\right)$$

$$+ \lambda v h^3$$

no symmetry

$$\Downarrow$$

$$\boxed{SSB}$$

$$\Phi_0 : \quad Q\,\Phi_0 \neq 0$$

$$(Q\,\phi = \phi) \qquad \Uparrow$$

$$\boxed{U\Phi_0 \neq \Phi_0}$$

$$\boxed{v = \text{measure of SSB}}$$

$m_h \propto v$!

$\Uparrow$

$$\boxed{\text{true } \cancel{\text{even}} \text{ if } \alpha \neq \alpha(x)}$$

(global symmetry)

$\Downarrow$ $\alpha = \alpha(x)$

$\Downarrow$

gauge symmetry

$$\mathcal{L}_{kin} = \frac{1}{2}|D_\mu \phi|^2$$

$$\rightarrow \frac{1}{2}|D_\mu \phi_0|^2 = \frac{1}{2}|-ig A_\mu Q \phi_0|^2$$

$$(Q\phi_0 \neq 0)$$

$$= \frac{1}{2}|-ig A_\mu v|^2$$

$$= \frac{1}{2} g^2 v^2 A_\mu A^\mu$$

$$\boxed{\text{Higgs mechanism}}$$

$$\Downarrow$$

$$\boxed{M_A = g v}$$

- global case: $\underset{(2)}{A} + \underset{(2)}{\phi \in \mathbb{C}} \Rightarrow 4$ d.o.f.

  $(m_A = 0)$

  (h, G)

- local case: $\underset{(2)}{A} + \underset{(2)}{\phi \in \mathbb{C}} \Rightarrow 4$ d.o.f.

  $m_A = 0$ (initially)

final (SSB): $\underset{(3)}{A\,(m_A \neq 0)}$

$+ \; \phi \, (2)$

(h, G)

5 d.o.f.

Unitary gauge

$$\phi = v + h + iG \qquad (2)$$

$\updownarrow$ why not?

$$\Rightarrow \phi = e^{iG/v}(v+h) \qquad (3)$$

$$= v + h + iG + \cdots$$

$$(3) \Longrightarrow \quad \phi \to e^{i\alpha(x)} \phi$$

$$= e^{i\underbrace{\left(\alpha(x) + \frac{G(x)}{v}\right)}_{0}} (v + h(x))$$

!! choice of $\alpha(x)$

$$\Longrightarrow \boxed{\phi \to \phi_{un} = v + h}$$

I gauged 6 away!

$\Downarrow$

massless

<u>in.</u> $A\,(2),\ \phi\,(2) \Longrightarrow 4$

<u>f.</u> $A\,(3),\ h \Longrightarrow 4$

d.o.f.

massive

· Trouble

find theory !

$$h \longleftrightarrow m_h = v\sqrt{2\lambda}$$

$$A \longleftrightarrow M_A = g v$$

(Proca)

$\Updownarrow$

(no gauge symmetry)

$\Downarrow$

$$\boxed{\Delta^{(P)}_{\mu\nu} \text{ is bad}}$$

A_μ (4 d.o.f.) ; $\partial_\mu A^\mu = 0$

$\Rightarrow$ 3 physical

Unitary gauge

only physical degrees of freedom

$$\boxed{\text{Not QED}}$$

$\Downarrow$

A_μ (4 d.o.f.)

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\mathcal{L} + \mathcal{L}_{g.f.} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

gauge parameter

QED : A_μ + gauge

$$4 - 2 = 2$$

d.o.f. physical

SSB QED : (A_μ, S) + gauge

$$5 - 2 = 3$$

d.o.f. physical

$$\phi = v + h + iG \Leftarrow e^{iG/v}(v+h)$$

not physical

$$D_\mu \phi = i\partial_\mu G - i g A_\mu v$$

$$\frac{1}{2}|D_\mu \phi|^2 = \frac{1}{2}\left(\partial_\mu G - g A_\mu v\right)^2$$

$$= \frac{1}{2}(\partial_\mu G)^2 \checkmark + \frac{1}{2} g^2 v^2 A_\mu A^\mu \checkmark$$

$$- g(\partial_\mu G) A_\mu v$$

// integrate by part

$$g v G \, \partial^\mu A_\mu$$

$$= \cancel{m_A G(\partial^\mu A_\mu)}$$ not good

$$\Downarrow$$

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi m_A G\right)^2$$

$$= -\frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{1}{2}\xi m_A^2 G^2$$

$$\cancel{- m_A G (\partial^\mu A_\mu)}$$

$$\Downarrow$$

$$\mathcal{L}_G = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{2} 3 m_A^2 G^2$$

$$\Longrightarrow \boxed{m_G = \sqrt{3}\, m_A}$$

$G \neq$ physical

(gauge dep. mass)

$$\Downarrow$$

$$D(q) = \frac{i}{q^2 - 3 m_A^2} \qquad (4)$$

$$\Delta_{\mu\nu}(A) = \frac{-i}{q^2 - m_A^2}\left[g_{\mu\nu} + (\xi - 1)\frac{q_\mu q_\nu}{q^2 - \xi m_A^2}\right] \quad (5)$$

$\Downarrow$

$$\boxed{\left.\begin{array}{l} D(G) \xrightarrow[q\to\infty]{} \dfrac{1}{q^2} \\[2em] \Delta_{\mu\nu}(A) \xrightarrow[q\to\infty]{} \dfrac{1}{q^2} \end{array}\right\} \text{ for any finite } \xi}$$

<u>unitary gauge</u> ?

$\Downarrow$

$$\xi \to \infty$$

$$\Downarrow$$

$$D(G) \to 0 \qquad (\text{no } G)$$

$$\Delta_{\mu\nu}(A) \to \frac{-i}{q^2 - m_A^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{m_A^2}\right]$$

Proca!

<u>Crux of it all:</u>

$$Q\,\Phi_0 \neq 0 \qquad Q\,\phi_0 = q\,\phi_0$$

$$\Downarrow$$

$$\boxed{|m_A| = |g\,v| = g\,q\,|\phi_0|}$$

global case $\qquad Q\phi_0 \neq 0$

$$\phi = v + h + iG$$

$$\Downarrow$$

$$\boxed{m_G = 0}$$ NG boson

local case $\qquad Q\phi_0 \neq 0$

$$\Rightarrow \quad m_A \neq 0$$

but NG is "eaten" by A

$$\boxed{\text{but we cannot do loops without } G}$$

$$\Downarrow$$

$\phi \to e^{i\alpha(x)}\phi$ $\therefore$ $\phi_{un} = (v+h)$

(G = gauged away)

$\Updownarrow$

$m_G \to \infty$ in $\phi_{ren} = v + h + iG$

(renormalisable)

$\boxed{\text{Choosing variables}}$

· global case α = const.

$$\phi_{un} = e^{iG/v} (v+h)$$

$$\Rightarrow \quad |\phi_{un}|^2 = (v+h)^2$$

$$\Rightarrow \quad m_G = 0 \quad (G \text{ \underline{not} in } V)$$

$$\partial_\mu \phi_{un} = \partial_\mu h \, e^{iG/v} +$$

$$(v+h) \partial_\mu iG/v \, e^{iG/v}$$

$$= \Big(\partial_\mu h + i \partial_\mu G \, (1 + h/v)\Big) e^{iG/v}$$

$\uparrow$

<u>non-trivial int.</u>