Neutrino Physics Course

Lecture V

SSB : Higgs mechanism

LMU

Spring 2023

1. $G_{SM} = \overset{g}{SU(2)_L} \times \overset{g'}{U_Y(1)}$

$$Q_{em} = T_3 + \frac{Y}{2}$$

2. matter

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, \; d_R$$

$$l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$$

$$\Downarrow$$

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L \right) + h.c.$$

$$+ e A_\mu \bar{f} \gamma^\mu Q_{em} f \quad (Q_{em} f = q_f f)$$

$$+ \frac{g}{\cos\theta_W} Z_\mu \bar{f} \gamma^\mu (T_3 - Q_{em} \sin^2\theta_W) f$$

(only parameter)

$$\boxed{e = g \sin\theta_W}$$

$M_W \simeq 80$ GeV, $M_Z \simeq 90$ GeV

$\theta_W \simeq 30^\circ$ $(\sin^2\theta_W = 0.25)$

$\tan\theta_W = g'/g$

However

if gauge symmetry good

$$\Downarrow$$

(a) $M_W = M_Z = 0$

(b) $m_f = 0$

why? $m_f \bar{f} f = m_f (\bar{f}_L f_R + h.c.)$

doublet

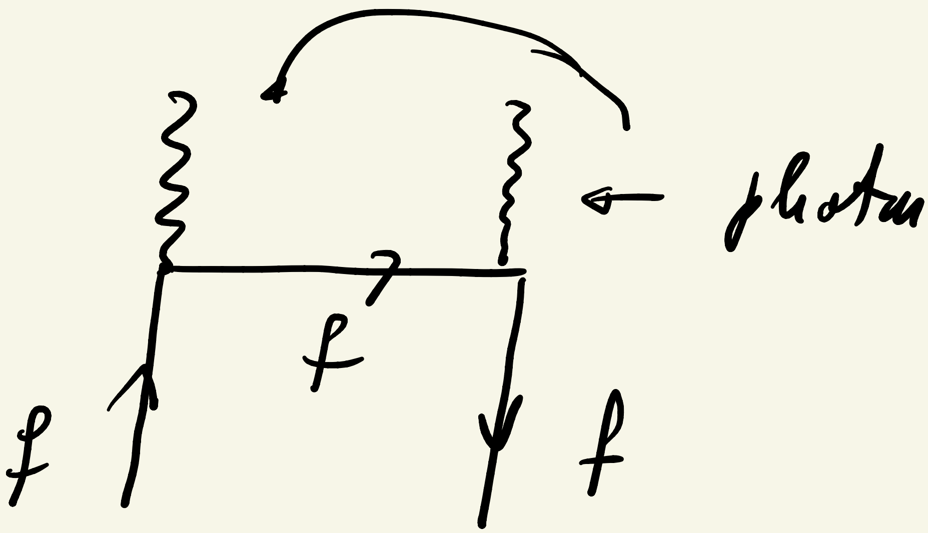
singlet

e.g. $\bar{q}_L \, d_R \longrightarrow \bar{q}_L U^\dagger \mathbb{1} \, d_R \neq \text{inv.}$

$$E \to \infty \Rightarrow {}^{m_f}/_{E} \to 0$$

of high E $\Rightarrow$ particle

masses are negligible!



$$\not{p} = p^\mu \gamma_\mu$$

$$S_F = \frac{i}{\not{p} - m} \xrightarrow[p \to \infty]{} 0$$

$$p \gg m \implies \frac{1}{\not{p} - m} \simeq \frac{1}{\not{p}}$$

$$= \frac{1}{\not{p}(1 - m/\not{p})} = \frac{1}{\not{p}}\left(1 + \frac{m}{\not{p}} + \dots\right)$$

What about gauge bosons?

- $m_A \neq 0 \Leftarrow$ Proca theory

$$\mathcal{L}_P = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha$$

m_A breaks this symmetry

$$\mathcal{L}_p = -\frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu +$$

$$\boxed{\Box \equiv \partial_\mu \partial^\mu}$$

$$+\frac{1}{2} m_A{}^2 A_\mu A^\mu$$

$$= -\frac{1}{2}(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\nu A_\mu \partial^\mu A^\nu) +$$

$$= \text{total derivative} + \frac{1}{2}(A_\nu \Box A^\nu - A_\mu \partial^\mu \partial^\nu A_\nu)$$

$$+$$

$$= \frac{1}{2} A_\mu \left[g^{\mu\nu}(\Box + m^2) A_\nu - \partial^\mu \partial^\nu\right] A_\nu$$

$$\Downarrow$$

$$\cancel{\partial_\mu}\ \boxed{\left[g^{\mu\nu}(\Box + m^2) - \partial^\mu \partial^\nu\right] A_\nu = 0}$$

$$\left[\cancel{\partial^\nu(\Box} + m^2) - \cancel{\Box\,\partial^\nu}\right] A_\nu = 0$$

$$\Rightarrow \quad m^2\,\partial^\nu A_\nu = 0$$

$$\Downarrow$$

$$\boxed{\partial^\nu A_\nu = 0}$$

A_μ = Proca field (= 4 comp)

but

3 physical degrees : $\vec{S}_A$

$$\mathcal{L}_p = \frac{1}{2} A_\mu \left[g^{\mu\nu} (\Box + m^2) - \partial^\mu \partial^\nu \right] A_\nu$$

$$A_\mu = e^{ipx} \varepsilon_\mu(p)$$

$$\mathcal{L}_p \xrightarrow{(p)} \frac{1}{2} \varepsilon_\mu \left[g^{\mu\nu} (-p^2 + m^2) + p^\mu p^\nu \right] A_\nu$$

$$\Downarrow$$

propagator = inverse of

$$\Delta_{\mu\nu} = A g_{\mu\nu} + B \frac{p_\mu p_\nu}{p^2}$$

$$\Downarrow$$

$$\boxed{\Delta^{(P)}_{\mu\nu} = \frac{-i}{p^2 - m_A^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}\right]}$$

$$m_A = 0 \Rightarrow \Delta^{(M)}_{\mu\nu} = \text{gauge dependent}$$

$$\Rightarrow \frac{-i}{p^2} g^{\mu\nu} \quad (\text{rel. gauge})$$

$$\boxed{\Delta^{(M)}_{\mu\nu} \xrightarrow[p\to\infty]{} 0}$$

$$\boxed{\begin{array}{c}\text{QED} \\ \text{renormalizable}\end{array}}$$

$$\Delta^{\mu\nu}_{(p)} \xrightarrow[p\to\infty]{} \propto \frac{p^\mu p^\nu}{p^2}\frac{1}{m_A^2}$$

$$\propto \frac{\cancel{p^2} g^{\mu\nu}}{\cancel{p^2}}\frac{1}{m_A^2}$$

$$\xrightarrow[p\to\infty]{} \frac{1}{m_A^2}$$

PROBLEM

$\Downarrow$

Higgs mechanism

a must!

$$\Delta_{\mu\nu} \propto \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{m_A^2}}{p^2 - m_A^2}$$

$$\xrightarrow[m_A \to 0]{} \frac{g_{\mu\nu}}{p^2} - \frac{p_\mu p_\nu}{p^2} \frac{1}{m_A^2}$$

ok

$\boxed{\text{bad}}$

$$\xrightarrow[m_A \to 0]{} \infty$$

$\boxed{\text{Why singularity?}}$

- $m_A \neq 0 \Longrightarrow 3$ d.o.f. $(\vec{S})$

$(+1, -1, 0)$

- $m_A = 0 \Longrightarrow 2$ d.o.f. $h = \vec{S} \cdot \hat{p}$

(± 1)

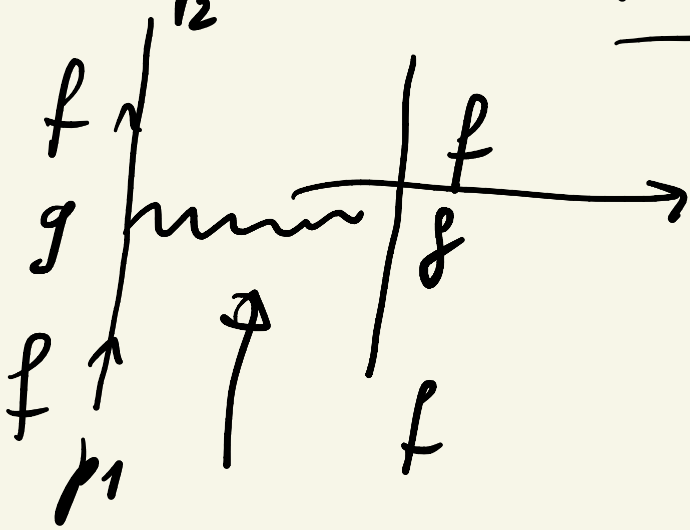
$\Downarrow$

jump in d.o.f.

$3 \longrightarrow 2$

Perturbation theory in a nutshell (renormalizability)

f - f scattering



$p = p_2 - p_1$

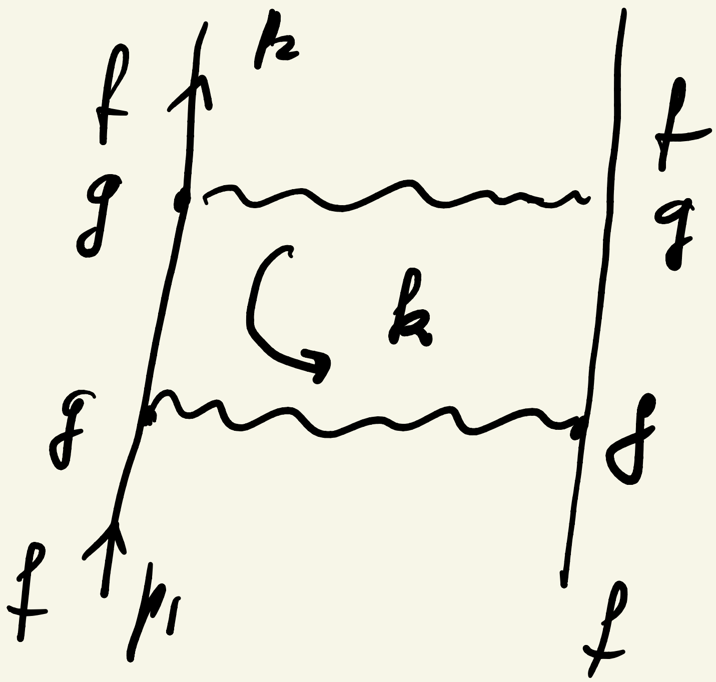
gauge boson propagator $\Delta_{\mu\nu}$

$$A \propto g^2\, \bar{f} \partial^\mu f \;\; \bar{f} \partial^\nu f \;\; \Delta_{\mu\nu}$$

$$\alpha = \frac{g^2}{4\pi} \ll 1$$

$$\Downarrow$$

loops



$$\mathcal{A} \propto g^4 \int_0^{\Lambda} d^4k \; \frac{1}{\Delta_A}\,\frac{1}{\Delta_A}\,\frac{1}{S_F}\,\frac{1}{S_F} \cdots$$

$$(\Lambda \to \infty)$$

QED

$$\boxed{\not k \not k = k^2}$$

$$A^{\mu} \propto \int_{-\pi^2}^{\Lambda} \frac{k^2\, dk^2}{} \; \frac{1}{k^2} \; \frac{1}{k^2} \; \frac{1}{\cancel{k}} \; \frac{1}{\cancel{k}}$$

$$\left(k \to \infty \quad (\Lambda \to \infty) \right)$$

$$\propto \int^{\Lambda} \frac{\cancel{k^2}\, dk^2}{\cancel{k^2}\, k^2\, k^2} = \frac{1}{\Lambda^2} +$$

(small err)

$$\left(A^{\mu}_{(\infty)} \xrightarrow[\Lambda \to \infty]{} 0 \right)$$

Proca

$$A_{\infty}^{(p)} \propto \int^{\Lambda} \frac{\cancel{k^2}\, dk^2}{\cdots \pi^2} \frac{1}{m_A^2} \frac{1}{m_A^2} \frac{1}{\cancel{k^2}}$$

$$\propto \frac{\Lambda^2}{m_A^4} \leftarrow \text{quadratic divergence}$$

$$\boxed{\text{UV catastrophy of Proca theory}}$$

$$\boxed{\begin{array}{l}\text{1960} \\ \text{Van Dam , Veltman}\end{array}}$$

- need a symmetry (compute)
- break –"– (exp)

$$\Downarrow$$

Spontaneous Symmetry Breaking

(SSB)

$$\Uparrow$$

Nambu '60

Goldstone '61

Brout, Englert '64

Higgs

Hidden Symmetry

example: gauge $U(1)$ symmetry

scalar field : $\phi \to e^{i\alpha(x)Q}\phi$

$$Qf = f \Rightarrow \phi \to e^{i\alpha(x)}\phi$$

$$\Downarrow$$

$$\mathcal{L} = \frac{1}{2} D_\mu \phi(x) D^\mu \phi(x) - V(\phi)$$

$$\therefore \boxed{d(V) \leq 4}$$

$$\Downarrow$$

$$V = \frac{1}{2} m^2 \phi^* \phi + \frac{\lambda}{4} (\phi^* \phi)^2 + \text{const.}$$

$$= \frac{\lambda}{4} \left(\phi^* \phi + v^2 \right)^2$$

dim. of mass

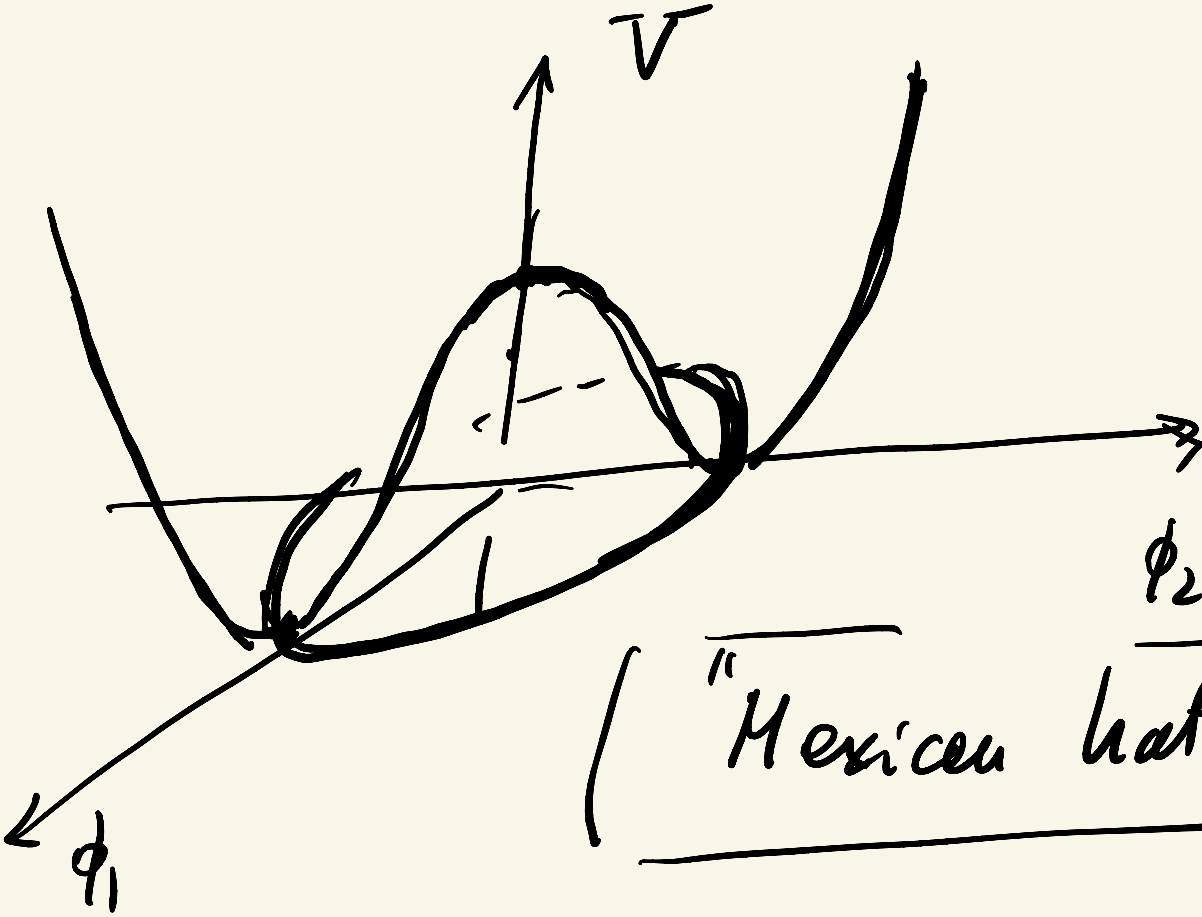
$$m^2 = \lambda v^2$$

Nambu - Goldstone

$$V_{NG} = \frac{\lambda}{4} \left(\phi^* \phi - v^2 \right)^2$$

$$m^2 = - \lambda v^2 \; ! ?$$

$$(\lambda > 0)$$



"Mexican hat"

$$\phi = \phi_1 + i\phi_2$$

<u>ground state</u> (vacuum manifold)

$$\mathcal{M}_0 = \{ \phi_0 : V = V_{min} = 0 \}$$

$$= \{ \phi_0 : |\phi_0|^2 = v^2 \}$$

$$\Downarrow$$

$$\boxed{M^0 = S_1}$$ circle

- $\phi_0^G = v$ choice

$$\boxed{\phi = \underbrace{v + h}_{\text{motion around } v} + iG}$$

$\Downarrow$

$$V = \frac{\lambda}{4}\left(|v + h + iG|^2 - v^2\right)^2$$

$$= \frac{\lambda}{4}\left[(v+h)^2 + \sigma^2 - v^2\right]^2$$

$$= \frac{\lambda}{4}\left[\cancel{v^2} + 2vh + h^2 + \sigma^2 - \cancel{v^2}\right]^2$$

$$= \frac{\lambda}{4}\left[h^4 + 4v^2h^2 + 4vh^3 + \right.$$

$$\left. + 2h^2\sigma^2 + \sigma^4 + 4vh\sigma^2 \ldots\right]$$

$$= \lambda v^2 h^2 + 0 \cdot \sigma^2 + \text{int.}$$

$$\Downarrow$$

$$\boxed{m_\sigma = 0}$$

Nambu – Goldstone boson

$$\boxed{m_h^2 = 2\lambda v^2} \quad (m_h^2 > 0)$$

$$V = \mu^2 \phi^2 + \lambda \phi^4 + \frac{\phi^6}{M^2}$$

$$\mathcal{L}_F = G_F \, J^\mu_W \, J_\mu^W = \frac{1}{\Lambda_F^2} J^\mu_W \, J_\mu^W$$

$\boxed{\text{non - renormalizable}}$