Neutrino Physics Course

Lecture IX

23/5/2023

LMU

Spring 2023

Higgs mechanism in imaginary P conserving world

SM in a nutshell

- $$G_{SM} = \overset{g}{SU(2)_L} \times \overset{g'}{\underset{Y}{U(1)}}$$

$$\downarrow \qquad \downarrow$$

$$Q = T_3 + \frac{Y}{2}$$

$\overset{\circ}{W} \circlearrowright \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R,\ d_R$

$W \circlearrowright \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R$

} matter

$$\Downarrow$$

$$f_L \to U f_L \qquad f_R \to U f_R$$

$$U = e^{i \frac{\sigma^a}{2} \theta_a} \qquad U = 1$$

- Higgs repr.

$$\to \quad \Phi \longrightarrow U \Phi \,, \quad Y(\Phi) = +1$$

$$\Downarrow \quad \Phi_{un} = \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$\uparrow$ Higgs boson

masses $v \longrightarrow v + h = v\left(1 + \frac{h}{v}\right)$

$$\Downarrow$$

$\boxed{\text{h couples to SM particles proportional to masses}}$

- $M_W = \frac{g}{2} v$

$$\longrightarrow \quad g M_W \, h \, W^+ W^-$$

- $M_Z = \frac{g}{2 \cos\theta_W} v = \frac{M_W}{\cos\theta_W}$

$$\longrightarrow \quad \frac{g}{2} \frac{M_Z}{\cos\theta_W} \, h \, Z_\mu Z^\mu$$

- $m_f = y_f v$

$$\longrightarrow \quad y_f\, h\, \bar{f} f = \frac{g}{2}\frac{m_f}{M_W}\, h\, \bar{f} f$$

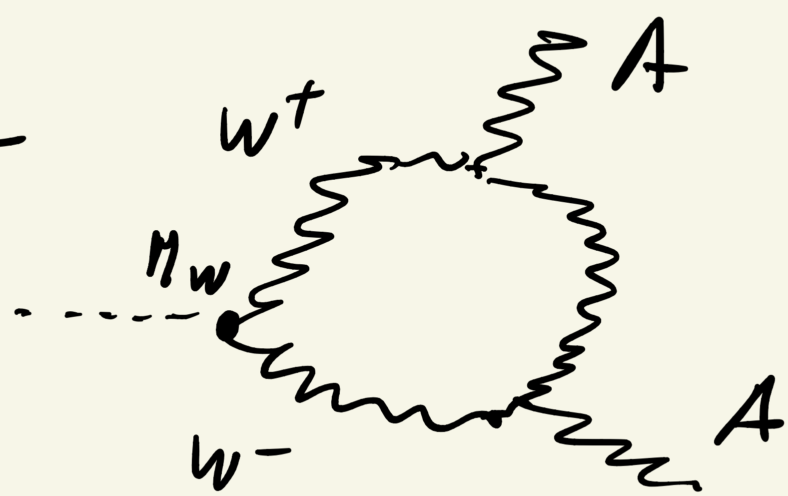
- $M_A = 0$

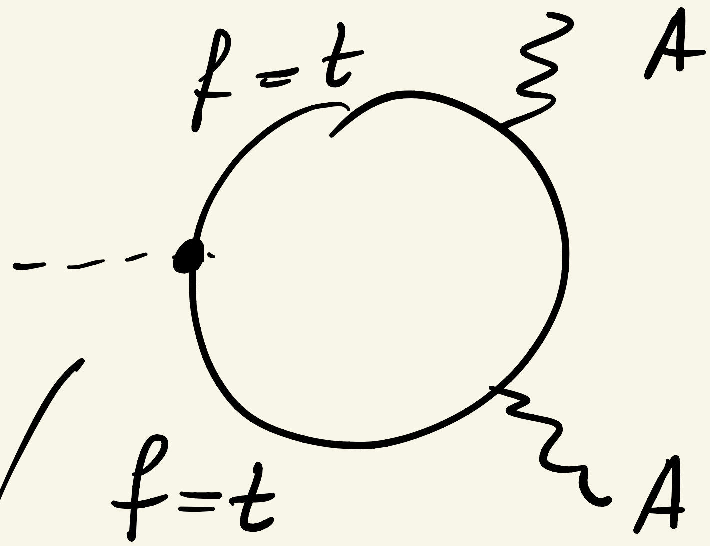
$$\longrightarrow \quad \cancel{0 \; h\, A_\mu A^\mu}$$

Higgs discovery 4/7/2012

$$h \longrightarrow A + A \quad (\gamma + \gamma)$$

$$\propto M_W\, h\, W^+ W^-$$





$m_t \simeq 175\,\mathrm{GeV}$

$m_b \simeq 5\,\mathrm{GeV}$

$m_f \longrightarrow$ top dominates

$$Br(h \to AA) = \frac{\Gamma(h \to AA)}{\Gamma_{total}} \simeq 10^{-3}$$

$$E_A \simeq \frac{m_h}{2}$$ rest frame

Why not $h \to b\bar{b}$?

$$\frac{g}{2}\,\frac{m_b}{M_W}\,h\,b\bar{b}$$

LHC = dirty machine

full of b quarks

$\Downarrow$

$\boxed{\text{NO good}}$

- $\boxed{m_\nu = 0}$

only incompleteness of SM

Imagine: 1956 Christmas

Wu et al: $\underline{P}$ is exact

$$\Downarrow$$

Construct "SM" ($\underline{P}$)

- $G_{SM(P)} = SU(2) \times U_Y(1)$

$$Q = T_3 + Y/2$$

$$\boxed{P = LR}$$

- matter

$$q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \xleftrightarrow{\underline{P}} \begin{pmatrix} u \\ d \end{pmatrix}_R \equiv q_R$$

$$l_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \overset{P}{\longleftrightarrow} \begin{pmatrix} \nu \\ e \end{pmatrix}_R \equiv l_R$$

- <u>Higgs sector</u>

quarks

$$\mathcal{L}_y^{(P)}(\text{mass}) = \bar{q}_L \, M \, q_R = \text{inv.}$$

$$\to \bar{q}_L U^\dagger U q_R \checkmark$$

$$m_f \bar{f} f = m_f (\bar{f}_L f_R + \bar{f}_R f_L)$$

$$q_L \to U q_L \,, \quad q_R \to U q_R$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\Rightarrow \quad \boxed{m_u = m_d} \qquad (m_d > m_u)$$

$u \; u$
d

p

$d \; d$
u

n

$$\boxed{m_n > m_p}$$

$$\Uparrow$$

$$n \to p + e + \bar{\nu}_e$$

$$\Downarrow$$

Higgs to break

$$m_u = m_d$$

$$q_L \xleftrightarrow{\mathcal{P}} q_R$$

$$\mathcal{L}_Y^{(P)} = \bar{q}_L M q_R + Y \bar{q}_L H q_R + h.c.$$

$$\Rightarrow \quad H \neq \Phi \quad (\text{doublet})$$

$$\bar{q}_L H q_R \to \bar{q}_L U^{+} H' U q_R = \text{inv.}$$

$$= \bar{q}_L H q_R$$

$$\Rightarrow \quad U^+ H' U = H$$

$$\Downarrow$$

$$\boxed{H' = U H U^+}$$

$$A_\mu = A_\mu^a T_a$$

adjoint rep.

$$A_\mu \to U A_\mu U^+ + \frac{i}{g} (\partial_\mu U) U^+$$

H = adjoint representation

How many components?

$H \sim A_\mu^a$ (global)

$\cdot\ H \to U H U^{+}$

(a) $H = H^{+} \Rightarrow$ presered

$\Rightarrow \boxed{H = \text{hermitian}}$

(b) $\operatorname{Tr} H \to \operatorname{Tr} U H U^{+} = \operatorname{Tr} H$

$\boxed{\operatorname{Tr} H = 0}$

$$\underline{M}_{2\times 2} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad a, b, c, d \in \mathbb{C}$$

$H = H^{+} \Rightarrow \boxed{\text{4 real componts}}$ ※

$$H = \begin{pmatrix} r_1 & z \\ z^* & r_2 \end{pmatrix} = H^{+}$$

$\Downarrow$

Adjoint $\Rightarrow$ 3 comp. $(\mathrm{Tr} = 0)$

$\Downarrow$

triplet T

$$T \longrightarrow U T U^{+} \qquad T = T^{+}, \quad \mathrm{Tr}\, T = 0$$

$\Downarrow$

$$\boxed{T = T_a \varphi_a} \qquad a = 1, 2, 3$$

$\uparrow$ $SU(2)$

$$U = e^{iH} \Leftarrow (UU^{+} = 1)$$

$$\mathrm{Tr}\, H = 0 \Leftarrow \det U = 1$$

$$\Updownarrow$$

$$H = T_a \theta_a \qquad T_a = \frac{\sigma_a}{2}$$

$$\Downarrow$$

$$\varphi_a T_a \to e^{iT_b\theta_b} \varphi_c T_c e^{-iT_b\theta_b}$$

$$= (1 + iT_b\theta_b) \; -''- \; (1 - iT_b\theta_b)$$

$$= \varphi_c T_c + i[T_b, T_c]\varphi_c \theta_b$$

$$\varphi_a T_a \to \varphi_a T_a + 1 \cdot i \varepsilon_{bca} \theta_b \varphi_c T_a$$

$$= T_a(\varphi_a + i \cdot i \varepsilon_{abc} \theta_b \varphi_c)$$

$$\boxed{\varphi_a \to \varphi_a - \varepsilon_{abc} \theta_b \varphi_c}$$

$\Uparrow$

vector

$$\boxed{\vec{V} \to \vec{V} - \vec{\theta} \times \vec{V}}$$

$\Updownarrow$

$$\boxed{(T_a)_{bc} = -i\,\varepsilon_{abc}}$$

$\Downarrow$

$$[T_a, T_b] = i\,\varepsilon_{abc} T_c$$

$\Downarrow$

$$T = T_a \varphi_a = \frac{\sigma_a}{2}\varphi_a = \begin{pmatrix} \varphi_3 & \varphi_1 - i\varphi_2 \\ \varphi_1 + i\varphi_2 & -\varphi_3 \end{pmatrix}\frac{1}{2}$$

$\Downarrow$ equivalence

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \longrightarrow O \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$O = e^{i L_a (T_a) \theta_a}$$

rotation

$$(L_a)_{bc} = -i\, \varepsilon_{abc}$$

ROT

$$\mathcal{L}_Y^{(D)} = \bar{q}_L M q_R + y_T\, \bar{q}_L T q_R$$

$$\Downarrow$$

$$V = V(T) \qquad \boxed{\begin{array}{c} T \to U T U^{\dagger} \\ Y(T=0) \end{array}}$$

$$\text{inv.} = \mathrm{Tr}\, T = 0 \quad (!)$$

$$\text{Inv.} = \mathrm{Tr}\, T^2 \neq 0 \qquad (2)$$

$$\text{Inv} = \mathrm{Tr}\, T^3 = ? \qquad (3)$$

$$\boxed{\text{check it}}$$

$$T \to U T U^\dagger \qquad (T = T^\dagger)$$

$$\xrightarrow[\text{basis}]{} \text{diagonal} = \begin{pmatrix} t_3 & 0 \\ 0 & -t_3 \end{pmatrix}$$

$$\Rightarrow \quad \mathrm{Tr}\, T = 0$$

$$\boxed{\mathrm{Tr}\, T^2 = 2 t_3^2} \quad \text{only inv.}$$

$$\mathrm{Tr}\, T^3 = 0$$

$$\mathrm{Tr}\, T^4 = 2 t_3^4 = \frac{1}{2} (\mathrm{Tr}\, T^2)^2$$

$\underline{SU(3)}$ $$T_3 = \frac{1}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

digression

$$T_8 = N \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$\boxed{\nu = 2}$ (# of diagonal gen.)

V : SSB

$\therefore$ $T_0 \neq 0$ $(\langle T \rangle)$

$$\boxed{T_0 \longrightarrow U T_0 U^{+} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}}$$

$\Uparrow$

$$V = \frac{\lambda}{4}\left(T_r\, T^2 - 2v^2 \right)^2$$

- Reminder:

$$\Phi \to U\,\Phi \quad (\text{doublet})$$

$$\Rightarrow \quad \Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \Rightarrow T_a\, \Phi_0 \neq 0$$

$$\Leftrightarrow$$

$$U\,\Phi_0 \neq \Phi_0$$

$$\boxed{SU(2) \xrightarrow[\Phi_0]{} \mathbb{1}}$$

$$SU(2) \xrightarrow[T_0]{} ???$$

$$T_0 = \mathrm{diag}(v, -v)$$

$$\Updownarrow$$

$$\phi_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \qquad \left\{ \begin{array}{l} \phi \to O\,\phi \\ O = 3\times 3 \text{ rot.} \end{array} \right.$$

SO(2) rotation symmetry

$\parallel$

U(1) $\qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to O_2 \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\Updownarrow$$

$$T_0 \to U T_0 U^\dagger = e^{i\sigma_3/2\,\theta_3}\, T_0\, e^{-}$$

$$= T_0$$

$$[\sigma_3, T_0] = 0$$

$$SU(2)_{L+R} \times U_Y(1)$$

$$\downarrow T_0$$

$$[\overline{U(1)_3} \times \overline{U_Y(1)}]$$

$$\downarrow \qquad \downarrow$$

$$T_3 \qquad Y/2$$

$$\downarrow U(1)_{em}$$

$$\boxed{Q = T_3 + Y/2}$$

$$\Downarrow$$

more Higgs needed

$$+ \Phi$$

$$\mathcal{L}_Y^{(P)} = \bar{Q}_L \left(M + y_T T \right) Q_R + h.c.$$

$$\Big\downarrow \text{ vacuum}$$

$$\bar{Q}_L \left(M + y_T T_0 \right) Q_R$$

$$\Downarrow$$

$$\boxed{\begin{aligned} m_u &= M + y_T v \\ m_d &= M - y_T v \end{aligned}}$$

$$\Downarrow$$

$\not{P}$ maximal $\longleftrightarrow$ needed for m_f

P maximal $\longleftrightarrow$ needed for m_d

$\Downarrow$ solution

$$\boxed{\text{SSB of } P}$$